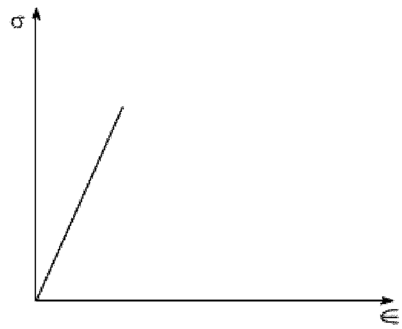
LECTURE 9

STRESS - STRAIN RELATIONS

Stress – Strain Relations: The Hook's law, states that within the elastic limits the stress is proportional to the strain since for most materials it is impossible to describe the entire stress – strain curve with simple mathematical expression, in any given problem the behavior of the materials is represented by an idealized stress – strain curve, which emphasizes those aspects of the behaviors which are most important is that particular problem.

(i) Linear elastic material:

A linear elastic material is one in which the strain is proportional to stress as shown below:

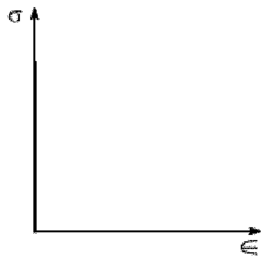


Linearly elastic material

There are also other types of idealized models of material behavior.

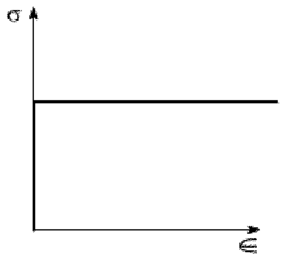
(ii) Rigid Materials:

It is the one which donot experience any strain regardless of the applied stress.



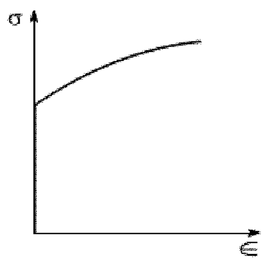
(iii) Perfectly plastic(non-strain hardening):

A perfectly plastic i.e non-strain hardening material is shown below:



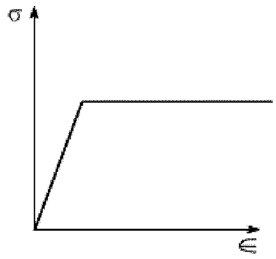
(iv) Rigid Plastic material(strain hardening):

A rigid plastic material i.e strain hardening is depicted in the figure below:



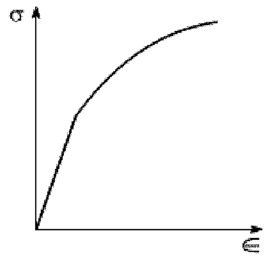
(v) Elastic Perfectly Plastic material:

The elastic perfectly plastic material is having the characteristics as shown below:



(vi) Elastic – Plastic material:

The elastic plastic material exhibits a stress Vs strain diagram as depicted in the figure below:



Elastic Stress – strain Relations :

Previously stress – strain relations were considered for the special case of a uniaxial loading i.e. only one component of stress i.e. the axial or normal component of stress was coming into picture. In this section we shall generalize the elastic behavior, so as to arrive at the relations which connect all the six components of stress with the six components of elastic stress. Futher, we would restrict overselves to linearly elastic material.

Before writing down the relations let us introduce a term ISOTROPY

ISOTROPIC: If the response of the material is independent of the orientation of the load axis of the sample, then we say that the material is isotropic or in other words we can say that isotropy of a material in a characteristics, which gives us the information that the properties are the same in the three orthogonal directions x y z, on the other hand if the response is dependent on orientation it is known as anisotropic.

