OPTIMAL PIPELINE SIZING TECHNIQUE

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(Reviewed by the Pipeline Division)

ABSTRACT: In this paper, the writers present a formula for sizing steel pipelines of optimal diameter with equally spaced, similar pumping units. The general case of an inclined pipeline with fitting is considered in the analysis. The sizing formula is based on a friction-factor formula that spans through all the flow regimes of the Moody diagram. Although the formula is implicit, the final solution is attained after a limited number of trial cycles. A simple computer program carries out design calculations and provides the optimal pipeline size as well as spacing between pumping units. Solved numerical examples are provided to demonstrate the simplicity and practicability of the proposed technique.

INTRODUCTION

A number of factors have to be considered in sizing a pipeline of minimum cost. Among these factors are the initial investment cost of pipes and pumps, the annual operating and maintenance costs for the life of the pipes and pumps, and the salvage value of the pipeline (Albertson et al. 1960).

In general, a pipe with a large diameter produces a small friction head loss against which each pump should act; on the other hand, though a pipe with a smaller diameter is cheaper, it corresponds to a greater friction head loss (Russel 1963). Accordingly, the optimal pipeline size is that for which the total annual cost of pipe, pumps, and power is a minimum (Hathoot 1984; Cheremisinoff et al. 1988).

In an earlier treatment (Daugherty and Franzini 1977), the total annual cost of a pipeline (K_t) was given by

$$K_t = aD^2 + b/D^5 \tag{1}$$

where D = the pipe diameter in meters and a and b are constants. In fact, b is not a constant since it contains the coefficient of friction (f) that varies with both the Reynolds number and pipe roughness. Hathoot (1980) presented two pipeline design formulas, one for smooth pipe flow and the other for completely rough pipe flow, where the actual variation of f is taken into account. A horizontal pipeline was investigated by Hathoot (1984), who presented a single design formula that represents a significant portion of the Moody diagram. Later, Hathoot (1986) presented a number of pipeline design formulas that represent the Moody diagram and deal with an inclined pipeline (Fig. 1). The objective of this paper is to present a general steel pipeline-sizing technique that covers all fluid flow regimes. It is worthy to note that SI units are used throughout this paper.

Pipe Losses

Friction losses constitute the major portion of pipe losses. Between two successive pumping units, friction losses (h_f) are given by the Darcy-Weisbach equation (Streeter and Wylie 1983) as

$$h_f = fLV^2/2gD \tag{2}$$

where f = coefficient of friction; L = spacing between two successive pumping units in meters; V = average pipeline velocity in meters per second; and g = acceleration due to gravity in meters per square second. For convenience, (2) is put into this form

$$h_f = 8fLQ^2/\pi^2 gD^5 \tag{3}$$

where Q = pipeline discharge in cubic meters per second. Losses in pipe fittings (h_{fit}) are generally given by

$$h_{fit} = kV^2/2g \tag{4}$$

where k = sum of the coefficients representing head loss in fittings between two successive pumping units. In terms of the discharge, (4) is put into this form

$$h_{fit} = 8kQ^2/g\pi^2 D^4 \tag{5}$$

Pumping Energy

The power required per pumping unit (p) is given by

$$P = \gamma Q H_p/\eta \tag{6}$$

where γ = specific weight of liquid in Newtons per cubic meter; H_p = total head provided per pumping unit in meters; and η = pump efficiency. For an inclined pipeline, the total head is given by

$$H_p = h_f + h_{fit} \pm LS_0 \tag{7}$$

where S_0 = slope of pipeline. The positive sign corresponds to

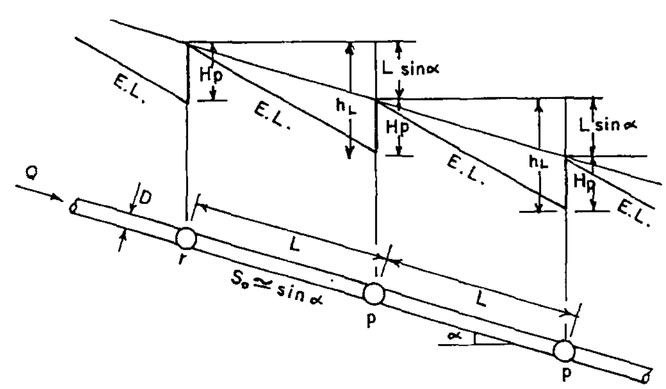

FIG. 1. Pipeline with Equally Spaced Pumping Units

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upward slopes, and the negative sign corresponds to negative slopes; therefore, the power required per pumping unit is given by

