UNIT–2
PRINCIPLES OF LIMIT STATE DESIGN AND ULTIMATE STRENGTH OF R.C. SECTION:

2.1 Introduction:

A beam experiences flexural stresses and shear stresses. It deforms and cracks are developed. ARC beam should have perfect bond between concrete and steel for composites action. It is primarily designed as flexural member and then checked for other parameters like shear, bond , deflection etc. In reinforced concrete beams, in addition to the effects of shrinkage, creep and loading history, cracks developed in tension zone effects its behavior. Elastic design method (WSM) do not give a clear indication of their potential strengths. Several investigators have published behavior of RC members at ultimate load. Ultimate strength design for beams was introduced into both the American and British code in 1950's. The Indian code ES456 introduced the ultimate state method of design in 1964. Considering both probability concept and ultimate load called as "Limit state method of design" was introduced in Indian code from 1978.

2.2 Behavior of Reinforced concrete beam

To understand the behavior of beam under transverse loading, a simply supported beam subjected to two point loading as shown in Fig. 2.1 is considered. This beam is of rectangular cross-section and reinforced at bottom.

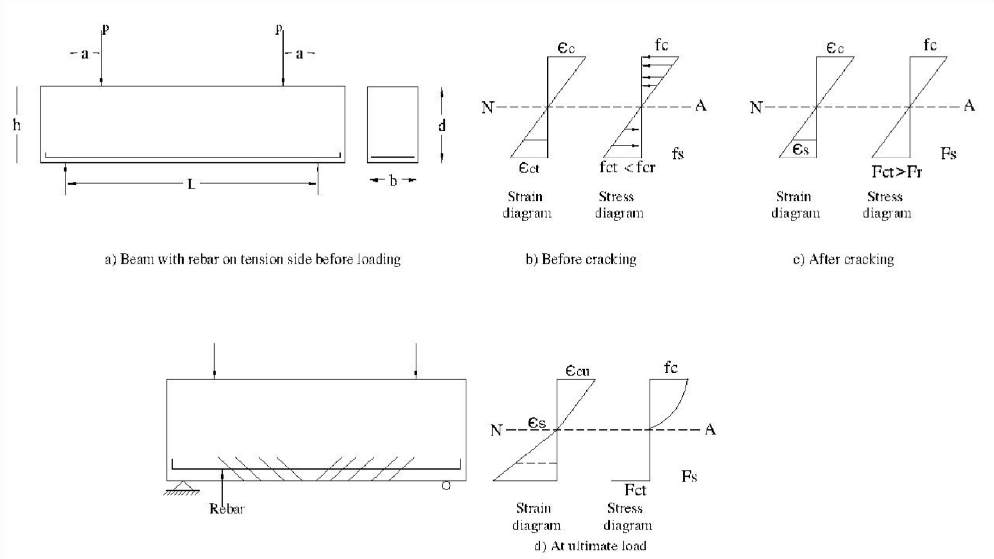

Fig 2.1 Behaviour of RC beam

When the load is gradually increased from zero to the ultimate load value, several stages of behavior can be observed. At low loads where maximum tensile stress is less than modulus of rupture of concrete, the entire concrete is effective in resisting both compressive stress and tensile stress. At this stage, due to bonding tensile stress is also induced in steel bars.

With increase in load, the tensile strength of concrete exceeds the modulus of rupture of concrete and concrete cracks. Cracks propagate quickly upward with increase in loading up ;to neutral axis. Strain and stress distribution across the depth is shown in Fig 4.1c. Width of crack is small. Tensile stresses developed are absorbed by steel bars. Stress and strain are proportional till fc< $\frac{fcn}{2}$. Further increase in load, increases strain and stress in the section and are no longer proportional. The distribution of stress – strain curve of concrete. Fig 41d shows the stress distribution at ultimate load.

Failure of beam depends on the amount of steel present in tension side. When moderate amount of steel is present, stress in steel reaches its yielding value and stretches a large amount whth tension crack in concrete widens. Cracks in concrete propagate upward with increases in deflection of beam. This induces crushing of concrete in compression zone and called as “secondary compression failure”. This failure is gradual and is preceded by visible signs of distress. Such sections are called “under reinforced” sections.

When the amount of steel bar is large or very high strength steel is used, compressive stress in concrete reaches its ultimate value before steel yields. Concrete fails by crushing and failure is sudden. This failure is almost explosive and occur without warning. Such reactions are called “over reinforced section”

If the amount of steel bar is such that compressive stress in concrete and tensile stress in steel reaches their ultimate value simultaneously, then such reactions are called “Balanced Section”.

The beams are generally reinforced in the tension zone. Such beams are termed as “singly reinforced” section. Some times rebars are also provided in compression zone in addition to tension rebars to enhance the resistance capacity, then such sections are called “Doubly reinforce section.

2.3 Assumptions

Following assumptions are made in analysis of members under flexure in limit state method

1. Plane sections normal to axis remain plane after bending. This implies that strain is proportional to the distance from neutral axis.
2. Maximum strain in concrete of compression zone at failure is 0.0035 in bending.

3. Tensile strength of concrete is ignored.
4. The stress-strain curve for the concrete in compression may be assumed to be rectangle, trapezium, parabola or any other shape which results in prediction of strength in substantial agreement with test results. Design curve given in IS456-2000 is shown in Fig. 2.2

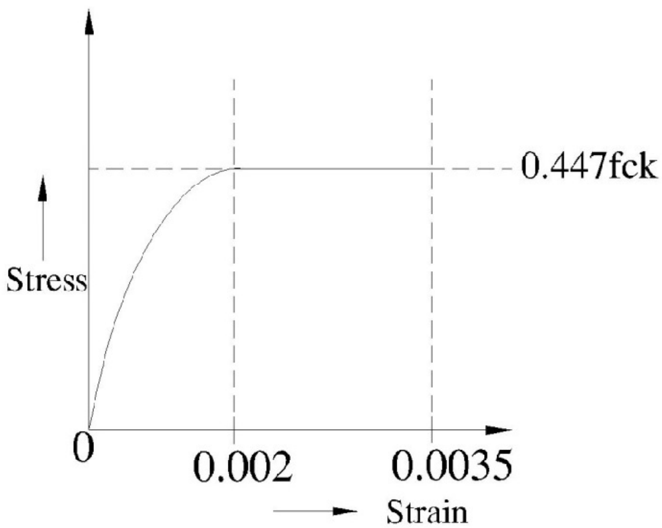

Fig 2.2 Stress-Strain Curve for Concrete

5. Stress – strain curve for steel bar with definite yield print and for cold worked deformed bars is shown in Fig 2.3 and Fig 2.4 respectively.

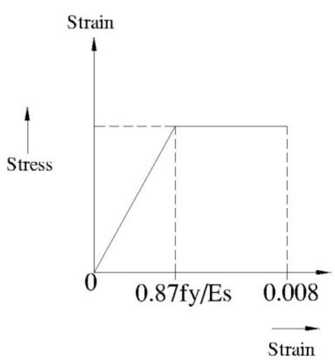

Fig 2.3 stress-strain curve for steel bar with defective yield point

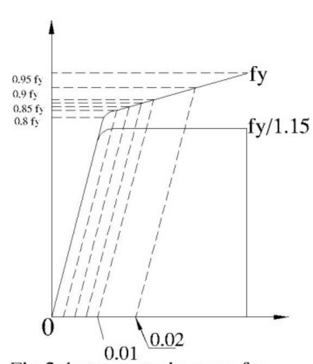

Fig 2.4 stress-strain curve for cold worked deformed bars

6. To ensure ductility, maximum strain in tension reinforcement shall not be less than $\frac{fy}{1.15Es} + \ 0.002.$

7. Perfect bond between concrete and steel exists.

2.4. Analysis of singly reinforced rectangular sections

Consider a rectangular section of dimension b x h reinforced with A_{st} amount of steel on tension side with effective cover Ce from tension extreme fiber to C.G of steel. Then effective depth d=h-ce, measured from extreme compression fiber to C.G of steel strain and stress distribution across the section is shown in Fig.2.4. The stress distribution is called stress block.

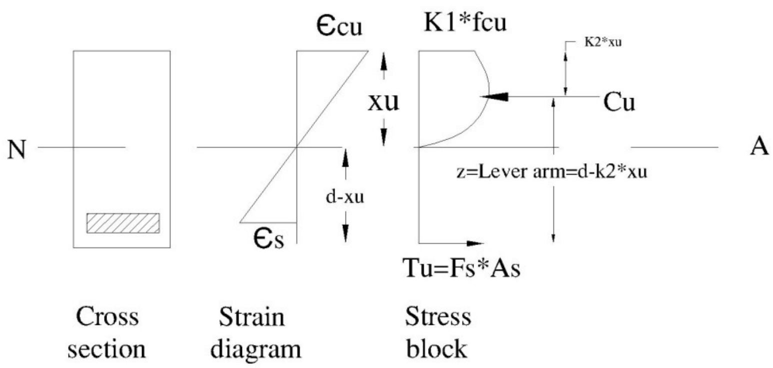

Fig 2.5 Stress Block

From similar triangle properly applied to strain diagram

$$\frac{\varepsilon cu}{xu} = \frac{\varepsilon s}{d - xu} \rightarrow (1)$$

$$\in s = \in cu \ \times \frac{d - xu}{xu} \rightarrow (2)$$

For the known value of x4 & 6cu the strain in steel is used to get the value of stress in steel from stress-strain diagram. Equation 4.4-1 can be used to get the value of neutral axis depth as

$$xu = \frac{\in cu}{\in s} \times (d - xu) = \frac{\in cu}{\in s} \times d - \frac{\in cu}{\in s} \times xu$$

$$xu\left(1+\frac{\in cu}{\in s}\right)=\frac{\in cu}{\in s}\times d$$

$$xu\left(\frac{\in s+\in cu}{\in s}\right)=\frac{\in cu}{\in s}\times d$$

$$\therefore xu=\frac{\in cu}{\in cu+\in s}\times d-(3)$$

Here $\frac{\in cu}{\in cu+\in s}$ is called neutral axis factor

For equilibrium Cu = Tu.

$K_1, k_3\, f_{cu}\, b\, x_u = f_s A_s$

$$\therefore fs=\frac{K1,k3fcubxu}{As}=\frac{k1k3fcub}{As}\times\frac{\in cu}{\in cu+\in s}\times d$$

$$fs = k1k3fcu\times\frac{Ecu}{Ecu+Es}\times\frac{bd}{As}\ Let\ p=steel\ raio=\frac{As}{bd}$$

$$\therefore fs=\frac{k1k3fcu}{p}\times\frac{Ecu}{Ecn+Es}\ or\ \frac{Ecu}{Ecu+Es}=\frac{fsp}{k1k3fcu}-(4)$$

Value of fs can be graphically computed for a given value of P as shown in Fig2.6

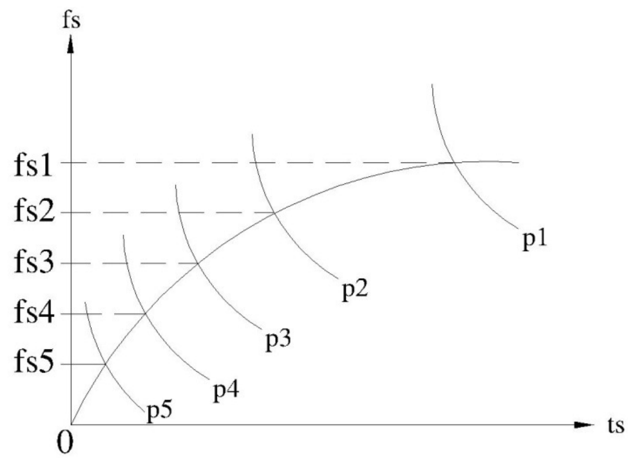

Fig 2.6

After getting fs graphically, the ultimate moment or ultimate moment of resistance is calculated as

Mu = T_u x Z = f_sA_s(d-k_2 xu)

Mu= Cu x Z = $k_1k_2f_{cu}bx_u$ x (d-$k_2$xu)