Examples of anisotropic materials, whose properties are different in different directions are

(i) Wood

(ii) Fibre reinforced plastic

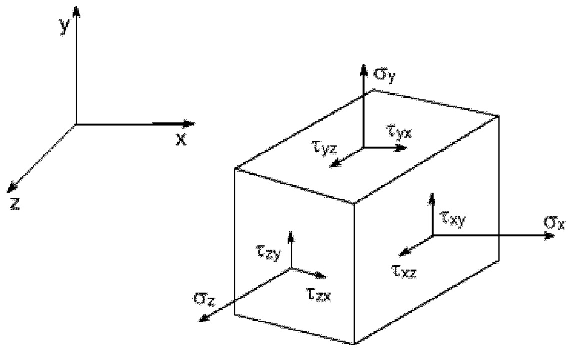
(iii) Reinforced concrete

HOMOGENIUS: A material is homogenous if it has the same composition through our body. Hence the elastic properties are the same at every point in the body. However, the properties need not to be the same in all the direction for the material to be homogenous. Isotropic materials have the same elastic properties in all the directions. Therefore, the material must be both homogenous and isotropic in order to have the lateral strains to be same at every point in a particular component.

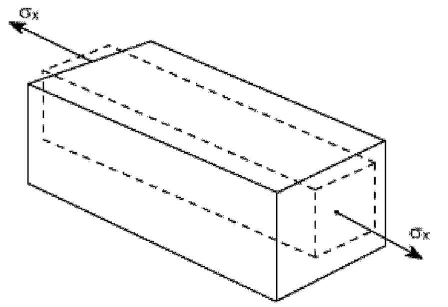
Generalized Hook's Law: We know that for stresses not greater than the proportional limit.

$$\epsilon = \frac{\sigma}{E} \text{ or } \mu = -\frac{|\epsilon_{lateral}|}{|\epsilon_{axial}|}$$

These equation expresses the relationship between stress and strain (Hook's law) for uniaxial state of stress only when the stress is not greater than the proportional limit. In order to analyze the deformational effects produced by all the stresses, we shall consider the effects of one axial stress at a time. Since we presumably are dealing with strains of the order of one percent or less. These effects can be superimposed arbitrarily. The figure below shows the general triaxial state of stress.



Let us consider a case when σ_x alone is acting. It will cause an increase in dimension in X-direction whereas the dimensions in y and z direction will be decreased.

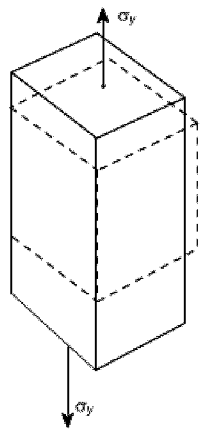


Therefore the resulting strains in three directions are

$$\epsilon_x = \frac{\sigma_x}{E} , \epsilon_y = -\mu\epsilon_x ; \epsilon_z = -\mu\epsilon_x$$

$$\epsilon_x = \frac{\sigma_x}{E} ; \epsilon_y = -\mu\frac{\sigma_x}{E} ; \epsilon_z = -\mu\frac{\sigma_x}{E}$$

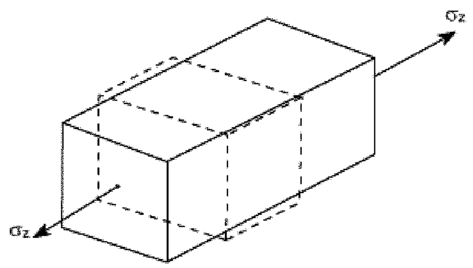
Similarly let us consider that normal stress σ_y alone is acting and the resulting strains are



$$\epsilon_y = \frac{\sigma_y}{E} , \epsilon_x = -\mu\epsilon_y ; \epsilon_z = -\mu\epsilon_y$$

$$\epsilon_y = \frac{\sigma_y}{E} ; \epsilon_x = -\mu\frac{\sigma_y}{E} ; \epsilon_z = -\mu\frac{\sigma_y}{E}$$

Now let us consider the stress σ_z acting alone, thus the strains produced are



$$\epsilon_z = \frac{\sigma_z}{E} , \epsilon_y = -\mu\epsilon_z ; \epsilon_x = -\mu\epsilon_z$$

$$\epsilon_z = \frac{\sigma_z}{E} ; \epsilon_y = -\mu\frac{\sigma_z}{E} ; \epsilon_x = -\mu\frac{\sigma_z}{E}$$

Thus the total strain in any one direction is

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\mu}{E}(\sigma_y + \sigma_z) \qquad (1)$$

In a similar manner, the total strain in the y and z directions become

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\mu}{E}(\sigma_x + \sigma_z) \qquad (2)$$

$$\epsilon_z = \frac{\sigma_z}{E} - \frac{\mu}{E}(\sigma_x + \sigma_y) \qquad (3)$$

In the following analysis shear stresses were not considered. It can be shown that for an isotropic material's a shear stress will produce only its corresponding shear strain and will not influence the axial strain. Thus, we can write Hook's law for the individual shear strains and shear stresses in the following manner.

