Search Wikibooks | Go

Vedic Mathematics/Techniques/Multiplication

Vedic Mathematics		
Previous: Addition And Subtraction	**Multiplication**	**Next: Division**

Introduction

There are a wide range of multiplication techniques, the one perhaps most familiar to the majority of people is the classic long multiplication algorithm, e.g.

```
      23958233
          5830 ×
  ------------
      00000000 (= 23,958,233 ×     0)
     71874699  (= 23,958,233 ×    30)
   191665864   (= 23,958,233 ×   800)
  119791165    (= 23,958,233 × 5,000)
  ------------
  139676498390 (= 139,676,498,390)
```

While this algorithm works for any pair of numbers, it is long winded, requires many intermediate stages, and requires you to record the results of each of the intermediate stages so you can sum them at the end to produce the final answer. However, when the numbers to be multiplied fall into certain categories, short-cuts can be used to avoid much of the work involved in long multiplication. There are many of these 'special cases' some of them allow seemingly difficult multiplications to be completed mentally, literally allowing you to just write down the answer.

Memory versus Calculation

Special Cases of Multiplication

There are a number of simple techniques that can be used when multiplying by certain numbers. They are very useful in their own right but can be even more useful when combined with other techniques where they can facilitate the solution of more difficult problems.

Multiplying by 11

To multiply any number by 11 do the following:
Working from **right to left**

1. Write the rightmost digit of the starting number down.
2. Add each pair of digits and write the results down, (carrying digits where necessary **right to left**).
3. Finally write down the left most digit (adding any final carry if necessary).

It's as simple as that, e.g.

- Multiply 712x11

712x11=7832

The reason for working from right to left instead of the more usual left to right is so any carries can be added in. e.g.

- Multiply 8738x11

8738x11=96118

Multiplying by 15

Multiplying by 15 can be broken down into a multiplication by 10 plus a multiplication by 5. Multiplying by 10 is just a matter of adding a 0 on the end of a number, multiplying by 5 is half of a multiplication by 10 as above e.g.

```
15 x 33 = [10 x 33] + [5 x 33] = 330 + [(10 x 33) / 2] = 330 + [330 / 2] = 330 + 165 = 495
```

This is easier to do in your head than it is to write down! e.g. to multiply 15 x 27 you first multiply 27 x 10 giving 270, then 'add half again', i.e. half of 270 is 135, add this to 270 to get 405 and that's your answer.

Multiplying two single digit numbers

Although most people have memorised multiplication tables from 1x1 to 10x10, one of the Vedic Sutras, (**Vertically and Crosswise**), allows you to multiply any pair of single digit numbers without using anything higher than the 5x multiplication table. While this may not be particularly useful, the algorithm is a good introduction to some of the ideas behind the Vedic techniques and so is worth taking the time to learn and understand as the basic idea is expanded upon later. Because of this I will go through the procedure in more detail than it probably deserves. The technique is as follows:

1. If either of the numbers are below 6 then just recall the answer from memory (you need to know your multiplication tables up to 5x, hopefully this shouldn't be too much of a problem!). If instead both numbers are above 5 then continue.
2. Write, (or imagine), the two single digit numbers one above the other with an answer line below.
3. Subtract each number from 10 and place the result to the right of the original number.
4. **Vertically**: Multiply the two numbers on the right (the answers to the subtractions in the previous step) and place the answer underneath them on the answer line, (this is the first part of the answer). Since the original numbers were above 5, these numbers will always be below 5 (because the original numbers were subtracted from 10) so you won't need anything above the 4x multiplication table. If the answer to the multiplication is 10 or more, just place the right-most digit on the answer line and remember to carry the other digit to the next step.
5. **Crosswise**: Select one of the original numbers, (it doesn't matter which one, the answer will be the same), and subtract the number diagonally opposite it. If there was no carry from the previous step just place the result on the answer line below the original numbers, if there was a carry add this to the result before you place it on the answer line.

That's it, the number on the answer line is the final answer. The technique is very simple but looks more complicated than it actually is when written down step by step. The following examples should clarify the procedure.

- **Multiply 8 x 6**

  Following steps 1 to 3 above, write the numbers down one above the other, subtract each from 10, and write the answer to the right of each number:

  Now, following step 4 (**Vertically**) multiply the two numbers on the right and write down the answer.