Consider

Mu = f_sA_s(d-k2 x $\frac{\epsilon cu}{\epsilon cu+\epsilon s}$ d) = fsAsd(1-kz $\frac{\epsilon cu}{\epsilon cu+\epsilon s}$)

From (4) $$\frac{Ecu}{Ecu+Es} = \frac{f_s\,p}{k1k3fcm}$$

$$\therefore\ Mu = fsAsd\left(1 - \frac{k2fsp}{k1k3fcu}\right) - \ (5)$$

Here the term $1 - \frac{k2fsp}{k1k3fcu}$ is called lever arm factor

Using As=pbd in (5), the ultimate moment of resistance is computed as

$$Mu = fs(pbd)d\left(1 - \frac{k2fsp}{k1k3fcu}\right) Let\ R = (1 - \frac{fsp}{fcu} \times \frac{k2}{k1k3})$$

$\frac{Mu}{bd^2} = pfs\ x\ R$ Dividing both sides by fcu we get or

$$\frac{Mu}{bd^2} = p\ \times \frac{fs}{fcu} \times R - (6)$$

A graph plotted between $\frac{Mu}{fcubd^2}$ as shown in fig 2.7 and can be used for design

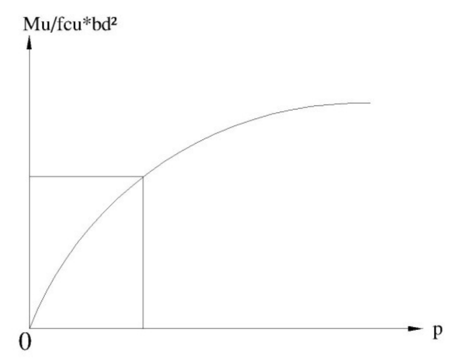

Fig 2.7

2.5 Stress Blocks

Stress blocks adopted by different codes are based on the stress blocks proposed by different investigators. Among them that proposed by Hog nested and Whitney equivalent rectangular block are used by most of the codes.

2.5.1 Stress block of IS456 – 2000

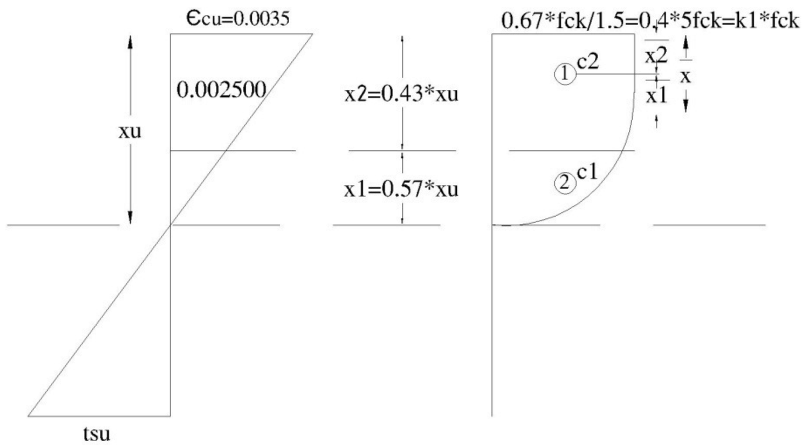

Fig 2.8

Stress block of IS456-2000 is shown in Fig 2.8. Code recommends ultimate strain ε_{cu}=0.0035 & strain at which the stress reaches design strength ε_0=0.002. Using similar triangle properties on strain diagram

$$\frac{0.0035}{xu} = \frac{0.002}{x1}$$

$$\therefore x_1 = 0.57xu \rightarrow (7)$$

and $x_2 = x_u - 0.57x_u = 0.43x_u$

Area of stress block is $A = A_1 + A_2$.

$$A = \frac{2}{3} \times 0.45fck \, * 0.57xu + 0.45fck \, \times 0.43xu$$

$= 0.171\ f_{ck}x_u + 0.1935\ f_{ck}x_u$.

$A = 0.3645\ f_{ck}x_u \rightarrow (8)$

Depth of neutral axis of stress block is obtained by taking moment of areas about extreme compression fiber.

$$\therefore \bar{x} = \frac{\sum aixi}{\sum ai}$$

$$\bar{x} = \frac{0.171fckxu\left(\frac{3}{8} \times 0.57xu + 0.43xu\right) + 0.1935fckxu \times \frac{0.43xu}{2}}{0.36fckxu}$$

$$\bar{x} = 0.42xu - (9)$$

The stress block parameters thus are

$$\left.\begin{aligned} K_1 &= 0.45 \\ K_2 &= 0.42 \\ K_3 &= \frac{0.3645}{0.45} = 0.81 \end{aligned}\right\} (10)$$

4.5.5 Analysis of rectangular beam using IS456-2000 stress block

Case 1: Balanced section

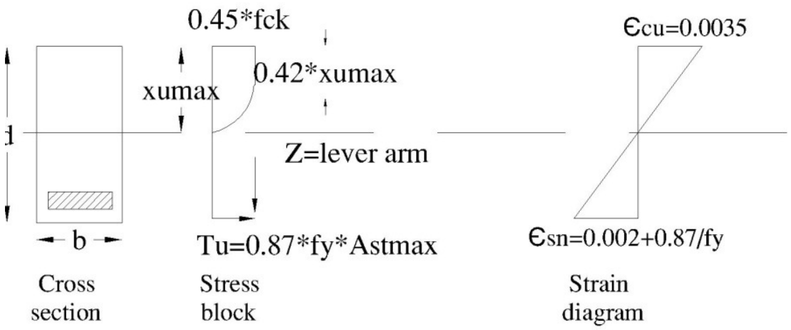

Fig 2.9

Balanced section is considered when the ultimate strain in concrete and in steel are reached simultaneously before collapse.

For equilibrium Cu=Tu

$\therefore$ $0.36 f_{ck} x_{umax}\, b = 0.87 f_y\, Ast_{\underline{max}}$.

$x_{umax} = \frac{0.87fy}{0.36fck}\,\frac{Ast_{max}}{b}$ dividing both sides by

$\frac{xumax}{d} = \frac{0.87fy}{0.36fck}\,\frac{Astmax}{bd}$ but $\frac{Ast\ max}{bd}$ = pt_{masc}.

$$\left.\begin{aligned} \therefore pt_{max} &= \left(\frac{xumax}{d}\right) \times \frac{0.36fck}{0.87fy} \\ pt_{max} &= \frac{xumax}{d} \times 0.414\,\frac{fck}{fu} - \end{aligned}\right\} (11)$$

From strain diagram

$$\frac{0.0035}{xumac} = \frac{0.002 + \frac{0.87fy}{Es}}{xumax - d}$$

$$\frac{xumax}{d} = \frac{0.0035}{\frac{0.87fy}{Es} + \ 0.0055} - (12)$$

Values of $\frac{xumax}{d}$ is obtained from equation (12). This value depends on grade of steel. Based on grade of steel this value is given in note of clause 38.1 as (pp70)

Fy	Xumax/d
250	0.53
415	0.48
500	0.46(0.456)

p_{tmax} given in equation (11) is called limiting percentage steel and denoted as pt lim.

To find moment of resistance, the internal moment of Cu & Tu is computed as

Mulim = Cu x Z = 0.36f_{ck} x_{ulim} b (d-0.42 x_{ulim})

From equation (11) $\frac{xumax}{d} = 2.42\ \frac{fy}{fck}$ p_{tmax}

Mulim = Tu x Z = 0.87fyAst [d-0.42xulim]

Mulim = 0.87fyAst[d-0.42 x 2.42 $\frac{fy}{fck} pt_{max} d$]

= 0.87fyAst [1- $\frac{fy}{fck} ptmax$]

$$\frac{Mulim}{fckbd^2} = 0.87\ \frac{fy}{fck}\ \frac{Ast}{bd}\ (1 - \frac{fy}{fck}\ ptlim)$$

$$\frac{Mulim}{fckbd^2} = 0.87\ \frac{fy}{fck}\ ptlim\left(1 - \frac{fy}{fck}\ ptlim\right) - \ (13)$$

From equation 4.5-5-2 ptlim can be expressed as

$$\frac{ptlimfy}{fck} = 0.414\ \frac{xumax}{d} \dashrightarrow (14)$$

Values of $\frac{mulim}{fckbd_2}$ & $\frac{ptlimfy}{fck}$

For different grade of Steel is given in Table (page 10 of SP -16. This table is reproduced in table 2.1.

Table 2.1 Limiting Moment resistance & limiting steel

Fy	250	415	500
$\frac{mulim}{fckbd_2}$	0.149	0.138	0.133
$\frac{ptlim\,fy}{fck}$	21.97	19.82	18.87

Where p_{tlim} is in%

Now considering $M_{ulim} = C_u$ x Z.

Mulim = $0.36 f_{ck} x_{ulim}$ b x (d-$0.42 x_{ulim}$)

$$\frac{Mulim}{fckbd^2} = 0.36 \times \frac{xulim}{d}\left[1 - 0.42\frac{xulim}{d}\right] - 15$$

Value of $\frac{Mulim}{fckbd_2}$ is abailalbe in table C of SP16 & Value of $\frac{Mulim}{bd_2}$ for different grade of concrete and steel is given in Tables. Value of pt lim for different grade of concrete and steel is given in table E of SP –'6'. Term $\frac{Mulim}{bd_2}$ is termed as limiting moment of resistance factor and denoted as Qlim

∴ Mulim = $Q_{lim}bd^2$.

Case 2: Under reinforced section

In under reinforced section, the tensile strain in steel attains its limiting value first and at this stage the strain in extreme compressive fiber of concern is less than limiting strain as shown in Fig 2.10

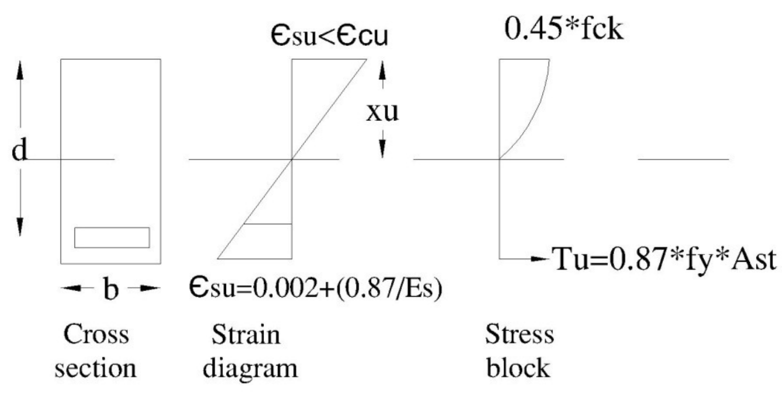

Fig 2.10

The neutral axis depth is obtained from equilibrium condition Cu=Tu

$\therefore$ $0.36 f_{ck} x_{ub}$ = 0.87fyAst

xu = $\frac{0.87 fyAst}{0.36 fck\, b} = 2.41\ \frac{fy}{fck}\ \frac{Ast}{b}$

or $\frac{xu}{d} = 2.41\ \frac{fy}{fck}\ \frac{Ast}{bd} -\ (16)$

Moment of resistance is calculated considering ultimate tensile strength of steel $\therefore$ $M_{uR} = T_u$ x Z or M_{ur} = 0.87fy Ast X (d-0.42xu)

= 0.87ftAst d(1-0.42 $\frac{xu}{d}$)

= 0.87fyAst d (1-0.42 x 2.41 $\frac{fy}{fck}\ \frac{Ast}{bd}$)

Considering p_t = 100 $\frac{Ast}{bd}$expressed as % we get

M_{uR} = 0.87fyAst d(1-1.0122 $\frac{fy}{fck}$ ($\frac{pt}{100}$))

Or $\frac{MuR}{0.87\ fybd^2} = \ \frac{Ast}{bd}$ (1-1.0122 $\frac{fy}{fck}$ ($\frac{pt}{100}$) , taking 1.0122 ≈ 1

$$\frac{MuR}{0.87\ fybd^2} = \left(\frac{pt}{100}\right) - \frac{fy}{fck}\left(\frac{pt}{100}\right)^2$$

Or $\frac{fy}{fck}\ \left(\frac{pt}{100}\right)^2 - \frac{pt}{100} + \frac{MuR}{0.87\ fybd^2} = 0 - (17)$

Equation (17) is quadratic equation in terms of (pt/100)

Solving for pt, the value of pt can be obtained as

P_t = 50 $\left[\frac{1-\sqrt{1-\frac{4.6MuR}{fckbd^2}}}{fy/fck}\right]$

Pt = 50 $\frac{fck}{fy}\left[1 - \sqrt{1 - \frac{4.6MuR}{fckbd^2}}\right] -\ (18)$

Let Ru = $\frac{4.6MuR}{fckbd^2}$ $then$

Pt = 50 $\frac{fck}{fy}\left[1-\sqrt{1-R_{\mathrm{u}}}\right]$

Case 3: Over reinforced section

In over reinforced section, strain in extreme concrete fiber reaches its ultimate value. Such section fail suddenly hence code does not recommend to design over reinforced section.

