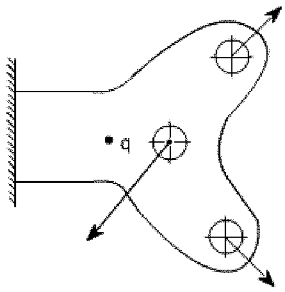
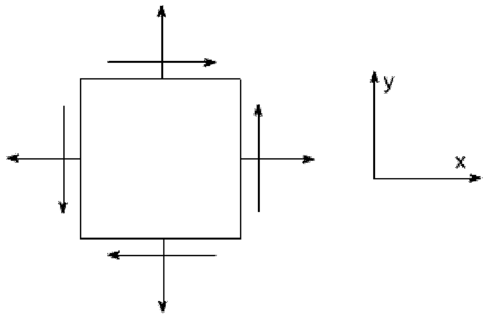
LECTURE 3

Analysis of Stresses:

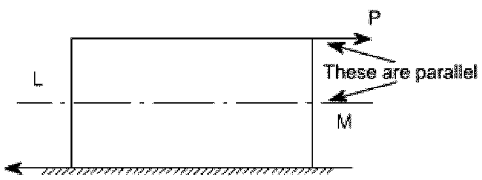


Consider a point ‘q' in some sort of structural member like as shown in figure below. Assuming that at point exist. ‘q' a plane state of stress exist. i.e. the state of state stress is to describe by a parameters σ_x, σ_y and τ_{xy} These stresses could be indicate a on the two dimensional diagram as shown below:



This is a commen way of representing the stresses. It must be realize a that the material is unaware of what we have called the x and y axes. i.e. the material has to resist the loads irrespective less of how we wish to name them or whether they are horizontal, vertical or otherwise further more, the material will fail when the stresses exceed beyond a permissible value. Thus, a fundamental problem in engineering design is to determine the maximum normal stress or maximum shear stress at any particular point in a body. There is no reason to believe apriori that σ_x, σ_y and τ_{xy} are the maximum value. Rather the maximum stresses may associates themselves with some other planes located at ‘θ'. Thus, it becomes imperative to determine the values of σ_θ and τ_θ. In order tto achieve this let us consider the following.

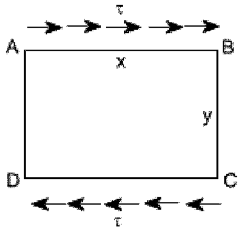
Shear stress:



If the applied load P consists of two equal and opposite parallel forces not in the same line, than there is a tendency for one part of the body to slide over or shear from the other part across any section LM. If the cross section at LM measured parallel to the load is A, then the average value of shear stress τ = P/A. The shear stress is tangential to the area over which it acts.

If the shear stress varies then at a point then τ may be defined as $\tau = \lim_{\delta A \to 0} \frac{\delta P}{\delta A}$

Complementary shear stress:



Let ABCD be a small rectangular element of sides x, y and z perpendicular to the plane of paper let there be shear stress acting on planes AB and CD

It is obvious that these stresses will from a couple (τ . xz)y which can only be balanced by tangential forces on planes AD and BC. These are known as complementary shear stresses. i.e. the existence of shear stresses on sides AB and CD of the element implies that there must also be complementary shear stresses on to maintain equilibrium.

Let τ' be the complementary shear stress induced on planes

AD and BC. Then for the equilibrium (τ . xz)y = τ' (yz)x

$$\boxed{\tau = \tau'}$$

Thus, every shear stress is accompanied by an equal complementary shear stress.

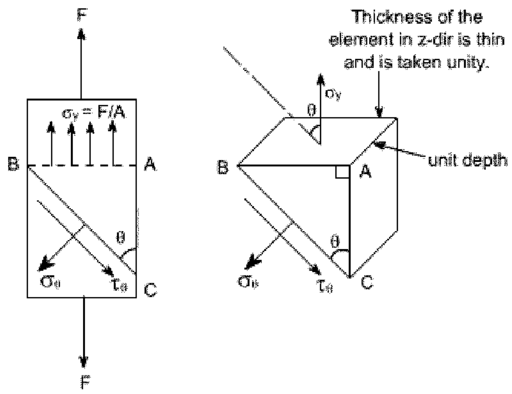
<u>Stresses on oblique plane:</u> Till now we have dealt with either pure normal direct stress or pure shear stress. In many instances, however both direct and shear stresses acts and the resultant stress across any section will be neither normal nor tangential to the plane.

A plane stse of stress is a 2 dimensional stae of stress in a sense that the stress components in one direction are all zero i.e

$\sigma_z = \tau_{yz} = \tau_{zx} = 0$

examples of plane state of stress includes plates and shells.