Finally, following step 5 (**Crosswise**) Subtract along any diagonal (the answer will be the same either way) and write down the answer.

So 8 x 6 is 48 as expected, but as you can see from the above sequence, to work this out you only needed to know know how to subtract small numbers and multiply 2 x 4.

- **Multiply 7 x 8**

Giving an answer of 56 as expected.
The next example involves a carry between the last two stages.

- **Multiply 7 x 6**

As you can see the multiplication in the second (**Vertically**) stage (3x4) results in 12, the 2 is written down and the 1 is carried. The subtraction in the **Crosswise** stage (6-3) results in 3, but you have to add the carried 1 from the previous step resulting in 4. The final answer is 42.

This may all seem like a lot of work for doing simple multiplications that you can just recall from memory, but the important thing here is to learn the technique as it is expanded upon later. If you are interested in improving your mental arithmetic skills, learn to do this procedure in your head by visualising the numbers laid out as above. Before going on to the next technique, try it now; close your eyes and multiply 6x6 using the procedure above...
You should have imagined the following:

- **Multiply 6 x 6**

Multiplying two numbers close to 100

The above technique actually works for any two numbers but it is only useful if it results in an easier process than traditional long multiplication. The key thing to remember is that with this technique you end up multiplying the subtracted numbers instead of the original numbers, i.e. it is only easier than normal multiplication if these subtracted numbers are smaller than the original numbers, hence the reason why we only used the above technique for numbers greater than 5, (since the subtracted numbers will then be 4 or smaller).

When the technique is extended to double digit numbers, you subtract each from 100 during the '**Vertically**' stage instead of subtracting them from 10 so the technique is only easier if the result of this subtraction is small for one or both of the numbers, this is obviously the case when one or both of the numbers are close to 100. Take a look at the following example:

- **Multiply 89 x 97**

So **89x97=8633**.
Now the power of the technique becomes clear. In the introduction I asked whether you wanted to be able

to multiply **89x97** quickly in your head, you should now be able to see that this is actually quite easy. You first visualise both numbers one on top of the other, you then subtract each from 100 giving **11** and **3**, mentally placing each result to the right of the original numbers. Next you multiply **11** and **3** together giving **33**. (Note that since we are now dealing with double digit numbers, we don't carry unless the answer to this multiplication is 100 or more). We now have the last two digits of the answer (**33**), all we have to do now is subtract along either of the diagonals to get the first digits. We can pick any diagonal as they will always give the same answer but 89-3=**86** is perhaps easier than 97-11=**86**. The final answer is the concatenation of the two parts giving **8633**
If you practice this technique you will find you can do two digit multiplications without writing anything down. Try it now, multiply 95x93 in your head, try to visualise the procedure above.
You should have come up with the following:

- **Multiply 95 x 93**

95x93=8835

Remember that the reason why this is easier than normal multiplication is because you only have to multiply the results of the subtractions. This means that you can usually use it in situations where only one of the numbers is close to 100 because the multiplication will still be easy in this case. e.g.

- **Multiply 97 x 69**

97x69=6693

- **Multiply 96 x 88**

96x88=8448

The same technique works for numbers slightly over 100 except you now have to add during the **Crosswise** step. e.g.

- **Multiply 105 x 107**

105x107=11235

There are a number of ways to remember the extension of the technique to numbers larger than 100. If you are familiar with the sign multiplication rules (i.e. -x-=+, -x+=-, etc.) then you don't have to alter the technique at all as you will understand that 100-105 = -5 and 100-107=-7, then (-7)x(-5)=**35**, and 107-(-5)=107+5=**112**.
If you are not comfortable with this then you can instead just reverse the initial subtractions when the original numbers are greater than 100 (or just remember that you need the difference between the original numbers and 100, i.e. 105-100 = 5 instead of 100-105 = -5), and then also remember that you have to change the **Crosswise** subtraction to an addition if the number you want to subtract came from an original number greater than 100.
Try to do the following examples in your head.

- **Multiply 109 x 108**

109x108=11772

- **Multiply 115 x 106**

115x106=12190

- **Multiply 123 x 103**

123x103=12669

As before you need to carry the first digit of the **Vertically** multiplication if it is more than 2 digits long. e.g.