Depth of neutral axis is computed using equation 4.5-6. Moment of resistance is calculated using concrete strength.

$\therefore$ M_{uR} = Cu x Z

= 0.36 fck xub (d-0.42xu) – 19

$$\frac{xu}{d} > \frac{xulim}{d}$$

Position of neutral axis of 3 cases is compared in Fig. 2.11

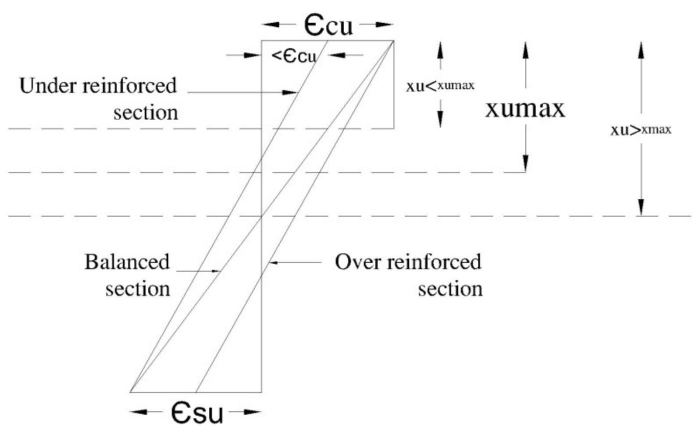

Fig 2.11

Worked Examples

1. Determine MR of a rectangular section reinforced with a steel of area 600mm² on the tension side. The width of the beam is 200mm, effective depth 600mm. The grade of concrete is M20 & Fe250 grade steel is used.

   Solve: f_{ck} = 20Mpa fy = 250Mpa, A_{st} = 600mm²

   Step . 1 To find depth of NA

   $$\frac{xu}{d} = 2.41\frac{fy}{f_{ck}}\frac{A_{st}}{bd}$$

   $$x_u = \left(2.41 \times \frac{250}{250} \times \frac{600}{200 \times 600}\right)600 = 90.375\text{mm}$$

   Step 2 Classification

   From clause 38.1, page 70 of IS456,

   For Fe 250 $\frac{x_{ulim}}{d} = 0.53$, x_{ulim} = 0.53 X 600 = 318mm

   $x_u < x_{ulim}$. Hence the section is under reinforced.

   Step . 3 MR for under reinforced section.

   $$MR = 0.87f_y A_{st} (d-0.42x_u) \qquad \frac{N-mm}{1000 \times 1000}$$

   $$= \frac{0.87 \times 250 \times 600\ (600-0.42 \times 90.375)}{10^6}$$

   =73.36kN-m.

2. Determine the MR of a rectangular section of dimension 230mm X 300mm with a clear cover of 25mm to tension reinforcement. The tension reinforcement consists of 3 bars of 20mm dia bars. Assume M20 grade concrete & Fe 415 steel.

   If cover is not given, refer code – 456 → page 47

   1 inch = 25mm → normal construction

   Effective depth, d= 300-(25+10)

   = 265mm.

   $A_{st} = 3\frac{\pi}{4} \times 20^2$ = 942.48mm².

(1) To find the depth of N.A

$$x_u = 2.41 \times \frac{fy}{f_{ck}} \times \frac{A_{st}}{bd} \times d$$

$$= 2.41 \times \frac{415}{25} \times \frac{603.18}{230} = 104.916mm$$

ii) For Fe415, $\frac{x_{ulim}}{d} = 0.48$

$x_{ulim} = 0.48 \times 442 = 212.16mm$

$x_u < x_{ulim}$

The section is under reinforced.

iii) $M_R = 0.87 f_y A_{st} (d - 0.42 x_u)$

$= 0.87 \times 415 \times 603.18 (442 - 0.42 \times 104.916)$

$= 86.66$ kN-m.

3. Find MR of the section with the following details.

Width of section: 230mm

Overall depth of section: 500mm

Tensile steel: 3 bars of 16mm dia

Grade of concrete: M25

Type of steel : Fe 415

Environmental condition: severe

Solve: b= 230, h= 500mm, $f_{ck} = 25$, $f_y = 415$

From table 16(page 47, IS 456-2000)

Min clear cover (CC) = 45mm

Assume CC 50mm.

Effective depth = 500-(50+8) = 442mm

$A_{st} = 3 \times \frac{\pi}{4} \times 16^2 = 603.18mm^2$

i) To find the depth of N-A, $x_u = 2.41 \frac{415}{20} \times \frac{942.47}{230 \times 230} = 204.915mm$

ii) For Fe 415, $\frac{x_{ulim}}{d} = 0.48$

$x_{ulim} = 0.48 \times 300 = 144mm.$

$x_u > x_{ulim}$ $\therefore$ over reinforced.

(1) $M_R = (0.36 f_{ck} x_u b)(d - 0.42 x_u)$.

$= 60.71$kN-m.

4. A R – C beam 250mm breadth & 500mm effective depth is provided with 3 nos. of 20mm dia bars on the tension side, assuming M20 concrete & Fe 415 steel, calculate the following:

(i) N-A depth (ii) compressive force (iii) Tensile force (iv) ultimate moment (v) safe concentrated load at mid span over an effective span of 6m.

Solve: d=500mm, b=250mm

$A_{st} = 3 \text{ X } \frac{\pi}{4} \text{X} 20^2 = 942.48\text{mm}^2$

$f_{ck} = 20\text{Mpa}$ $f_y = 415\text{Mpa}$.

Step – 1

$$\frac{x_u}{d} = 2.41 \times \frac{fy}{f_{ck}} \times \frac{A_{st}}{bd}$$

$$= 2{,}41 \times \frac{415}{20} \times \frac{942.48}{250 \times 250} \times 500$$

$x_u = 188.52\text{mm}$

Step – 2: For Fe 415 $\frac{x_{ulim}}{d} = 0.48$; $x_{ulim} = 0.48 \text{X } 500$

$= 240\text{mm}$

$\therefore$ $x_u < x_{ulim}$, the section is under reinforced.

$C_u = 0.36 f_{ck} x_u b = 0.36\text{X}20\text{X}188.52 \text{ X } 250/10^3 = 339.34\text{kN}$.

Step – 3 MR for under reinforced section is

$M_u = MR = 0.87 f_y A_{st} (d - 0.42 x_u)$

$$= \frac{0.87 \times 415 \times 942.48\ (500 - 0.42 \times 188.52)}{10^6}$$

$= 143.1\text{kN-m}$.

Step – 4

$$M_u = \frac{w_u \times L}{4} = \frac{w_u \times 6}{4}$$

Equating factored moment to MR

$$\frac{W_u \times 6}{4} = 143.1$$

$W_u = 95.5\text{KN}$.

Safe load, $W = \frac{W_u}{1.5}$ load factor/factor of safety

$= 63.67\text{kN}$

Step .2 $T_u = 0.87 f_y A_{st} = \frac{0.87 \times 415 \times 942.48}{10^3}$ = 340.28kN.

$C_u \approx T_u$.

5. In the previous problem, determine 2 point load value to be carried in addition to its self weight, take the distance of point load as 1m.

Solve: Allowable moment, $\frac{Mu}{1.5} = \frac{143.1}{1.5} = 95.4\ KN - m$

Considering self weight & the external load,

M=MD+ML: MD = dead load moment, ML = live load moment, qd = self weight of beam = volume X density: density = 25kN/m³ for R C C IS 875 –part – 1, plain concrete = 29kN/m³

Volume = b X h X 1m

Let CC= 25mm, Ce = 25 + $\frac{20}{2}$ = 3

H= 500+35 = 535mm

$$q_d = \frac{250 \times 535}{1000^2} \times 1 \times 25 = 3.34 KN/m.$$

$$M_D = \frac{q_d \times l^2}{8} = \frac{3.34 \times 6^2}{8} = 15.03 KN - m.$$

M= 95.4= 15.03 + ML

ML = 80.37 kN-m

80.37 = WLX1

WL = 80.37kN

6. A singly reinforced beam 200mm X 600mm is reinforced with 4 bars of 16mm dia with an effective cover of 50mm. effective span is 4m. Assuming M20 concrete & Fe 215 steel, let the central can load p that can be carried by the beam in addition to its self weight max 5m $= \frac{W.L}{4}$,

Solve: $A_{st} = 4 \times \frac{\pi}{4} \times 16^2$ = 804.25mm²

B=200mm, d=550mm, h=600mm, fck = 20Mpa, fy = 250Mpa.

Step (1) $\frac{x_u}{d} = 2.41 \frac{fy}{fck} \frac{A_{st}}{bd}$

$x_u = 2.41 \times \frac{250}{20} \times \frac{804.25}{200}$

=121.14mm

Step (2)

$\frac{x_{ulim}}{d}$ for Fe 250 us 0.53

$\therefore$ $x_u$lim = 0.53 X 550 = 291.5mm

x_u < $x_u$lim .'. section is under reinforced.

Step – 3

M_u= MR = $0.87f_y A_{st}[d-0.42x_u]$

$$\frac{0.87 \times 250 \times 804.25\,[550 - 0.42(121.14)]}{10^6}$$

M_u=87.308kNm.

M= Allowable moment = $\frac{M_u}{1.5} = \frac{87.308}{1.5}$ = 58.20kN-m

M=MD+ML

q_d=self weight of beam

q_d = volume X density

density = 25kN/m³ for R C C

Volume = b X h X 2 = 200 X 600 X 2 = 2,40,000m³

q_d= 0.2 X 0.6 X 1 X 25 = 3kN/m

$q_d = \frac{240000 \times 25}{(1000)^2}$

q_d= 6KN/m

Md = $\frac{q_d l^2}{8}$

$$= \frac{3 \times 4^2}{8}$$

Md = 6kN.m

M= 58.2 = 6 + ML

ML= 52.2kN-m

ML= $\frac{p.l}{4}$

P=$\frac{52.2 \times 4}{4}$

P=52.2kN.

7. Determine N-A depth & MR of a rectangular beam of section 300mm X 600mm. The beam is reinforced with 4 bars of 25mm having an effective cover of 50mm, assume grade of concrete & steel as M20 & Fe 415 respectively

Solve: b=300mm, h=600mm, d=550mm.

f_{ck} = 20Mpa fy=415Mpa, A_{st}= 1963mm²

Step- 1 Neutral axis depth

$$\frac{x_u}{d} = 2.41 \frac{fy}{fck} \frac{A_{st}}{b} = \frac{2.41 \times 415 \times 1963}{20 \times 300}$$

x_u = 327.21mm

<u>Step – 2</u> <u>Classification</u>

$\frac{x_{ulim}}{d}$= 0.48 → x_{ulim} = 0.48X550 = 264mm

$x_u > x_{ulim}$. Hence the section is over – reinforced.

<u>NOTE</u>: Whenever the section is over reinforced, the strain in steel is less than the ultimate strain (0.002 + $\frac{0.87fy}{Es}$). Hence actual N_A depth has to be computed, by trial and error concept because in the above equation of x_u, we have assumed the stress in steel as yield stress & this is not true.