$$P = (\gamma Q/\eta)(h_f + h_{fit} \pm LS_0) \quad (8)$$

The power required per unit length of pipeline (w) may be written as

$$W = \frac{8f\gamma Q^3}{g\eta\pi^2D^5} + \frac{8k\gamma Q^3}{g\eta\pi^2LD^4} \pm \frac{\gamma QS_0}{\eta} \quad (9)$$

Cost of Energy

The levelized net annual cost of pumping energy per unit pipe length (K_{en}) (Cheremisinoff et al. 1988) is written as

$$K_{en} = \frac{8fC_2\gamma Q^3}{9\eta\pi^2D^5} + \frac{8kC_2\gamma Q^3}{9\eta\pi^2LD^4} \pm \frac{C_2\gamma QS_0}{\eta} \quad (10)$$

where C_2 = levelized net annual cost of pumping energy per watt.

Cost of Pipes

The levelized net annual cost of pipe per unit pipe length (K_p) (Cheremisinoff et al. 1988) is given by

$$K_p = \pi Dt\gamma_pC_1 \quad (11)$$

where t = pipe wall thickness in meters; γ_p = specific weight of pipe material; and C_1 = levelized net annual cost of pipes per unit weight of pipe material. In practice, C_1 is constant for a suitable range of pipe diameters. The pipe wall thickness t is roughly proportional to the pipe diameter (Davis and Sorensen 1969; Russel 1966) so that

$$t = CD \quad (12)$$

where C = constant of proportionality that depends upon the expected pressure and diameter ranges of the pipe. Substitution of (12) in (11) yields

$$K_p + CC_1\gamma_p\pi D^2 \quad (13)$$

Total Cost of Pipeline

According to (10) and (13), the levelized total annual cost of pipeline per unit length (K_{tu}) is

$$K_{tu} = \frac{8fC_2\gamma Q^3}{g\eta\pi^2D^5} + \frac{8kC_2\gamma Q^3}{g\eta\pi^2LD^4} \pm \frac{C_2\gamma QS_0}{\eta} + CC_1\gamma_p\pi D^2 \quad (14)$$

Since the coefficient of friction, f, is a function of the pipe diameter, it is important to discuss it prior to any minimum cost-analysis.

Coefficient of Friction

Swamee and Jain (1976) presented a coefficient of friction equation that covers a significant portion of the turbulent zone in the Moody diagram, $5{,}000 < \mathsf{R} < 10^8$ and $10^{-6} < \varepsilon/D < 10^{-2}$, where R = Reynolds number and ε = absolute roughness of the pipe. However, Churchill (1977) provided a more general coefficient of friction equation that satisfactorily covers both the turbulent and laminar zones of the Moody diagram (Scaloppi and Allen 1993). According to Churchill (1977), the coefficient of friction is given by

$$f = 8\left[\left(\frac{8}{\mathsf{R}}\right)^{12} + \frac{1}{(A+B)^{1.5}}\right]^{1/12} \quad (15)$$

where A and B are given by

$$A = \left\{2.457 \ln\left[\frac{1}{\left(\frac{7}{\mathsf{R}}\right)^{0.9} + 0.27\left(\frac{\varepsilon}{D}\right)}\right]\right\}^{16} \quad (16)$$

and

$$B = \left(\frac{37{,}530}{\mathsf{R}}\right)^{16} \quad (17)$$

In computing f, A, and B, double precision should be used since these quantities are calculated to very large and very small numbers, therefore allowing them to be sensitive to round-off error (Scaloppi and Allen 1993).