Consider the general case of a bar under direct load F giving rise to a stress σ_y vertically



The stress acting at a point is represented by the stresses acting on the faces of the element enclosing the point.

The stresses change with the inclination of the planes passing through that point i.e. the stress on the faces of the element vary as the angular position of the element changes.

Let the block be of unit depth now considering the equilibrium of forces on the triangle portion ABC

Resolving forces perpendicular to BC, gives

$\sigma_\theta.BC.1 = \sigma_y \sin\theta \,.\, AB \,.\, 1$

but $AB/BC = \sin\theta$ or $AB = BC\sin\theta$

Substituting this value in the above equation, we get

$\sigma_\theta.BC.1 = \sigma_y \sin\theta \,.\, BC\sin\theta \,.\, 1$ or $\boxed{\sigma_\theta = \sigma_y \,.\sin^2 2\theta}$ (1)

Now resolving the forces parallel to BC

$\tau_\theta.BC.1 = \sigma_y \cos\theta \,.\, AB\sin\theta \,.\, 1$

again $AB = BC\cos\theta$

$\tau_\theta.BC.1 = \sigma_y \cos\theta \,.\, BC\sin\theta \,.\, 1$ or $\tau_\theta = \sigma_y \sin\theta\cos\theta$

$$\boxed{\tau_\theta = \frac{1}{2}.\sigma_y.\sin 2\theta} \quad (2)$$

If $\theta = 90^0$ the BC will be parallel to AB and $\tau_\theta = 0$, i.e. there will be only direct stress or normal stress.

By examining the equations (1) and (2), the following conclusions may be drawn

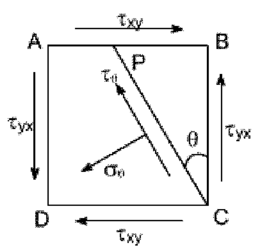
(i) The value of direct stress σ_θ is maximum and is equal to σ_y when $\theta = 90^0$.

(ii) The shear stress τ_θ has a maximum value of 0.5 σ_y when $\theta = 45^0$

(iii) The stresses σ_θ and σ_θ are not simply the resolution of σ_y

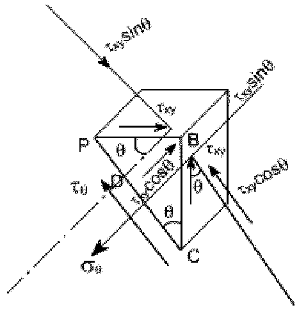
Material subjected to pure shear:

Consider the element shown to which shear stresses have been applied to the sides AB and DC



Complementary shear stresses of equal value but of opposite effect are then set up on the sides AD and BC in order to prevent the rotation of the element. Since the applied and complementary shear stresses are of equal value on the x and y planes. Therefore, they are both represented by the symbol τ_{xy}.

Now consider the equilibrium of portion of PBC



Assuming unit depth and resolving normal to PC or in the direction of σ_θ

$$\sigma_\theta.PC.1 = \tau_{xy}.PB.\cos\theta.1 + \tau_{xy}.BC.\sin\theta.1$$

$$= \tau_{xy}.PB.\cos\theta + \tau_{xy}.BC.\sin\theta$$

Now writing PB and BC in terms of PC so that it cancels out from the two sides

$$PB/PC = \sin\theta \quad BC/PC = \cos\theta$$

$$\sigma_\theta.PC.1 = \tau_{xy}.\cos\theta\sin\theta PC + \tau_{xy}.\cos\theta.\sin\theta PC$$

$$\sigma_\theta = 2\tau_{xy}\sin\theta\cos\theta$$

$$\sigma_\theta = \tau_{xy}.2.\sin\theta\cos\theta$$

$$\boxed{\sigma_\theta = \tau_{xy}.\sin 2\theta} \qquad (1)$$

Now resolving forces parallel to PC or in the direction τ_θ.then $\tau_{xy}PC . 1 = \tau_{xy} . PB\sin\theta - \tau_{xy} . BC\cos\theta$

–ve sign has been put because this component is in the same direction as that of τ_θ.

again converting the various quantities in terms of PC we have

$\tau_{xy}PC . 1 = \tau_{xy} . PB.\sin^2\theta - \tau_{xy} . PC\cos^2\theta$

$= -[\ \tau_{xy} (\cos^2\theta - \sin^2\theta)\ ]$

$= -\tau_{xy}\cos 2\theta$ or $\boxed{\tau_\theta = -\tau_{xy} \cos 2\theta}$ (2)

the negative sign means that the sense of τ_θ is opposite to that of assumed one. Let us examine the equations (1) and (2) respectively

From equation (1) i.e,

$\sigma_\theta = \tau_{xy} \sin 2\theta$

The equation (1) represents that the maximum value of σ_θ is τ_{xy} when $\theta = 45^0$.