Step – 3   Actual N-A depth (which lies b/w x_u = 327 mm & xulim = 264mm)

Trial 1: Let $x_u = \frac{264+327}{2}$ = 295.5mm

$$\frac{\epsilon cu}{x_u} = \frac{\epsilon_s}{500 - x_u}$$

$$\epsilon_s = \frac{0.0035}{295.5}(500 - 295.5)$$

= 0.00303

For HYSD bar, the stress strain curve is given in the code, however for definite strain values, the stress is given in SP – 16

$$\frac{360.9 - 351.8}{0.0038 - 0.00276} = \frac{y^1}{(0.00303 - 0.002 + 6)} \quad y^1 = 2.36$$

f_s=351.8+y^1 = 351.8 + 2.36 = 3.54.16Mpa

Equating compressive force to tensile force,

$C_u = T_u$

0.36fckx_ub= $f_s A_{st}$

$x_u = \frac{354.16 \times 1963}{0.36 \times 20 \times 300}$ = 321.86mm

Compared to the earlier computation, this value is less than 327. However to confirm we have to repeat the above procedure till consecutive values are almost same.

Trial 2 Let $x_u = \frac{295+321.8}{2} \approx$ 308mm $\frac{T-2}{x_u}$ = 317.7

Repeat the computation as in trial 1.

$$\frac{\epsilon_{cu}}{x_u} = \frac{\epsilon_u}{550 - x_u}$$

$$\epsilon_s = \frac{0.0035}{308}(550 - 308)$$

For HYSD bars, the stress strain curve is given in the code, however for definite strain values, the stress is given in SP-16.

$$\frac{351.8-342.8}{0.00276-0.00241}=\frac{y^1}{0.00275-0.00241}$$

$$y^1=8.74$$

fs= 342.8 + 8.74 = 351.54 Mpa

Equating compressive forces to tensile forces,

$C_u=T_u$

$0.36f_{ck}b\ xu = f_s\ A_{st}$

0.36 X 20 X 300 X x_u =351.54 X 1963.

x_u=319.48mm

Compared to earlier computation this value is lesser than 321.8. However to confirm we have to repeat the above procedure till consecutive values are almost same.

Trial 3 Let $x_u = \frac{308+319.48}{2} \approx$ 313.74mm

$$\frac{\epsilon_{cu}}{x_u}=\frac{\epsilon_u}{550-x_u}$$

$$\epsilon_s=\frac{0.0035}{313.74}(550-313.74)$$

= 0.00263

For HYSD bars, the stress strain curve is given in the code, however for definite strain values, the stress is given in SP-16.

$$fs=\frac{351.8-342.8}{0.00276-0.00241}(0.00263-0.00241)+342.8$$

$$fs=348.46$$

Equating compressive forces to tensile forces,

$C_u=T_u$

$0.36f_{ck}xub = f_s\ A_{st}$

$0.36 \times 20 \times 300 \times x_u = 348.46 \times 1963.$

$x_u = 316.7mm$

Step – 4   MR

Dia (mm)	Area
8	50
10	78.5
12	113
16	201
20	314
25	490

$MUR = 0.36 f_{ck} x_u b(d-0.92x_u)$

$$= \frac{0.36 \times 20 \times 300 \times 375(550 - 0.42 \times 315)}{1 \times 10^6}$$

=284.2kN-m

A rectangular beam 20cm wide & 40cm deep up to the center of reinforcement. Find the reinforcement required if it has to resist a moment of 40kN-m. Assume M20 concrete & Fe 415 steel.

NOTE: When ever the loading value or moment value is not mentioned as factored load, assume then to be working value. (unfactored).

Solve: b=200mm, d=400mm, fck=20Mpa, fy = 415Mpa

$M = 40kN\text{-}m$, $M_u = 1.5 \times 40 = 60kN\text{-}m = 60 \times 10^6$

$M_u = MuR = 0.87 f_y \times A_{st} \times (d-0.42x_u) \rightarrow$ ①

$$\frac{x_u}{d} = 2.41 \frac{fy}{f_{ck}} \frac{A_{st}}{bd}$$

$$x_u = \frac{2.41 \times 415 \times A_{st}}{20 \times 200} = 0.25A_{st}$$

Substituting in 1.

$60 \times 10^6 = 0.87 \times 415 \times Ast (400 - 0.42 \times 0.25 \times Ast)$

$60 \times 10^6 = 1,44,420\ Ast - 37.91\ Ast^2$

$Ast = 474.57mm^2$ [take lower value → under reinforced]

- In beams, dia of reinforcement is taken above 12.
  Provide 2-#16 & 1-#12

(Ast) provided $= 2 \times \frac{\pi}{4} \times 16^2 + 2 \times \frac{\pi}{4} \times 12^2 = 515.2mm^2 > 474.57mm^2$

<u>Check for type of beam</u>

$$x_u = 2.41 \frac{fy}{fck} \frac{A_{st}}{bd}$$

$$= 2.41 \times \frac{415}{20} \times \frac{515.2}{200} = 128.82mm$$

From code, xumax = 0.48d = 192mm : $x_u < x_u max$

For M20 & Fe 415

∴ Section is under reinforced.
Hence its Ok . . . .

9. A rectangular beam 230mm wide & 600mm deep is subjected to a factored moment of 80kN-m. Find the reinforcement required if M20 grade concrete & Fe 415 steel is used.

Solve: b=230mm, h=600mm, M_u=80kN-m,fck = 20Mpa, fy=415Mpa,Ce=50mm,

$= 80 \times 10^6$ N-mm

$M_u = MuR = 0.87fy\ Ast (d - 0.42x_u)$ →①

$$\frac{x_u}{d} = 2.41 \frac{fy}{fck} \frac{A_{st}}{bd}$$

$$x_u = \frac{2.41 \times 415 A_{st}}{20 \times 230} = 0.217 A_{st}$$

Substituting in ①

$80 \times 10^6 = 0.87 \times 415 \times Ast\ (\quad 0.42 \times 0.217 Ast)$

Procedure for design of beams

1. From basic equations.

Data required: a) Load or moment & type of support b) grade of concrete & steel.

Step . 1 If loading is given or working moment is given, calculate factored moment. (M_u)

$$M_u = \frac{wl^2}{2} \qquad\qquad M_u = \frac{1.5(wd + wl)l^2}{8}$$

Step – 2 Balanced section parameters

x_{ulim}, Q_{lim}, p_{tlim}, (table A to E SP 16) $\qquad Q_{lim} = \frac{M_u}{bd^2}$

Step – 3 Assume b and find

$$d_{lim} = \sqrt{\frac{M_u}{Q_{lim} \times b}}$$

Round off d_{lim} to next integer no.

$h = d + C_e$: C_e = effective cover : assume C_e.

$Ce = 25mm$: $Ce = Cc + \frac{\emptyset}{2}$

Step – 4   To determine steel.

Find $P_t = R_u = \frac{4.6M_u}{f_{ck}bd^2}$ $p_t = \frac{100A_{st}}{bd}$

Ast = $\frac{p_t bd}{100}$: assume suitable diameter of bar & find out no. of bars required.
N=$\frac{Ast}{Q_{st}}$: ast = area of 1 bar.

Using design aid SP16

Step 1 & Step 2 are same as in the previous case i.e using basic equation

Step – 3 Find $\frac{M_u}{bd^2}$and obtain pt from table 1 to 4 → page 47 – 50

Which depends on grade of concrete.

From pt, calculate Ast as Ast = $\frac{p_t.b.d}{100}$

Assuming suitable diameter of the bar, find the no. of re-bar as N= $\frac{A_{st}}{Q_{st}}$

where Qst = $\frac{\pi}{4}\emptyset^2$

NOTE: 1 .   To find the overall depth of the beam, use clear cover given in IS 456 – page 47 from durability & fire resistance criteria.

2. To take care of avoiding spelling of concrete & unforcy tensile stress, min. steel has to be provided as given in IS 456 – page 47 $\frac{A_s}{bd} = \frac{0.85}{fy}$

If Ast calculated, either by method 1 or 2 should not be less than (As)min .

If   Ast < Asmin :   Ast = Asmin

1. Design a rectangular beam to resist a moment of 60kN-m, take concrete grade as M20 & Fe 415 steel.

Solve: M = 60 KN-m

M_u = 1.5 X 60 = 90kN-m   fck = 20Mpa , fy = 415 Mpa

Step 1: Limiting Design constants for M_{20} concrete & Fe 415 steel.

$\frac{Z_{umax}}{d} = 0.48$: From Table – c of SP-16, page 10

$Q_{lim} = 2.76$

$P_{t\ lim} = 0.96$ column sizes 8 inches= 200mm

9 inches = 230mm

Step - 2 : $d_{bal} = \sqrt{\frac{M_u}{Q_{lim} \times b}}$

Let b = 230mm

$$d_{col} = \sqrt{\frac{90 \times 10^6}{2.76 \times 230}} = 376.5mm$$

Referring to table 16 & 16 A, for moderate exposure & 1½ hour fire resistance, let us assume clear cover Cc=30mm & also assume 16mm dia bar ∴ effective cover Ce = 30 + 8 = 38mm

h_{bal} =376.5 + 38 = 414.5 18 inches = 450mm

Provide overall depth, h= 450mm.

'd' provided is 450-38 = 412mm

Step .3 Longitudinal steel

Method . 1 → using fundamental equations

Let p_t be the % of steel required

$$p_t = \frac{50fck}{fy}\left[1-\sqrt{1-Ru}\right]; Ru = \frac{4.6Mu}{f_{ck}bd^2} = \frac{4.6 \times 90 \times 10^6}{20 \times 230 \times 412^2} = 0.53$$

$$= \frac{50 \times 20}{415}\left[1-\sqrt{1-0.53}\right]$$

$$= 0.758$$

Method .2 → using SP 16. Table – 2 page – 47 use this if it is not Specified in problem

$$K = \frac{M_u}{bd^2} = \frac{90 \times 10^6}{230 \times 412^2} = 2.305$$

K=2.3 → pt = 0.757

K= 2.32 → pt = 0.765

For k = 2.305, pt= 0.757+ $\frac{(0.765-0.757)}{(2.32-2.3)} \times (2.305 - 2.3)$

= 0.759

Step 4 : detailing

Area of steel required, $A_{st} = \frac{p_t bd}{100} = \frac{0.76 \times 230 \times 412}{100}$ = 720mm²

- M20 → combination 12mm & 20mm aggregate (As) size.
- Provide 2 bars of 20mm & 1 bar of 12mm,
- ∴ Ast provided = 2 X ¾ + 113
  = 741 > 720mm²

[To allow the concrete flow in b/w the bars, spacer bar is provided]

1.Design a rectangular beam to support live load of 8kN/m & dead load in addition to its self weight as 20kN/m. The beam is simply supported over a span of 5m. Adopt M25 concrete & Fe 500 steel. Sketch the details of c/s of the beam.

Solve: q_L =8kN/m b=230mm q_d^1 = 20kN/m f_{ck} = 25Mpa fy = 500Mpa. l= 5m=5000mm

Step .1: c/s

NOTE; The depth of the beam is generally assumed to start with based on deflection criteria of serviceability. For this IS 456-2000 – page 37, clause 23.2-1 gives $\frac{l}{d}$ = 20

with some correction factors. However for safe design generally l/d is taken as 12

$\frac{l}{12} = d \quad d = \frac{5000}{12}$ = 416mm. (no decimal)

Let Ce = 50mm, h=416+50 = 466mm

Step 2 Load calculation

i) Self weight =0.23 X 0.5 X 1 X 25 = 2.875kN/m $=q_d^{11}$

ii) dead load given q_d^1 = 20kN

qd= $q_d^1 + q_d^{11}$ = 22.875kN/m 25kN/m [multiple of 5]

[Take dead load as x inclusive of dead load, don't mention step . 2]

iii) Live load = 8kN/m

$M_D = \frac{q_d \times l^2}{8} = \frac{25 \times 5^2}{8}$ = 78kN-m(no decimal)

Dead load Live load moment = $\frac{q_l \times l^2}{8} = \frac{8 \times 5^2}{8} = 25KN - m$

M_u= 1.5MD+1.5ML =1.5 X 78 + 1.5 X 25 = 154.5kN-m

Step -3 Check for depth

$d_{bal} = \sqrt{\frac{M_u}{Q_{lim} \times b}}$

From table.3, Qlim = 3.33 (page – 10) SP-16

(h bal < h assumed condition for Safe design)

$$= \sqrt{\frac{154.5 \times 10^6}{3.33 \times 230}} = 449.13$$

Hbal = 449.13 + 50 = 499.13

Hence assumed overall depth of 500mm can be adopted.