- **Multiply 133 x 120**

133x120=15960

Combining techniques

One of the key ideas of the Vedic system is that you can combine techniques to solve problems. You should look at problems in a flexible way and use the combination of techniques that best suits a particular problem, (and the way your own brain works!). It is perhaps a little early to be discussing this since we have only covered one major technique so far, but even at this stage it is possible to combine the **Vertically and Crosswise** multiplication technique with the special case multiplication techniques already described to handle the situation when both numbers are further away from 100, 1000, etc.

For example, using the 'by 10 and half again' rule to multiply by 15 lets you easily deal with numbers further away from 10, 100, 1000, etc., if one of the numbers is 15 away from your 'base' number e.g.

- **Multiply 66 x 85**

66x85=5610

You can see here that the **Vertically** multiplication results in 510 (34x15 = 340 'and half again' = 340 + 170 = 510). 510 has 3 digits so we write down the last two digits (10) and carry the leading 5. We then do the **Crosswise** subtraction along the easiest diagonal (66-15 = 51) and add the carry (5) before writing down the final answer (56).

Extending the Multiplication Technique

The same **Vertically and Crosswise** technique described above works for any numbers but it is particularly useful for numbers near a power of 10, i.e. 10, 100, 1000, 10000, 100000, etc. As long as the initial subtraction results in numbers that are 'easier' to multiply it is a useful technique. e.g.

- **Multiply 1232 x 1003**

1232x1003=1235696

Since we are dealing with numbers near 1000 here we find the initial differences from 1000 instead of 100 and we only carry if the **Vertically** multiplication is 1000 or greater.

- **Multiply 9960 x 9850**

9960x9850=98106000

In this case the numbers are slightly below 10000 so we initially subtract from 10000, we also subtract in the **Crosswise** stage, and we only carry if the **Vertically** multiplication is 10000 or more. (In this particular case it would have perhaps been simpler to do 996x981 using the **Vertically and Crosswise** technique and then add two zeroes to the end of the answer.)

- **Multiply 89684 x 99989**

89684x99989=8967413476

Note the combination of techniques in the above example, i.e. the difference between 89684 and 100000 is easily derived using the special case technique for subtracting from a power of 10, (i.e. using the sutra **All from 9 and the last from 10**). The special case technique for multiplication by 11 is also used.

- **Multiply 98688 x 99997**

98688x99997=9868503936

In this example the numbers are slightly less than 100000 so the initial subtraction is from 100000. The thing to watch out for here is that the result of the **Vertically** multiplication must be padded out to 5 digits by adding an extra zero on the left (03936 instead of 3936). This is also true generally, i.e. the number of digits in the result of the **Vertically** multiplication must always be the same as the number of zeroes in the 'base' number (i.e. 100, 1000, 100000, etc.).

It is worth remembering how far we have come even at this early stage. Even if you are writing the calculations down, it is still much more efficient to do the multiplication above the Vedic way than using traditional long multiplication. e.g.

- **Multiply 98688 x 99997 using long multiplication**

```
     98688
     99997 ×
----------
    690816 (= 98688 ×     7)
   8881920 (= 98688 ×    90)
  88819200 (= 98688 ×   900)
 888192000 (= 98688 ×  9000)
8881920000 (= 98688 × 90000)
----------
9868503936
```

This multiplication technique can be extended further to cover cases where one number is slightly above a power of 10 and one slightly below the same power of 10. In this case it is advantageous to note whether the original numbers are greater or smaller than the 'base' power of 10 using **+** or **-** symbols accordingly. e.g.:

- **Multiply 111 x 88**

Note that the result of the **Vertically** multiplication is now negative because the signs of the two numbers you are multiplying are different. Additionally, since our 'base' is 100, we can only write 2 digits down in the answer section so the leading **-1** of the **-132** must be carried; thus:

Now we have **-32** in the answer section. We must convert this negative number by replacing it with it's 'compliment', i.e. that number which when added would result in 100. In this case **32+68=100** so we replace the **-32** with **68**. Whenever we do a replacement of this sort we must also subtract one from the carry (i.e. in this case the carry changes from **-1** to **-2**). Once this is done we continue as before; thus:

111x88=9768
Note that in this example we have added the **11** to **88** during the **Crosswise** step because the **11** was written down as **+11**. It is important to note this sign. If we had instead used the other diagonal the calculation would have been **111-12**, (resulting in the same answer **99**), because the **12** is written as **-12**.
Now all of this may seem a bit convoluted, but you would not explicitly write down each of the steps above, I have only done this to clearly illustrate what is going on. In practice the 'compliment and carry' steps would be written down directly, e.g.