MINIMUM-COST PIPELINE SIZING

For the minimum cost of design, the levelized total annual cost of pipeline per unit length, K_{tu}, should be a minimum. In other words, the first derivative of K_{tu} (with respect to the pipe diameter D) should equal zero (Hathoot 1986). Reynolds number may be written as

$$\mathsf{R} = 4Q/\pi D\upsilon \quad (18)$$

where υ = kinematic viscosity of liquid. Substitution of R as given in (18), into (15), (16), and (17) yields

$$f = 8\left[\left(\frac{2\pi\upsilon D}{Q}\right)^{12} + \frac{1}{(A+B)^{1.5}}\right]^{1/12} \quad (19)$$

$$A = \left\{2.457 \ln\left[\frac{1}{\left(\frac{1.75\pi D\upsilon}{Q}\right)^{0.9} + 0.27\left(\frac{\varepsilon}{D}\right)}\right]\right\}^{16} \quad (20)$$

and

$$B = \left(\frac{9{,}382.5\pi D\upsilon}{Q}\right)^{16} \quad (21)$$

For convenience, (14) is put into the following form:

$$K_{tu} = \frac{Mf}{D^5} + \frac{N}{D^4} \pm T + UD^2 \quad (22)$$

where

$$M = \frac{8C_2\gamma Q^3}{g\eta\pi^2};\; N = \frac{8kC_2\gamma Q^3}{g\eta\pi^2L} \quad (23, 24)$$

$$T = \frac{C_2\gamma QS_0}{\eta} \text{ and } U = CC_1\gamma_p\pi \quad (25, 26)$$

Substitution of f in (19) into (22), differentiating (22) with respect to D, and equating to zero leads to

$$0 = \frac{-16M}{(J)^{11/12}D^6}\left[2\left(\frac{8}{\mathsf{R}}\right)^{12} + \frac{2.457(E)^{15}(F)(G) + 2.5A + 3.5B}{(A+B)^{2.5}}\right] - 4\frac{N}{D^5} + 2UD \quad (27)$$

where

$$J = \left(\frac{8}{\mathsf{R}}\right)^{12} + \frac{1}{(A+B)^{1.5}} \quad (28)$$

$$E = 2.457 \ln(1/F);\; F = (7/\mathsf{R})^{0.9} + 0.27(\varepsilon/D) \quad (29, 30)$$

$$G = 0.9(7/\mathsf{R})^{0.9} - 0.27(\varepsilon/D) \quad (31)$$

solving (27) for D

$$D = \left(\frac{1}{2U}\left\{\frac{16M}{(J)^{11/12}}\left[2\left(\frac{8}{\mathsf{R}}\right)^{12} + \frac{2.457(E)^{15}(F)(G) + 2.5A + 3.5B}{(A + B)^{2.5}}\right] + 4ND\right\}\right)^{1/7} \quad (32)$$

Eq. (32) is the governing equation for minimum cost pipeline sizing. The preceding equation is of the implicit type and is to be solved through a trial-and-error procedure.

Two unknowns exist in (32): D and L, and (8) is to be solved simultaneously with (32). The power per pumping unit, W, [contained in (8)] should be known in advance for solving such problems.

COMPUTER PROGRAM

The flowchart in Fig. 2 illustrates a simple computer program for estimating the most economical pipeline diameter. At the beginning of the calculations, a rational average velocity is assumed, 1.0 m/s say, and the corresponding diameter is estimated. The spacing L is then calculated by applying (8).

For the second trial, the pipe diameter is estimated by applying (32). A new trial-cycle begins after calculating the spacing L and considering the last calculated pipe diameter. A third diameter is then calculated. Trial-cycles continue until the difference between two successive calculated values of D decreases. The following are numerical examples when the aforementioned program is applied.

NUMERICAL EXAMPLE 1

It is necessary to determine a size of pipeline that can deliver 1.2 m^3/s of liquid up a slope of 1:20,000 for the following data: $C = 0.012$; $C_1 = 0.0026$ \$/N; $C_2 = 0.11$ \$/W; $\gamma = 9{,}810$ N/m^3; $\upsilon = 1.01 \times 10^{-6}$ m^2/s; $\gamma_p = 76{,}518$ N/m^3; and $\varepsilon = 7.2 \times 10^{-5}$ m. Pumping units are 80 kW each with an efficiency of 0.67. Losses in fittings are such that $k = 10$.