Let us assume 20mm dia bar & Cc=30mm (moderate exposure & 1.5 hour fire resistance)

∴ Ce provided = 30+10 = 40, $d_{provided}$ = 500-40 = 460mm

Step .4 Longitudinal steel

$$\frac{M_u}{bd^2} = \frac{154.5 \times 10^6}{230 \times 460^2} = 3.17$$

Page 49 – SP-16 M_u/bd^2 pt

3.15	0.880
3.20	0.898

For $\frac{M_u}{bd^2} = 3.17, p_t = 0.880 + \frac{(0.898-0.880)}{(3.20-3.15)} \times (3.17 - 3.15)$

= 0.8872

$$A_{st} = \frac{p_t bd}{100} = \frac{0.8872 \times 230 \times 460}{100} = 938.66mm^2$$

No. of # 20 = $\frac{938.66}{314} = 2.98 \approx 3\, Nos.$

$(A_{st})_{provided}$ = 3 x 314 = 942mm^2 > 938.66mm^2.

Page – 47 – IS456

$(A_{st})_{min} = \frac{0.85bd}{fy} = \frac{0.85 \times 230 \times 460}{500} = 179mm^2$

∴ $(A_{st})_{provided} > (A_{st})_{min}$. Hence o.k.

Step . 5 Detailing

3. Design a rectangular beam to support a live load of 50 kN at the free end of a cantilever beam of span 2m. The beam carries a dead load of 10kN/m in addition to its self weight. Adopt M30 concrete & Fe 500 steel.

l=2m=2000mm, q_d^1 =10, fck= 30Mpa, fy=500Mpa, qL=50kN, b=230mm

Step – 1 c/s

NOTE: The depth of the beam is generally assumed to start with based on deflection criteria of serviceability. For this IS 456-2000-Page – 37, clause 23.2.1 gives

$\frac{l}{d}$ = 5 with some correction factor.

$\frac{l}{s}$ = d → d=$\frac{2000}{5}$ = 400mm

Let Ce=50mm, h=400 + 50 = 450mm

However we shall assume h=500mm

Step – 2 Load calculation.

(i) Self wt= 0.23 X 0.5 X 1 X 25 = 2.875kN/m = q_d^{11}

(ii) Dead load given = q_d^1 =10kN/m

qd=q_d^1 + q_d^{11} = 10+2.875 = 12.87kN/m ≈ 15kN/m [multiply of5]

(iii) Live load = 50kN

$$M_D = \frac{q_D l^2}{2} = \frac{15 \times 2^2}{2} = 50KN - m$$

ML=W X 2 = 50 X 2 = 100kN-m

M_u = 1.5MD +1.5ML = 195kN-m = 88

Step – 3 Check for depth

dbal = $\sqrt{\frac{M_u}{Q_{lim} \times b}}$ From table – 3,SP-16 (page – 10) Q_{lim} = 3.99

$$= \sqrt{\frac{195 \times 10^6}{3.99 \times 230}} = 460.96mm$$

hbal = 460.96+50= 510.96mm

hassumed = 550mm.

Let us assume 20mm dia bars & Cc=30mm constant

∴ Ce provided = 30+10 = 40, dprovided= 550-40 =510mm

Step – 4 Longitudinal steel

$$\frac{Mu}{bd^2} = \frac{195 \times 10^6}{230 \times 510^2} = 3.26$$

Page – 49m SP-16m

M_u/bd^2	pt
3.25	0.916
3.30	0.935

For $M_u/bd^2 = 3.26$,

Pt = $0.916 + \frac{(0.935-0.916)}{(3.30-3.25)} \times (3.26 - 3.25)$

= 0.9198.

Ast = $\frac{p_t bd}{100} = \frac{0.9198 \times 230 \times 510}{100}$ = 1078.9mm²

3-# 20, 1-#16 = 1143

2- #25 2=#20 = 1382

$(A_{st})_{Provided}$ =

$(A_{st})_{min} = \frac{0.85bd}{fy} = \frac{0.85 \times 230 \times 510}{500} = 199.41mm^2$

Step – 5 Detailing

Design of slabs supported on two edges

Slab is a 2 dimensional member provided as floor or roof which directly supports the loads in buildings or bridges.

In RCC, it is reinforced with small dia bars (6mm to 16mm) spaced equally.

Reinforcement provided in no. RCC beam → 1 dia width is very (12mm – 50mm) small compared to length. Element with const. Width: fixed width.

It is subjected to vol.

RCC slabs → reinforcement are provided with equally spaced. No fixed width & length are comparable, dia -6mm to 16mm. It is subjected to pressure.

Beams are fixed but slabs are not fixed. For design, slab is considered as a beam as a singly reinforced beam of width 1m

Such slabs are designed as a beam of width 1m & the thickness ranges from 100mm to 300mm. IS456-2000 stipulates that $\frac{l}{d}$ for simply supported slabs be 35 & for continuous slab 40 (page – 39). For calculating area of steel in 1m width following procedure may be followed.

$$Ast = N \times \frac{\pi}{4} \times \emptyset^2 \quad ; \quad A_{st} = \frac{1000 \times Q_{st}}{s} \qquad N = \frac{1000}{s}$$

For every 10cm there is a bar

$$S = \frac{a_{st}}{A_{st}} \times 1000$$

(The loading on the slab is in the form of pr. expressed as kN/m^2)

As per clause 26.5.2-1 (page 48 min steel required is 0-15% for mild steel & 0-12% for high strength steel. It also states that max. dia of bar to be used is $1/8^{th}$ thickness of the slab. To calculate % of steel we have to consider gross area is 1000Xh.

The slabs are subjected to low intensity secondary moment in the plane parallel to the span. To resist this moment & stresses due to shrinkage & temp, steel reinforcement parallel to the span is provided. This steel is called as distribution steel. Min. steel to be provided for distribution steel.

Practically it is impossible to construct the
Slab as simply supported bozo of partial bond

b/w masonry & concrete, also due to the parapet wall constructed above the roof slab. This induces small intensity of hogging BM. Which requires min. % of steel in both the direction at the top of the slab as shown in fig. 2 different types of detailing is shown in fig.

<u>Method – 1</u>

Crank → for the change in reinforcement.

(1) Alternate cranking bars.

Dist. Steel is provided.

- To take care of secondary moment, shrinkage stresses & temp. stress steel is provided parallel to the span.

<u>Method – 2</u>

1. Compute moment of resistance of a 1- way slab of thickness 150mm. The slab is reinforced with 10mm dia bars at 200mm c/c. Adopt M20 concrete & Fe 415 steel. Assume Ce=20mm.

<u>Solve</u>: h=150mm,Ce=20mm, ϕ=10mm

d=150-20 =130mm, sx=200mm

$$A_{st} = \frac{1000}{sx} \times \frac{\pi}{4} \times \emptyset^2$$

$$= \frac{1000}{200} \times \frac{\pi}{4} \times 10^2$$

$= 392.7mm^2$

Step - 1 N-A depth

$$\frac{x_u}{d} = 2.41 \frac{fy}{fck} \frac{Ast}{bd}$$

$x_u = 2.41 \times \frac{415}{20} \times \frac{392.7}{1000}$

=19.64mm.

x_{umax} = 0.48d= 0.48X130 = 62.4mm.

Step – 2 Moment of resistance

$x_u < x_{umax}$. hence section is under reinforced.

MuR= 0.87fy Asta(d-0-42x_u)

$$\frac{0.87 \times 415 \times 392.7\,(130 - 0.42 \times 19.64)}{10^6}$$

=17.26 kN-m/m.

Doubly reinforced Beams.

Limiting state or Balanced section.

Cuc = 0.36fck b_{xulim}

T_{u1}= 0.87$f_y$$A_{st}$

Mulim = 0.36f_{ck}bx_{ulim}(d-0.42x_{ulim})

$p_{tlim} = 0.414 \frac{x_{ulim}}{d} \times \frac{fck}{fy}$

$A_{st} = \frac{P_{tlim}}{100} \times bd$

$M_u > M_{ulim}$ M_u = applied factored moment.

$M_{u_2} = M_u - Mulim$

For M_{u2} we require Ast in compression zone & Ast_2 in tension zone for equilibrium

$Cus = T_{u2}$

Cus = fsc .Asc : Fsc is obtained from stress – strain curve of corresponding steel.

In case of mild steel, it is $fsc = \frac{fy}{1.15} = 0.87fy$

In case of high strength deformable bars,

fsc corresponding to strain εsc should be obtained from table A-SP-16(Page – 6)

$Z_2 = d - d^1$ If εsc< 0.00109,

fsc = εsXεsc
else fst = fy/1.15

$$\frac{\epsilon cu}{x_{ulim}} = \frac{\epsilon sc}{x_{ulim} - d^1}$$

$$\epsilon sc = \frac{\epsilon cu(x_{ulim} - d^1)}{x_{ulilm}}$$

$T_{u2} = 0.87 f_y A_{st2} \rightarrow 1$

Couple $M_{u2} = Cus\ XZ_2 = T_{u2} Z_2$

$M_{u2} = (fsc\ Asc)(d - d^1)$

$$A_{sc} = \frac{M_{u2}}{fsc(d - d^1)} \rightarrow 2$$

$Cus = T_{u2}$

fsc X Asc = 0.87fy Ast_2

$$A_{st2} = \frac{FscAsc}{0.87fy} \rightarrow 3$$

Procedure for design of doubly reinforced section.

Step. 1 Check for requirement of doubly reinforced section

1. Find xulim using IS456
2. Find Mulim for the given section as Mulim = QlimXbd² Refer table – D, page – 10, of SP – 16 for Qlim
3. Find ptlim from table – E page 10 of SP – 16 & then compute.

$$A_{st} = \frac{p_{tlim} \times b \times d}{100}$$

Step – 2 If $M_u > M_u lim$ then design the section as doubly reinforced section, else design as singly reinforced section.

Step – 3 $M_{u2} = M_u - M_u lim$

Step – 4 Find area of steel in compression zone using the equation as

Asc = $\frac{M_{u2}}{fsc(d-d^1)}$ fsc has to be obtained from stress – strain curve or from table – A, page – 6 of SP-16.

The strain εsc is calculated as $\frac{\epsilon_{cu}(x_{ulim}-d^1)}{x_{ulim}}$

Step - 5 Additional tension steel required is computed as

$$A_{st2} = \frac{f_{sc}A_{sc}}{0.87fy}$$

∴ Total steel required Ast = $Ast_1 + Ast_2$ in tension zone.

Use of SP-16 for design of doubly reinforced section table – 45-56, page 81-92 provides pt & pc:

pt = $\frac{Ast}{bd} \times 100$

Pc = $\frac{Asc}{bd}$ X100 for different values of M_u/bd^2 corresponding to combination of fck & fy. Following procedure may be followed.

Step – 1 Same as previous procedure.

Step – 2 If M_u > Mulim , find M_u/bd^2 using corresponding table for given fy & fck obtain pt & pc . Table – 46.

NOTE: An alternative procedure can be followed for finding fsc in case of HYSD bars i.e use table – F, this table provides fsc for different ratios of d'/d corresponding to Fe 415 & Fe 500 steel

Procedure for analysis of doubly reinforced beam

Data required: $b, d, d^1, A_{st}(A_{st_1}+A_{st_2}), A_{sc}, f_{ck}, fy$

Step – 1 Neutral axis depth

$C_{uc}+C_{us} = T_u$

$0.36f_{ck} < bx_u+f_{sc} A_{sc} = 0.87fy. A_{st}$.

$$x_u = \frac{0.87fyA_{st}-f_{sc}A_{sc}}{0.36f_{ck}b}$$

This is approximate value as we have assumed the tensile stress in tension steel is $0.87f_y$ which may not be true. Hence an exact analysis has to be done by trial & error. (This will be demonstrated through example).