- **Multiply 97 x 104**

97x104=10088
In the above case, after the initial differences are found, (**-3** and **+4**), they are multiplied to give **-12** but then you remember that a negative number cannot be written down so it is complemented giving **88** (12+88=100) and the carry is reduced by one, (since there is no carry in this example, the carry becomes **-1**). Finally the **Crosswise** step is performed, (in this case 97+4=101), and the carry subtracted (101-1=100), restulting in **100**. The final answer being the concatenation of the two parts as usual, i.e. **10088**.
Some further examples should make the process clear. Try doing them mentally before reading through the working.

- **Multiply 103 x 87**

103x87=8961

- **Multiply 998 x 1004**

998x1004=1001992
In the above example the base is now 1000, the result of the vertically multiplication is -8 resulting in a compliment of **992**.

- **Multiply 1234 x 989**

1234x989=1220426
The above example deserves some explanation as it shows the use of multiple techniques. First the initial residuals are written down, i.e. **+234** and **-11**. We then use the special case technique to multiply 234 by -11 which results in **-2574**. This gives a carry of **-2** and a remainder of **-574**. The remainder is complemented by subtracting it from 1000, this is done using the special case method for subtracting from a power of 10, (i.e. using the sutra **All from 9 and the last from 10**), resulting in **426** and the carry is reduced by one changing it from **-2** to **-3**. Finally the **Crosswise** subtraction is performed taking account of the carry, (1234-11-3) resulting in **1220** giving a final answer of **1220426**.

Multiplying Two Numbers that are 'Closely Related'

We are now ready to extend the multiplication technique described above to the most general case, i.e. the multiplication of any two numbers that are 'closely related'. The precise definition of 'closely related' is:

> "Numbers that are a small distance away from a 'proportional power of 10' such that the differences between the original numbers and this proportional power of 10 are simple to multiply".

This may sound very complicated, but it is actually quite simple. Firstly the 'proportional power of 10' is just a simple multiple or division of a power of 10, powers of 10 being 10, 100, 1000, 10000, etc. So, proportional powers of 10 are 10, 20, 25, 30, 40,... 80, 90,100, 200, 250, 300,... 800, 900, etc. etc. (Note

that 25, 250, etc. are proportional powers of 10 in this definition because they are a simple division of a power of 10, e.g. 100/4=25).
So If both of the numbers you want to multiply are 'close' to one of these numbers (i.e. the residuals are small enough so that you can multiply them easily), then the extended technique can be used. This extended technique is simply the addition of the sub-sutra *Anurupyena* or **Proportionately** to the technique already described above. The complete description of the extended **Vertically and Crosswise** technique is:

1. Place the two numbers you wish to multiply one on top of the other, leave an answer line below.
2. Choose a '**working base'** that is close to both numbers, this must be a '**proportional power of 10'**. The '**theoretical base'** is the actual power of 10 before you have multiplied or divided it to get your '**working base'**. e.g. If the '**working base'** is 25 then the '**theoretical base'** would be 100 (the '**theoretical base'** of 100 would have been divided by 4 in this case to get the '**working base'** of 25). Remember the '**proportionality'** of what you have done, (e.g. if you have divided by 4 or multiplied by 3 etc.), you will use this '**proportionality'** correct the left hand side of the answer.
3. Subtract the '**working base'** from each of the original numbers and place the results to the right of each, (remember to include the sign of the result, e.g. 21-25=-4, 28-25=+3). We will call these results the '**residuals'**.
4. **Vertically** multiply the '**residuals'** obtained above noting the sign of the result, (i.e. if the signs of the '**residuals'** are **different** then the sign of the multiplication result will be **negative**, if the signs of the '**residuals'** are **the same** then the result will be **positive**. We will call this the '**Vertically Result'**.
5. Add or Subtract **Crosswise** following the sign present on the particular diagonal chosen, (you will get the same answer no matter which diagonal is selected so pick the easiest calculation to create the '**Crosswise Result'**.
6. 'Correct' the '**Crosswise Result'** by repeating the '**proportionality'** used to create the '**working base'**, (e.g. if you divided by 4 to get your '**working base'** then divide the '**Crosswise Result'** by 4 also). Note that if the proportionality was a division, and if this resulted in a result with a fractional part, then this fractional part must be transferred to the right hand side of the answer by adding the same fractional proportion of the theoretical base to the previously calculated '**Vertically result'**.
7. If there are now too many digits in '**Vertically Result'** (i.e. more than the number of zeroes in the '**theoretical base'**), then you have to carry the leading digit(s) to the next stage remembering to preserve the signs. We will call the result of the **Vertically** multiplication after any carry has been removed the '**remainder'**.
8. If the '**remainder'** is positive you can just place it on the answer line, if it is negative, it must be replaced with it's compliment before placing it on the answer line, (remember the compliment of a number is the result of subtracting the number from the '**theoretical base'**). If you have to compliment the '**remainder'** to make it positive then you must also reduce the carry by 1, (if there is no carry yet then the carry becomes -1)
9. Add or Subtract any carry (according to it's sign) to the corrected Crosswise result above, place this result in the answer line to the left of the '**remainder'** part of the answer.
10. That's it, the digits on the answer line are the results of the original multiplication.

Now, that all sounds terribly complicated, but it really is much harder to write (and read) the steps than to actually follow them! In fact you have already followed all the steps in the many previous examples, the only additional steps are the '**proportionately** calculations. Some examples will clarify.

- **Multiply 489x512**

As you can see above, first a '**working base'** of **500** is chosen as it is close to both numbers, in this case we use a '**theoretical base'** of **100** and a '**proportionality'** of **x5**, (we could also have used 1000 and ÷2 as will be seen in the next example). The '**residuals'** are then found i.e. **-11** and **+12** and these are then multiplied giving **-132** (the multiply by 11 rule is used here giving 132 but as the residual signs are different the answer is -132). We now subtract **Crosswise** 512-11 resulting in **501** (we could have chosen the other diagonal if we wanted i.e. 489+12=501, both give the same answer). Then we 'correct' the Crosswise subtraction by repeating the original **proportionately** step, i.e. 501x5=**2505**.

We can only have 2 digits in the '**remainder'** result because our '**theoretical base'** is 100 so we must carry the -1 leaving **-32** on the answer line, however we can't put a negative number on the answer line either so we compliment **-32** resulting in **68** (and then reduce the carry by 1 to **-2**). Finally we subtract the carry giving 2503. Thus the final answer is the contactination of both parts, i.e. **250368**

This calculation is as complicated as it gets and every step has been split out into it's constituent parts intentionally so you can more easily see and understand each step, but remember that you would not write each step down like this, you would usually either do it all mentally or just write down the part answers directly.

You can see below that choosing the alternative **proportionality** (i.e. 1000÷2) results in the same answer:

- **Multiply 489x512**

You can see above that we are performing the same multiplication (**489x512**) and we are still using a '**working base'** of 500 but this time the '**theoretical base'** is 1000 and the '**proportionality'** is ÷2. The calculation proceeds as before, with the the diagonally subtraction resulting in **501**, but this time the '**proportionality'** is ÷2 so we have to divide 501 by 2 resulting in 250½.

We can't write ½ in the answer so we transfer it back to the remainder column as half of the '**theoretical base'**, i.e. 1000÷2=500 added to the -132 already present giving **368**.

The many varied ways you can tackle a numerical problem with the Vedic techniques is a key advantage to the system. With practice you will become more confident when working with numbers and achieve a deeper understanding of arithmetic. However, one essential point in the above technique must be remembered, that is:

> You must **NOT** process any carries until you have 'corrected' the **Crosswise** calculation for any initial **proportionality** you applied at the beginning of the process.

It is a common mistake to try to deal with the carries before 'correcting' the left hand side of the answer for **proportionality**. If you do this the answer will be wrong.

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