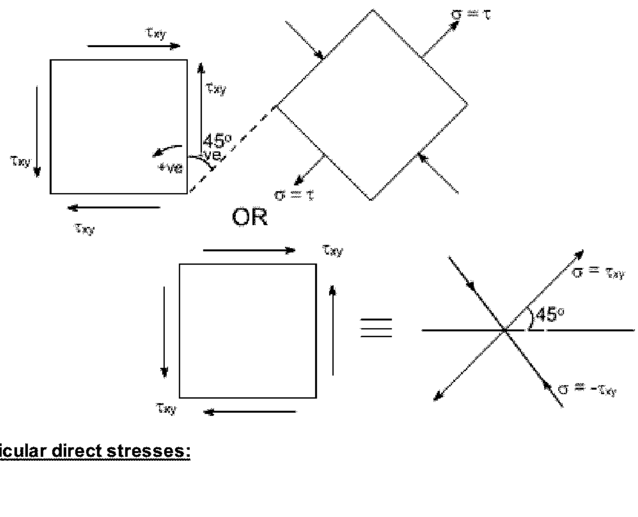
Let us take into consideration the equation (2) which states that

$\tau_\theta = -\tau_{xy} \cos 2\theta$

It indicates that the maximum value of τ_θ is τ_{xy} when $\theta = 0^0$ or 90^0. it has a value zero when $\theta = 45^0$.

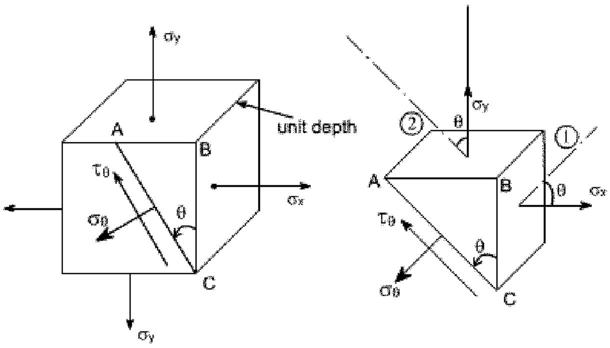
From equation (1) it may be noticed that the normal component σ_θ has maximum and minimum values of $+\tau_{xy}$ (tension) and $-\tau_{xy}$ (compression) on plane at $\pm 45^0$ to the applied shear and on these planes the tangential component τ_θ is zero.

Hence the system of pure shear stresses produces and equivalent direct stress system, one set compressive and one tensile each located at 45^0 to the original shear directions as depicted in the figure below:



Material subjected to two mutually perpendicular direct stresses:

Now consider a rectangular element of unit depth, subjected to a system of two direct stresses both tensile, σ_x and σ_y acting right angles to each other.



for equilibrium of the portion ABC, resolving perpendicular to AC

σ_θ . AC.1 = $\sigma_y \sin\theta$. AB.1 + $\sigma_x \cos\theta$. BC.1

converting AB and BC in terms of AC so that AC cancels out from the sides

$\sigma_\theta = \sigma_y \sin^2\theta + \sigma_x \cos^2\theta$

Futher, recalling that $\cos^2\theta - \sin^2\theta = \cos 2\theta$ or $(1 - \cos 2\theta)/2 = \sin^2\theta$

Similarly $(1 + \cos 2\theta)/2 = \cos^2 q$

Hence by these transformations the expression for σ_θ reduces to

$= 1/2\sigma_y (1 - \cos 2\theta) + 1/2\sigma_x (1 + \cos 2\theta)$

On rearranging the various terms we get

$$\sigma_\theta = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta \qquad (3)$$

Now resolving parallal to AC

s_q.AC.1= $-\tau_{xy}$..cosθ.AB.1+ τ_{xy}.BC.sinθ.1

The – ve sign appears because this component is in the same direction as that of AC.

Again converting the various quantities in terms of AC so that the AC cancels out from the two sides.

$$\tau_\theta .AC.1 = [\tau_x \cos\theta \sin\theta - \sigma_y \sin\theta \cos\theta] AC$$

$$\tau_\theta = (\sigma_x - \sigma_y)\sin\theta\cos\theta$$

$$= \frac{(\sigma_x - \sigma_y)}{2}\sin 2\theta \qquad (4)$$

or $$\tau_\theta = \frac{(\sigma_x - \sigma_y)}{2}\sin 2\theta$$

Conclusions :

The following conclusions may be drawn from equation (3) and (4)

(i) The maximum direct stress would be equal to σ_x or σ_y which ever is the greater, when $\theta = 0^0$ or 90^0

(ii) The maximum shear stress in the plane of the applied stresses occurs when $\theta = 45^0$

$$\tau_{max} = \frac{(\sigma_x - \sigma_y)}{2}$$

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