Step – 2: Using the exact analysis for N-A depth the MR can be found as.

$M_{ulim} = 0.36f_{ck} .b\, x_u(d\text{-}0.4\ 2x_u)+f_{st}\ A_{sc}\ X(d^2\text{-}d^1)$

1. Design a doubly reinforced section for the following data.
   b=250mm, d= 500mm, d^1=50mm, M_u=500kN-m con-, M_{30}, steel = Fe 500.
   $\frac{d^1}{d}$= 0.1, f_{ck} = 30Mpa, fy= 500Mpa.
   $M_u = 500 \text{ X } 10^6$ N-mm

Step – 1 Moment of singly reinforcement section.

Page – 70 –IS-456 $\frac{x_{ulim}}{d} = 0.46$

x_{ulim} = 0.46 X 500 = 230mm.

$M_{ulim} = Q_{lim}$ X $bd^2 = \frac{3.99 \times 250 \times 500^2}{10^6}$ = 249kN-m.

Page – 10 SP – 16

$P_{tlim} = 1.13$: $A_{st_1} = A_{stlim} = \frac{1.13 \times 250 \times 500}{100} = 1412.5mm^2$

$M_u > M_{ulim}$

Step – 2 $M_{u2} = M_u - M_{ulim}$ = 500-249 = 251kN-m.

$A_{sc} = \frac{M_{u2}}{f_{sc}(d-d^1)}$ page – 13. SP-16. Table – F

$= \frac{251 \times 10^6}{412(500-50)} = 1353.83mm^2$

From equilibrium condition,

$$A_{st2} = \frac{f_{sc} A_{sc}}{0.87 fy} = \frac{412 \times 1353.83}{0.87 \times 500} = 1282.25 mm^2$$

$A_{st} = A_{st_1} + A_{st_2}$ = 1412.5 + 1282.25 = 2694.75mm²

Step – 3: Detailing.

A_{sc} = 1353mm²

A_{st} = 2694mm²

Tension steel

Assume # 20 bars = $\frac{2694}{314}$ = 8.5

However provide 2 - #25 + 6 - #20

$(A_{st})_{provided}$ = 2 X 490 + 6 X 314 = 2864mm² > 2694mm²

Compression steel

Assume #25, No = $\frac{1353}{490}$ = 2.7

Provide 3 - #25 $(A_{st})_{provided}$ = 3 X 490 = 1470mm² > 1353mm²

C_e = 30 + 25 + 12.5 = 67.5mm

2. Design a rectangular beam of width 300mm & depth is restricted to 750mm(h) with a effective cover of 75mm. The beam is simply supported over a span of 5m. The beam is subjected to central con. Load of 80kN in addition to its self wt. Adopt M30 concrete & Fe 415 steel.

W_d = 0.3 X 0.75 X 1 X 25 = 5.625

$M_D = \frac{Wd^2}{8}$ = 17.6kN-m

$ML = \frac{w_l \times l}{4} = 100kN\text{-}m$

$Mu = 1.5(M_D + M_L) = 176.4$

3. Determine areas of compression steel & moment of resistance for a doubly reinforced rectangular beam with following data.

b= 250mm, d= 500mmm, d^1=50mm, A_{st} = 1800mm², f_{ck} = 20Mpa, f_y= 415Mpa. Do not neglect the effect of

Compression reinforcement for calculating Compressive force.

Solve: $C_{c_1} \rightarrow$ introduce a negative force

Note: $C_u = C_{sc} + C_c - C_{c_1}$

$= f_{sc} A_{sc} + 0.36 f_{ck} bx_u - 0.45 f_{ck} X A_{sc}$

$= 0.36 f_{ck} bx_u + A_{sc} (f_{sc} - 0.45 f_{ck})$

For calculating compressive force then,

Whenever the effect of compression steel is to be considered.

Step – 1 Depth of N-A.

From IS – 456 for M20 concrete and Fe 415 steel is

$\frac{x_{umax}}{d} = 0.48$ & $x_{umax} = 0.48 X 500 = 240mm$

From table – 6 , SP – 16 page- 10, $p_{tlim} = 0.96$

$A_{st_1} = A_{stlim} = \frac{0.96 \times 250 \times 500}{100} = 1200mm^2$

$A_{st_2} = A_{st} - A_{st_1} = 1800 - 1200 = 600mm^2$

Step – 2 : Asc

For equilibrium, $C_u = T_u$.

In the imaginary section shown in fig.

$C_{u1} = A_{sc}(f_{sc}-0.45f_{ck})$

$\frac{d^1}{d} = \frac{500}{500} = 0.1$, Table – F, SP – 16, P – 13, $f_{sc} = 353$Mpa

Ast₂

$A_{sc}(353 - 0.45 \text{ X } 20) = 600 \text{ X } 0.87 \text{ X } 415$

$A_{sc} = 629\text{mm}^2$

Step – 3 MR $A_{sc}(f_{sc} - 0.45f_{ck})\,(d - d^1)$

$Mur_2 = C_{u1} \text{ x } Z_2 = \frac{629 \text{ X } (353-0.45\text{X}20) \text{ X } (500-50)}{10^6}$

$= 97.6$kN-m

$Mur_1 = M_{ulim} = Q_{lim}bd^2 \quad Q_{lim} = 2.76$

$= \frac{2.76\text{X}250\text{X}500^2}{10^6} = 172$kN-m.

$M_{ur} = M_{ur1} + M_{ur2} = 97.6 + 172 = 269.8$kN-m

$A_{sc} = 629\text{mm}^2$, $M_{ur} = 269.8$kN-m

4. A rectangular beam of width 300mm & effective depth 550mm is reinforced with steel of area 3054mm² on tension side and 982mm² on compression side, with an effective cover of 50mm. Let MR at ultimate of this beam is M20 concrete and Fe 415 steel are used. Consider the effect of compression reinforcement in calculating compressive force. Use 1st principles No. SP – 16.

Solve: Step: 1 N-A depth

$\frac{\text{xulim}}{d} = 0.48$

$x_{ulim} = 0.48 \text{ X } 550 = 264$mm

Assuming to start with $f_{sc} = 0.87fy = 0.87 \text{ X } 415 = 361$Mpa.

Equating the total compressive force to tensile force we get,

$C_u = T_u = C_u = 0.36f_{ck}\, b\, x_u + A_{st}(f_{sc} - 0.45f_{ck})$

$T_u = 0.87fy\, A_{st}$.

$$x_u = \frac{0.87 \text{ X } 415 \text{ X } 3054 - 982(361 - 0.45 \text{ X } 20)}{0.36 \text{ X } 20 \text{ X } 300} \rightarrow 1$$

$=350.45$ mm $> x_{ulim}$.

Hence the section is over reinforced.

The exact N-A depth is required to be found by trial & error using strain compatibility, for which we use equation ① in which the value of fsc is unknown, hence we get,

$$x_u = \frac{\text{fst X } 3054 - 982(\text{fsc} - 0.45\text{X}20)}{0.36 \text{ X } 20 \text{ X } 300}$$

$$= \frac{3054\text{fst} - 982\text{fsc} + 8779}{2172} \longrightarrow ②$$

Range of x_u & 264 to 350.4

<u>Cycle – 1</u> Try $(x_u)_1 = \frac{264+350.4}{2} = 307$mm

From strain diagram $\varepsilon sc = 0.0035(1-\frac{50}{307}) = 0.00293$ (similar triangle)

$\varepsilon_{st} = \frac{0.0035 \text{ X } 550}{307} - 1) = 0.00279$

$f_{sc} = 339.3$mm. The difference b/w $(x_u)_1$ & $(x_u)_2$ is large, hence continue cycle 2.

<u>Cycle – 2</u> Try $(x_u)_3 = \frac{307+339.3}{2} = 323$mm

From strain diagram, $\varepsilon sc = 0.0035(1 - \frac{50}{323}) = 0.00296$.

$\varepsilon_{st} = 0.0035(\frac{550}{323} - 1) = 0.00246$

$f_{sc} = 353.5$, $f_{st} = 344.1$, $(x_u)_4 = 328$

The trial procedure is covering, we shall do 1 more cycle.

<u>Cycle – 3</u> Try $(x_u)_5 = \frac{323+328}{2} = 325.5$mm.

From strain diagram, $\varepsilon_{sc} = 0.0035(1 - \frac{50}{325.5}) = 0.00296$.

$\varepsilon_{st} = 0.0035(\frac{550}{325.5} - 1) = 0.00241$

$f_{sc} = 353.5$, $f_{st} = 342.8$. $(x_u)_6 = 326.2$

$\therefore$ $x_u = 326.2$mm.

Step . 2

$$M_{ur} = \frac{(0.36 \times 20 \times 300 \times 326)\,(550 - 0.42 \times 326) + 982(353.5 - 0.45 \times 20)(d - d')(550 - 50)}{10^6} = 463 \text{kN-m}$$

5. Repeat the above problem for Fe 450.

Note: $x_u < x_{umax}$, hence cyclic procedure is not required.

$x_u = 211.5$, $M_{ur} = 315$kN-m.

6. A rectangular beam of width 300mm & effective depth of 650mm is doubly reinforced with effective cover $d^1 = 45$mm. Area of tension steel 1964, area of compression steel = 982mm². Let ultimate MR if M- 20 concrete and Fe 415 steel are used.

Ans: x_u= 11734mm, M_{ur} = 422kN-m

Flanged sections.

T- section L- section

- Concrete slab & concrete beam are
  Cast together → Monolithic construction
- Beam → tension zone, slab in comp. zone
- Slab on either side → T beam

  Slab on one side → 1 beam

- bf → effective width bf>b

T- beam

$b_f = \frac{lo}{6} + bw + 6D_f$

- l_o=0.7le: continuous & frames beam.
- A & B paints of contra flexure (point of zero moment)

<u>L-Beam</u>

$b_f = \frac{lo}{12} + bw + 3Df$

<u>Isolated T- beam</u>

It is subjected to torsion & BM

- If beam is resting on another beam it can be called as L – beam.
- If beam is resting on column it cannot be called as L- beam. It becomes –ve beam.

<u>Analysis of T – beam</u> :- All 3 cases NA is computed from $C_u = T_u$.

<u>Case-1</u> – neutral axis lies in flange.

Case (ii) : NA in the web & $\frac{Df}{d} \leq 0.2$

Whitney equivalent rectangular stress block.

Case (iii): NA lies in web & $\frac{Df}{d} > 0.2$

1. All three cases NA is computed from $C_u = T_u$.
2. x_{ulim} same as in rectangular section.
   - Depth of NA for balanced s/n depends on grade of steel.
3. Moment of resistance

   Case . (1) $C_u = 0.36f_{ck}b_f x_u$ $T_u = 0.87f_y A_{st}$

   $M_{ur} = 0.36f_{ck}\, b_f\, x_u(d-0.42x_u)$ or $0.87f_yA_{st}(d - 0.42x_u)$

Case – (ii) $C_u = \underbrace{0.36f_{ck}b_wx_u}_{1} + \underbrace{0.45f_{ck}(b_f-b_w)}_{2}Df\,(d - \frac{Df}{2})$

$T_u = 0.87f_yA_{st}$

$M_{ur} = 0.36f_{ck}\, b_w\, x_u(d - 0.42x_u) + 0.45f_{ck}(b_f - b_w)D_f\,(d - \frac{Df}{2})$

Case (iii)

$C_u = 0.36f_{ck}\, b_wx_u + 0.45f_{ck}(bf - bw)y_f.$

$T_u = 0.87f_yA_{st}.$

Page . 97, $M_{ur} = 0.36f_{ck}b_wx_u(d-0.42x_u) + 0.45f_{ck}(b_f-b_w)\, y_f\,(d - \frac{yt}{2})$

Where, $y_f = 0.15\, x_u + 0.65D_f$

Obtained by equating areas of stress block.