Solution

Trial cycles results are listed in Table 1.

In this case, the diameter is taken to be $D = 1.0$ m. According to (8), the corresponding spacing between pumps L = 2,170 m. The flow is turbulent with $\mathsf{R} = 1.51 \times 10^6$; $\varepsilon/D = 7.2 \times 10^{-5}$; and $V = 1.53$ m/s.

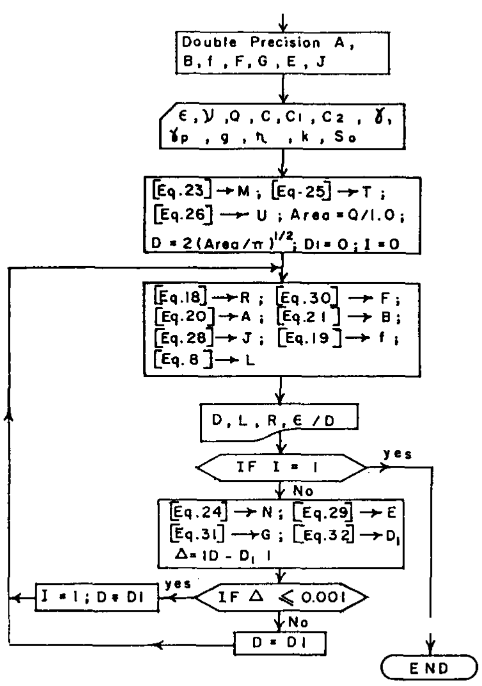

FIG. 2. Flowchart for Computer Program

TABLE 1. Results of Trial Cycles of Example 1

D (m) (1)	L (m) (2)	R (3)	ε/D (4)
1.2361	7,110.67	1.223839×10^6	5.825×10^{-5}
1.0098	2,306.45	1.498061×10^6	7.130×10^{-5}
1.0305	2,611.96	1.467918×10^6	6.987×10^{-5}
1.0273	2,562.41	1.472569×10^6	7.009×10^{-5}
1.0278	2,569.71	1.512760×10^6	7.005×10^{-5}

TABLE 2. Results of Trial Cycles of Example 2

D (m) (1)	L (m) (2)	R (3)	ε/D (4)
0.6676	1,909.71	1,056.26	4.4940×10^{-4}
0.8320	4,888.07	847.48	3.6057×10^{-4}
0.8553	5,504.07	824.44	3.5077×10^{-4}
0.8584	5,591.60	821.43	3.4949×10^{-4}
0.8588	5,603.37	821.03	3.4932×10^{-4}

NUMERICAL EXAMPLE 2

It is necessary to determine the size of a pipeline that can deliver 0.35 m^3/s of a liquid ($\upsilon = 6.32 \times 10^{-4}$ m^2/s and $\gamma = 12{,}500$ N/m^3) down a constant slope of 1:10,000 using equally spaced pumping units each producing 60 kW with an efficiency of 0.66. A system of fittings is used so that $k = 8$. The following data are available: $C = 0.01$; $C_1 = 0.002$ \$/N; $C_2 = 0.15$ \$/W; $\gamma_p = 73{,}575$ N/m^3; and $\varepsilon = 3.0 \times 10^{-4}$ m.

Solution

Trial-cycle results are shown in Table 2.

For convenience, the chosen diameter is $D = 0.85$ m and application of (8) yields $L = 5{,}359$ m. In this case, the flow is laminar with $\mathsf{R} = 829.55$ and $V = 0.62$ m/s.

CONCLUSIONS

The minimum cost pipeline sizing formula presented in this paper spans all flow regimes, namely laminar and turbulent.

Although the pipeline sizing formula is implicit, its convergent characteristics make the final solution attainable after a limited number of trial cycles. The computer program provided by the writers proves to be simple and practical. The solution of two practical examples yields reasonable pipeline sizing results.