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \qquad (4)$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G} \qquad (5)$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G} \qquad (6)$$

The Equations (1) through (6) are known as Generalized Hook's law and are the constitutive equations for the linear elastic isotropic materials. When these equations isotropic materials. When these equations are used as written, the strains can be completely determined from known values of the stresses. To engineers the plane stress situation is of much relevance (i.e. $\sigma_z = \tau_{xz} = \tau_{yz} = 0$), Thus then the above set of equations reduces to

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E}$$

$$\epsilon_z = -\mu\frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} \text{ and } \tau_{xy} = \frac{\gamma_{xy}}{G}$$

Their inverse relations can be also determined and are given as

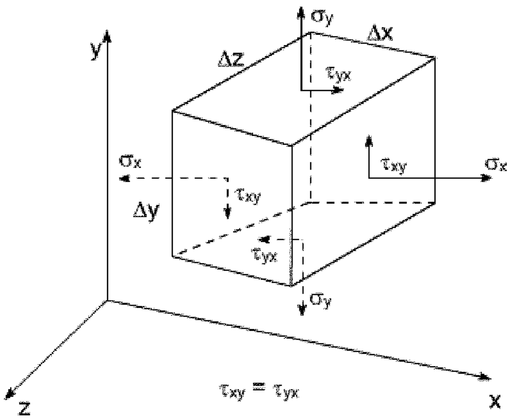
$$\sigma_x = \frac{E}{(1-\mu^2)}(\epsilon_x + \mu\epsilon_y)$$

$$\sigma_y = \frac{E}{(1-\mu^2)}(\epsilon_y + \mu\epsilon_x)$$

$$\tau_{xy} = G.\gamma_{xy}$$

Hook's law is probably the most well known and widely used constitutive equations for an engineering materials." However, we can not say that all the engineering materials are linear elastic isotropic ones. Because now in the present times, the new materials are being developed every day. Many useful materials exhibit nonlinear response and are not elastic too.

Plane Stress: In many instances the stress situation is less complicated for example if we pull one long thin wire of uniform section and examine – small parallepiped where x – axis coincides with the axis of the wire



So if we take the xy plane then σ_x , σ_y , τ_{xy} will be the only stress components acting on the parrallepiped. This combination of stress components is called the plane stress situation

A plane stress may be defined as a stress condition in which all components associated with a given direction (i.e the z direction in this example) are zero

$$\sigma_z = \tau_{zx} = \tau_{zy} = 0$$

Plane strain: If we focus our attention on a body whose particles all lie in the same plane and which deforms only in this plane. This deforms only in this plane. This type of deformation is called as the plane strain, so for such a situation.

$\epsilon_z = \gamma_{zx} = \gamma_{zy} = 0$ and the non – zero terms would be ϵ_x, ϵ_y & γ_{xy}

i.e. if strain components ϵ_x, ϵ_y and γ_{xy} and angle θ are specified, the strain components ϵ_x', ϵ_y' and γ_{xy}' with respect to some other axes can be determined.

ELASTIC CONSTANTS

In considering the elastic behavior of an isotropic materials under, normal, shear and hydrostatic loading, we introduce a total of four elastic constants namely E, G, K, and γ .

It turns out that not all of these are independent to the others. In fact, given any two of them, the other two can be foundout . Let us define these elastic constants

(i) E = Young's Modulus of Rigidity

= Stress / strain

(ii) G = Shear Modulus or Modulus of rigidity

= Shear stress / Shear strain

(iii) γ = Possion's ratio

= – lateral strain / longitudinal strain

(iv) K = Bulk Modulus of elasticity

= Volumetric stress / Volumetric strain

Where

Volumetric strain = sum of linear stress in x, y and z direction.

Volumetric stress = stress which cause the change in volume.

Let us find the relations between them

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