- When $D_f/x_u \leq 0.43$ & $D_f/x_u > 0.43$ for the balanced section & over reinforced section use x_{umax} instead of x_u.

Problem.

1. Determine the MR of a T – beam having following data.
   a) flange width = 1000mm = bf
   b) Width of web = 300mm = bw
   c) Effective depth = , d= 450mm
   d) Effective cover = 50mm
   e) A_{st} = 1963mm²
   f) Adopt M20 concrete & Fe 415 steel.

Solve: Note:

In the analysis of T – beam, assume N-A
To lie in flange & obtain the value of
x_u. If $x_u > D_f$ then analyses as
case (2) or (3) depending on the
ratio of D_f/d. If $D_f/d \leq 0.2$ case (2)
or $D_f/d > 0.2$ case (3)

Step – 1

Assume NA in flange
$C_u = 0.36 f_{ck}\, b_f\, x_u$ $\quad T_u = 0.87 f_y\, A_{st}$
$C_u = T_u$.

$0.36 f_{ck}\, b_f\, x_u = 0.87 f_y\, A_{st}$

$$x_u = \frac{0.87 \text{ X } 415 \text{ X } 1963}{0.36 \text{ X } 20 \text{ X } 1000}$$

$= 98.4\text{mm} < D_f = 1000\text{mm}.$

Assumed NA position is correct i.e(case . 1)

Step – 2 : $M_{ur} = 0.36 f_{ck}\, b_f\, x_u(d - 0.42 x_u)$

$= 0.36 \text{ X } 20 \text{ X } 1000 \text{ X } 98.4(450 - 0.42 \text{ X } 98.4)/10^6$

$= 290$ kN-m

Or $M_{ur} = 0.87 f_y A_{st} (d - 0.42 x_u)$

$= 0.87 \times 415 \times 1963(450 - 0.42 \times 98.4)$

$= 290$kN-m

<u>Use of SP 16 for analysis</u> p: 93 - 95

For steel of grade Fe 250, Fe 415 & Fe 500, SP – 16 provides the ratio $\frac{Mu}{f_{ck} b_w d^2}$ for combinations of $\frac{Df}{d}$ and $\frac{bf}{bw}$ using this table the moment of resistance can be calculated as $M_u = K_T f_{ck} b_w d^2$ where K_T is obtained from SP -16.

<u>Solve</u>: $\frac{Df}{d} = \frac{100}{450} = 0.22 > 0.2$

$\frac{bf}{bw} = \frac{1000}{300} = 3.33$

For Fe 415m P:94

$\frac{bf}{bw}$	3	4
K_T	0.309	0.395

K_T for $\frac{bf}{bw} = 3.3 \Rightarrow 0.309 + \frac{(0.395-0.309)}{(4-3)} \times (3.3\text{-}3)$

$= 0.337$

$M_{ulim} = \frac{0.337 \times 20 \times 300 \times 450^2}{10^6} = 410$kN-m.

This value corresponds to limiting value. The actual moment of resistance depends on quantity of steel used.

2. Determine area of steel required & moment of resistance corresponding to balanced section of a T – beam with the following data, bf = 1000, D_f = 100mm, b_w = 300mm, effective cover = 50mm, d= 450mm, Adopt M20 concrete & Fe 415 steel.

Use 1st principles.

<u>Solve</u>: <u>Step –1</u> $\frac{Df}{d} = \frac{100}{450} = 0.22 > 0.2$ case (iii)

<u>Step – 2</u> $y_f = 0.15 x_{umax} + 0.65 D_f$

$C_u = 0.36 f_{ck} b_w x_{umax} + 0.45 f_{ck} (b_f - b_w) y_f$.

$T_u = 0.87 f_y A_{stlim}$

For Fe 415, $x_{umax} = 0.48d = 216\text{mm} > D_f$.

C_u= 0.36 X 20 X 300 X 216 + 0.45 X 20(1000 - 300)97.4 = 1.0801 X 10^6Nmm

Y_f = 0.15 X 216 + 0.65 X 100 = 97.4mm

C_u=T_u

C_u= 0.87 X 415 A_{st}.

1. 0801 X 10^6 = 0.87 X 415 A_{stlim}
   <u>A_{stlim} = 2991.7mm²</u>

<u>Step – 3</u> $M_{ur} = 0.36f_{ck}\, b_w\, x_{umax}(d - 0.42x_{umax}) + 0.45f_{ck}(b_f-b_w)y_f(d - \frac{yf}{2})$.

$= 0.36 \text{ X } 20 \text{ X } 300 \text{ X } 216\ (450-0.42 \text{ X } 216) + 0.45 \text{ X } 20\ (1000-300)\ 97.4\ (450 - \frac{97.4}{2})$

M_{ur} = 413.27kN-m

3. Determine M R for the c/s of previous beam having area of steel as 2591mm²

<u>Step – 1</u> Assume N-A in flange

$C_u = 0.36f_{ck}b_fx_u$

$T_u = 0.87\ f_y\ A_{st}$

For equilibrium $C_u = T_u$

0.36 X 20 X 1000 x_u = 0.87 X 415 X 2591

x_u = 129.9mm > D_f

∴ N-A lies in web

$\frac{Df}{d} = \frac{100}{450}$ = 0.22 > 0.2 case (iii)

∴ $y_f = 0.15x_u + 0.65D_f$ = 0.15 X x_u + 0.65 X 100 = 0.15 x_u + 65

$C_u = T_u$

$C_u=0.36\ f_{ck}\ b_w\ x_u + 0.45f_{ck}\ (b_f- b_w)y_f$ =

$T_u = 0.87f_yA_{st}$ = 0.87 X 450 X 2591 = 1014376.5

1014376.5 = 0.36 X 20 X 300 x_u + 0.45 X 20(1000-300)84.485

x_u = 169.398mm < x_{umax} = 0.48 X 450 = 216mm

It is under reinforced section.

Step – 2 MR for under reinforced section depends on following.

(1) $\frac{Df}{d}$ = 0.22 > 0.2 (case iii)

(2) $\frac{Df}{xu} = \frac{100}{169.398}$ = 0.59 > 0.43

Use yf instead of D_f in computation of MR

$\therefore$ $y_f = 0.15x_u + 0.65D_f$

= 0.15 X 169.398 + 0.65 X 100

= 90.409mm.

$M_{ur} = 0.36f_{ck}\ x_u\ b_w\ (d-0.42x_u) + 0.45f_{ck}\ (b_f - b_w)\ y_f\ (d - \frac{yf}{2})$

= 0.36 X 20 X 169.398 X 300 (450-0.42 X 169.398) + 0.45 X 20 (1000-300) 90.409 (450-$\frac{90.409}{2}$)

= M_{ur} = 369.18kN-m

Design procedure

Data required:

1. Moment or loading with span & type of support
2. Width of beam
3. Grade of concrete & steel
4. Spacing of beams.

Step – 1 Preliminary design

From the details of spacing of beam & thickness of slabs the flange width can be calculated from IS code recommendation.

$b_f = \frac{lo}{6} + bw + 6D_f$. P-37 : IS 456

Approximate effective depth required is computed based on l/d ratio d $\approx \frac{l}{12}$ to $\frac{l}{15}$

Assuming suitable effective cover, the overall depth, h= d + C_e round off 'h' to nearest 50mm integer no. the actual effective depth is recalculated, $d_{provided}$ = h- C_e

Approximate area of steel i.e computed by taking the lever arm as Z= d - $\frac{Df}{2}$

$M_u = 0.87 f_y A_{st} X Z$

$A_{st} = \frac{Mu}{0.87fy(d-\frac{df}{2})}$

Using this A_{st}, no. of bars for assumed dia is computed.

Round off to nearest integer no. & find actual A_{st}.

NOTE: If the data given is in the form of a plan showing the position of the beam & loading on the slab is given as 'q' kN/m² as shown in the fig.

W=q X S X 1 also, $b_f \leq$ s

Step – 2 N –A depth

The N-A depth is found by trial procedure to start with assume the N-A to be in the flange. Find N-A by equating C_u & T_u if $x_u < D_f$ then NA lies in flange else it lies in web.

In case of NA in the web then find $\frac{Df}{d}$. If $\frac{Df}{d} \leq 0.2$, use the equations for

C_u & T_u as in case (II) otherwise use case (III).

Compute x_{ulim} & compare with x_u. If $x_u > x_{ulim}$ increase the depth of the beam & repeat the procedure for finding x_u.

Step . 3 Moment of resistance

Based on the position of NA use the equations given in cases (I) or case (II) or case (III) of analysis. For safe design $M_{ur} > M_u$ else redesign.

Step . 4 Detailing.

Draw the longitudinal elevation & c/s of the beam showing the details of reinforcement.

1. Design a simply supported T – beam for the following data. (I) Factored BM = 900kN-m (II) width of web = 350mm (III) thickness of slab = 100mm (IV)spacing of beams = 4m (V) effective span = 12m (VI) effective lover = 90mm, M20 concrete & Fe 415 steel.

Step . 1 Preliminary design.

$bf = \frac{lo}{6} + bw + 6D_f$ $\qquad l_o = l_e = 12000mm$

$= \frac{12000}{6} + 350 + 6 X 100 = 2950 < S = 4000$

$h \approx \frac{le}{12}$ to $\frac{le}{15}$ (1000 to 800mm)

Assume h = 900mm

$d_{provided} = 900\text{-}90 = 810mm$

Approximate $A_{st} = \frac{M_u}{0.87fy(d - \frac{d_f}{2})} = \frac{900X10^6}{0.87X415(810 - \frac{100}{2})} = 3279mm^2$

Assume 25mm dia bar.

No. of bars $= \frac{3279}{490} \approx 6.7$

Provide 8 bars of 25mm dia

$(A_{st})_{provided} = 8 X 491 = 3928mm^2$

$x_{umax} = 0.48 X 810 = 388.8$

<u>Step. 2</u> N-A depth, Assume NA to be in flange.

$C_u = 0.36f_{ck}\, x_u\, b_f$; $T_u = 0.87f_y\, A_{st}$.

$0.36 \text{ X } 20 \text{ X } x_u \text{ X } 2950 = 0.87 \text{ X } 415 \text{ X } 3927$.

$x_u = 66.75 < D_f < x_{umax}$

hence assumed position of N-A is correct.

<u>Step . 3</u> . $M_{ur} = 0.36f_{ck}b_fx_u(d-0.42x_u)$

$= 0.36 \text{ X } 20 \text{ X } 2950 \text{ X } 66.75(810-0.42 \text{ X } 66.75)$

$M_{ur} = 1108.64\text{kN-m} > 9.01\text{kN-m } M_u$

Hence ok

<u>Step . 4</u> Detailing

2. Design a T-beam for the following data. Span of the beam = 6m (effective) & simply supported spacing of beam -3m c/c, thickness of slab = 120mm, loading on slab -5kN/m² exclusive of self weight of slab effective cover = 50mm, M20 concrete & Fe 415 steel. Assume any other data required.

3. A hall of size 9mX14m has beams parallel to 9m dimension spaced such that 4 panels of slab are constructed. Assume thickness of slab as 150mm & width of the beam as 300mm. Wall thickness = 230mm, the loading on the slab (I) dead load excluding slab weight 2kN/m² (2) live load 3kN/m². Adopt M20 concrete & Fe 415 steel. Design intermediate beam by 1st principle. Assume any missing data.

* 1 inch = 25.4 or 25mm

* 'h' should be in terms of multiples of inches.

Step . 1 Preliminary design

$S = \frac{14.23}{4} = 3.55m$

Effective span, $le = 9 + \frac{2 \text{ X } 0.23}{2} = 9.23m$

Flange width, $b_f = \frac{lo}{6} + b_w + 6D_f = 9.23/6 + 0.300 + 6X0.15 = 2.74m < S = 3.55$

$h \approx \frac{le}{12} \text{ to } \frac{le}{15} \approx \frac{9230}{12} \text{ to } \frac{9230}{15} = 769.17 \text{ to } 615.33$

Let us assume h=700mm, C_e=50mm.