APPENDIX I. REFERENCES

Albertson, M., Barton, J., and Simons, D. (1960). *Fluid mechanics for engineers*. Prentice-Hall, Inc., New York, N.Y.

Cheremisinoff, P. N., Cheremisinoff, N. P., and Cheng, S. L., eds. (1988). *Civil engineering practice*. 2-Hydraulics/Mechanics., Technomic Publishing Co., Lancaster, Pa.

Churchill, S. W. (1977). "Friction-factor equation spans all fluid-flow regimes." *Chem. Engrg.*, 84(24), 91–92.

Daugherty, R., and Franzini, J. (1977). *Fluid mechanics with engineering applications*. 5th Ed., McGraw-Hill Book Co., New York, N.Y.

Davis, C., and Sorensen, K. (1969). *Handbook of applied hydraulics*. 3rd Ed., McGraw-Hill Book Co., New York, N.Y.

Hathoot, H. M. (1980). "Optimum design of irrigation pipelines." *Bull., Int. Commission on Irrig. and Drain. (ICID)*, 29(2), 73–76.

Hathoot, H. M. (1984). "Minimum cost design of horizontal pipelines." *J. Transp. Engrg.*, ASCE, 110(3), 382–389.

Hathoot, H. M. (1986). "Minimum cost design of pipelines." *J. Transp. Engrg.*, ASCE, 112(2), 465–480.

Russel, G. (1963). *Hydraulics*. 5th Ed., Holt, Rinehart and Winston, New York, N.Y.

Russel, G. (1966). *Standard handbook for mechanical engineering*. McGraw-Hill Book Co., New York, N.Y.
Scaloppi, E. J., and Allen, R. G. (1993). "Hydraulics of center-pivot laterals." *J. Irrig. and Drain. Engrg.*, ASCE, 119(3), 554–567.
Streeter, V. L., and Wylie, B. E. (1983). *Fluid mechanics*. Int. Stud. Ed., McGraw-Hill Book Co., New York, N.Y.
Swamee, P., and Jain, A. (1976). "Explicit equations for pipe flow problems." *J. Hydr. Div.*, ASCE, 657–664.

APPENDIX II. NOTATION

The following symbols are used in this paper:

A = quantity defined by Eq. (16);
a = constant contained in Eq. (1);
B = quantity defined by Eq. (17);
b = constant contained in Eq. (1);
C = constant contained in Eq. (12);
C_1 = levelized net annual cost of pipes per unit weight of pipe material (\$/N);
C_2 = levelized net annual cost of pumping energy per watt (\$/W);
D = pipe diameter (m);
E = quantity defined by Eq. (29);
F = quantity defined by Eq. (30);
f = coefficient of friction;
G = quantity defined by Eq. (31);
g = acceleration due to gravity (m/s^2);
H_p = total head per pumping unit (m);
h_f = friction head loss (m);
h_{fit} = head loss in pipe fittings (m);
J = quantity defined by Eq. (28);
K_{en} = levelized net annual cost of pumping energy per unit pipe length (\$/m);
K_p = levelized net annual cost of pipe per unit pipe length (\$/m);
K_t = total annual cost of pipeline (\$);
K_{tu} = levelized total annual cost of pipeline per unit pipe length (\$/m);
k = sum of coefficients representing head loss in fittings between two successive pumping units;
L = spacing between pumping units (m);
M = quantity defined by Eq. (23);
N = quantity defined by Eq. (24);
P = power provided by each pumping unit (w);
Q = pipeline discharge (m^3/s);
R = Reynolds number;
S_0 = slope of pipeline;
T = quantity defined by Eq. (25);
t = wall thickness of pipe (m);
U = quantity defined by Eq. (26);
V = average velocity of liquid (m/s);
W = power required per unit length of pipeline (w/m);
γ = specific weight of liquid (N/m^3);
γ_p = specific weight of pipe material (N/m^3);
ε = absolute roughness of pipe material (m);
η = pumping unit efficiency; and
υ = kinematic viscosity of liquid (m^2/s).