

Water & Wastewater Engineering

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Pipe Network Analysis

Analysis of water distribution system includes determining quantities of flow and head losses in the various pipe lines, and resulting residual pressures. In any pipe network, the following two conditions must be satisfied:

1. The algebraic sum of pressure drops around a closed loop must be zero, i.e. there can be no discontinuity in pressure.
2. The flow entering a junction must be equal to the flow leaving that junction; i.e. the law of continuity must be satisfied.

Based on these two basic principles, the pipe networks are generally solved by the methods of successive approximation. The widely used method of pipe network analysis is the Hardy-Cross method.

Hardy-Cross Method

This method consists of assuming a distribution of flow in the network in such a way that the principle of continuity is satisfied at each junction. A correction to these assumed flows is then computed successively for each pipe loop in the network, until the correction is reduced to an acceptable magnitude.

If Q_a is the assumed flow and Q is the actual flow in the pipe, then the correction δ is given by

$$\delta = Q - Q_a; \text{ or } Q = Q_a + \delta$$

Now, expressing the head loss (H_L) as

$$H_L = K \cdot Q^x$$

we have, the head loss in a pipe

$$= K \cdot (Q_a + \delta)^x$$

$$= K \cdot [Q_a^x + x \cdot Q_a^{x-1} \delta + \dots \dots \dots \text{negligible terms}]$$

$$= K \cdot [Q_a^x + x \cdot Q_a^{x-1} \delta]$$

Now, around a closed loop, the summation of head losses must be zero.

$$\therefore \Sigma K \cdot [Q_a^x + x \cdot Q_a^{x-1} \delta] = 0$$

$$\text{or } \Sigma K \cdot Q_a^x = - \Sigma K x Q_a^{x-1} \delta$$

Since, δ is the same for all the pipes of the considered loop, it can be taken out of the summation.

$$\therefore \Sigma K \cdot Q_a^x = - \delta \cdot \Sigma K x Q_a^{x-1}$$

$$\text{or } \delta = - \Sigma K \cdot Q_a^x / \Sigma x \cdot K Q_a^{x-1}$$

Since δ is given the same sign (direction) in all pipes of the loop, the denominator of the above equation is taken as the absolute

sum of the individual items in the summation. Hence,

$$\text{or } \delta = -\Sigma K \cdot Q_a^x / \Sigma |x \cdot K Q_a^{x-1}|$$

$$\text{or } \delta = -\Sigma H_L / x \cdot \Sigma |H_L / Q_a|$$

where H_L is the head loss for assumed flow Q_a .

The numerator in the above equation is the algebraic sum of the head losses in the various pipes of the closed loop computed with assumed flow. Since the direction and magnitude of flow in these pipes is already assumed, their respective head losses with due regard to sign can be easily calculated after assuming their diameters. The absolute sum of respective KQ_a^{x-1} or H_L/Q_a is then calculated. Finally the value of δ is found out for each loop, and the assumed flows are corrected. Repeated adjustments are made until the desired accuracy is obtained.

The value of x in Hardy- Cross method is assumed to be constant (i.e. 1.85 for Hazen-William's formula, and 2 for Darcy-Weisbach formula)

[Worked-out Example](#)