$d_{provided} = 700-50 = 650mm$

Loading

1. on slab

a) self weight of slab = 1m X 1m X 0.15m X 25 = 3.75kN/m²

b) Other dead loads (permanent) = 2kN/m²

c) Live load (varying) = 3kN/m²
(It is known as imposed load) q = 8.75 9kN/m²

2. Load on beam

a> From slab = 9 X 3.55 = 31.95kN/m
b> Self weight of beam = 0.3 X 0.55 X 1 X 25 = 4.125kN/m
↓ ↘depth of web
width of beam

w= 36.D75 36kN/m

$M = \frac{36 \text{ X } 9.23^2}{8} = 383.37kN\text{-}m$

M_u= 1.5 X 383.4 = 575kN-m.

$(A_{st})_{app.} = \frac{Mu}{0.87fy(d - \frac{D_l}{2})}$

$= \frac{575 \text{ X } 10^6}{0.87 \text{ X } 415(650 - \frac{150}{2})}$

$= 2769mm^2$ $(A_{st})_{actual} = 491 \text{ X } 6 = 2946$

No. of # 25 bars = $\frac{2769}{490}$ = 5.65 ≈ 6 1ft of span → depth is 1 inch1m = 3.28ft

Provide 6 bars of 25mm dia in 2 rows.

Step .2 N-A depth

x_{umax} = 0.48 X 650 = 312mm

Assuming N-A to lie in flange,

$x_u = \frac{0.87fy\ Ast}{0.36fck\ bf} = \frac{0.87 X 415 X 2946}{0.36 X 20 X 2740}$ = 53.91mm < D_f < x_{umax}

$M_{ur} = 0.36\ f_{ck} b_f\ x_u(d\text{-}0.42x_u)$

= 0.36 X 20 X 2740 X 53.91(650-0.42 X 53.91)/10^6

= 667.23kN-m.

Design a T-beam for a simply supported span of 10m subjected to following loading as uniformly distributed load of 45KN/m excluding self wt of the beam by a point load at mid-span of intensity 50KN due to a transverse beam. Assume the width of the beam=300mm & spacing of the beam=3m. Adopt M-20 concrete &Fe 415 steel.

Sol: M = $\frac{wl^2}{8} + \frac{p \times l}{4}$

Self cut = 1 x 1 x 0.3 x 25 = 78.dKW/M.

W= 7.5+45 = 52.KN/M.

Shear, Bond & tension in RCC Beams

Shear

- Types of cracks @ mid span → flexural crack beoz Bm is zero, SF is max
- Type of crack away from mid span →shear f flexural crack.
- Principal tensile stress at supports = shear stress

$$\tau_v = \frac{Vx(Ay)}{Ib}$$ A= area above the point consideration

If (As) hanger<(Ast)min does not contribute to compression as in doubly reinforced beams. $(Ast)_{min} = \frac{0.85bd}{fy}$

RCC – Heterogonous material → Distribution of shear stress in complex

$$\tau v = \frac{Vx(Ay)}{Ib} \rightarrow \text{Normal shear stress} \quad \tau v = \frac{vx}{bwd}$$

$V_u = V_{cb} + V_{ay} + V_d + V_s$

$V_u = V_{cu} + V_s$.

- Shear reinforcement →Vertical stirrup & Bent - up bars

Truss analogy

$$\boxed{V_{cu} = \tau_{cc} \times b \times d}$$

τ_c= Design shear stress

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$\tau_v \leq \tau_{cmax}$

V_s — vertical stirrups
— Inclined stirrups
— Bent up bars.

$(A_{sv})_{min} \geq \frac{0.4bsv}{0.87fy}$ $or\ (sv)\ \max \leq \frac{0.87fy\ Asv}{0.4b}$

2) $Vs=Vus = \frac{0.87fyAsv\ d}{sv}\ (\sin\alpha + cos\alpha)$

3) $V_{us} = 0.87f_yA_{sv}\ \sin\alpha$

No(6)

Whenever bent up bars are provided its strength should be taken as less than or equal to $0.5V_{us}$ (shear strength of reinforcement).

Procedure for design of shear in RCC

Step; 1 From the given data calculate the shear force acting on the critical section where critical section is considered as a section at a distance ‘d’ from the face of the support. However in practice the critical section is taken at the support itself.

Step – 2 For the given longitudinal reinforcement calculate pt=$\frac{100Ast}{bd}$, for this calculate T_c & T_v calculate from Pg. 73 $\tau v=\frac{Vu}{bd}$; $Vu = applied\ shear\ force$ calculated in step 1.

If $\tau_v > \tau_{cmax}$ (page 73) then in crease ‘d’

$V_{cu} = \tau_c bd$

If $V_{cu} \leq V_u$, Provide min. vertical stirrup as in page 48, clause 26.5.1.6 ie$(Sv)_{max} \leq$

$$\frac{0.87fyAsv}{0.4b}$$

Else calculate $V_{us}=V_u-V_{cu}$

Step 3 Assume diameter of stirrup & the no. of leg to be provided & accordingly calculates A_{sV} then calculate the spacing as given in P-73 clause 40.4(IS-456) This should satisfy codal requirement for (Su)Max. If shear force is very large then bent-up bars are used such that its strength is less than or equal to calculated V_{us}.

1. Examine the following rectangular beam section for their shear strength & design shear reinforcement according to IS456-2000. B=250mm, s=500mm, Pt=1.25, V_u=200kN, M20 concrete & Fe 415 steel

   Step 1: Check for shear stress

   Nominal shear stress, $\tau v = \frac{200 \times 10^3}{250 \times 500} = \frac{Vu}{bd} = 1.6N/mm^2$

   From table 19, p=73, $\tau_c = 0.67 N/mm^2$.

   From table 20, p=73, $\tau_{cmax} = 2.8 N/mm^2$.

   $\tau_c < \tau_v < \tau_{cmax}$.

   The depth is satisfactory & shear reinforcement is required.

   Step 2. Shear reinforcement

   $V_{cu} = \tau_c bd = \frac{0.67 \times 250 \times 500}{1000} = 83.75KN.$

   $V_{us} = V_u – V_{cu} = 200 – 83.75 = 116.25KN$

Assume 2 leg -10mm dia stirrups, A_{sv} =2 X $\frac{\pi}{4}$ X10^2 =157mm^2.

Spacing of vertical stirrups, obtained from IS456-2000

Sv = $\frac{0.87fyAsvd}{Vus} = \frac{0.87 \times 415 \times 157 \times 500}{116.25 \times 10^3} = 243.8 = 240mmc/c$

Check for maximum spacing

i) $S_{vmax} = \frac{0.87Asvfy}{0.4b} = \frac{0.87 \times 157 \times 415}{0.4 \times 250} = 566.8mm$

ii) 0.75d = 0.75 x 500 = 375mm.

iii) 300mm

S_{vmax} = 300mm(Least value)

2. Repeat the previous problem for the following data
   1) b=100mm, d=150mm, P_t=1%, V_u=9kN, M20 concrete & Fe 415 steel
   2) b=150mm, d=400mm, P_t=0.75%, V_u = 150KN, M25 concrete & Fe 915 steel
   3) b=200mm, d=300mm, P_t=0.8%, Vu=180kN, M20 concrete & Fe 415 steel.

3. Design the shear reinforcement for a T-beam with following data: flange width = 2000mm. Thickness of flange = 150mm, overall depth = 750mm, effective cover = 50mm, longitudinal steel = 4 bars of 25mm dia, web width = 300mm simply supported span=6m, loading =50kN/m, UDL throughout span. Adopt M20 concrete & Fe 415 steel

   <u>Step;</u> [Flange does not contribute to shear it is only for BM]

   <u>Step -1</u> Shear stress

   V= $\frac{50 \times 6}{2}$ = 150KN

   Vu= 1.5 X 150 = 225KN

   $\tau_v = \frac{v_u}{b_w d} = \frac{225 \times 10^3}{300 \times 700} = 1.07$

   $A_{st} = 4 \times 491 = 1964mm^2$. Pg – 73 0.75 → 0.56

   Pt = $\frac{100 \times 1964}{300 \times 700} = 0.93$ 1.00 → 0.62

- $\tau_c = 0.56 + \frac{(0.62-0.56)}{(1.00-0.75)}(0.93-0.75)$

= 0.6N/mm^2.

From table 20, max = 2.8

$\therefore \tau_c \leq \tau_v < \tau_{cmax} = 2.8$

Design of shear reinforcement is required

Step – 2 Design of shear reinforcement

$V_{cu} = \tau_c b_w d = \frac{0.6 \times 300 \times 700}{1000} = 126KN$

$V_{us} = V_u - V_{cu} = 225 - 126 = 99KN$

Assume 2-L, 8 dia stirrups $A_{su} = 2 \text{ X } \frac{\pi}{4} \text{ X } 8^2 = 100mm^2$

Spacing of vertical stirrups,

$S_v = \frac{0.8fyA_{su}d}{v_{us}} = \frac{0.87 \times 415 \times 100 \times 700}{99 \times 1000} = 255.28 = 250mm\ c/c$

Check for Max spacing

i) $Svmax = \frac{0.87f_yA_{su}}{0.4b_w} = \frac{0.87 \times 415 \times 100}{0.4 \times 300} = 300.87mm$

ii) $0.75d = 0.75 \times 700 = 525mm.$

iii) 300mm

$S_v < S_{vmax}$ $\therefore$ provide 2l -#8mm @ 250c/c

Step – 3 curti cement

From similar triangle

$\frac{225}{3} = \frac{126}{x} \quad \chi\ x = 1.68m = 1.6m.$

∴ provide (i) 2L – 3 8@ 250 c/c for a distance of 1.4m

(ii) 2L-#8@300 c/c for middle 3.2m length

Step.4 Detailing

Use 2- #12mm bars as hanger bars to support stirrups as shown in fig

A reinforced concrete beam of rectangular action has a width of 250mm & effective depth of 500mm. The beam is reinforced with 4-#25 on tension side. Two of the tension bars are bent up at 45° near the support section is addition the beam is provided with 2 legged stirrups of 8mm dia at 150mm c/c near the supports. If f_{ck} =25Mpa & f_y = 415Mp2. Estimate the ultimate shear strength of the support s/n

$(A_{st})_{xx} = 2 \times \frac{\pi}{4} \times 25^2 = 982mm^2.$

$P_t = \frac{100 \times 982}{250 \times 500} == 0.78\%$

$P_t = 0.75 \rightarrow 0.57$

$P_t = 1.00 \rightarrow 0.64$

For $P_t = 0.78 \Rightarrow \tau_c = 0.57 + \frac{(0.64-0.57)}{(1-0.75)}(0.78-0.75)$

$\tau_c = 0.5784.$

1) Shear strength of concrete

$V_{cu} = \tau_c bd = \frac{0.5784 \times 250 \times 500}{1000} = 72.3KN.$

2) Shear strength of vertical stirrups

$(A_{sv})_{stirrup} = 2 \times \frac{\pi}{4} \times 8^2 = 100mm^2$

$(V_{su})_{st} = \frac{0.87fyA_{su}d}{su}$ $\quad Su = 150mm$

$= \frac{0.87 \times 415 \times 100 \times 500}{150} = 120.35KN$

3) Shear strength of bent up bars

$(A_{su})_{bent} = 2 \times \frac{\pi}{4} \times 25^2 = 982mm^2$

$(V_{us})_{bent} = 0.87fy(A_{su})_{bent} \sin\propto$

$= \frac{0.87 \times 415 \times 982 \times sin45}{1000}$

= 250.7KN.

$V_u = V_{cu} + (V_{us}/_{st} + (V_{us})_{bent}$

= 72.3+120.35+250.7

V_u = 443.35KN

5. A reinforced concrete beam of rectangular s/n 350mm wide is reinforced with 4 bars of 20mm dia at an effective depth of 550m, 2 bars are bent up near the support s/n. The beam has to carry a factored shear force of 400kN. Design suitable shear reinforcement at the support s/n sing M20 grade concrete & Fe 415 steel.

V_u = 400KN, b=350mm, d=550mm, f_{ck}=20Mpas

f_y = 415Mpa, $(A_{st})_{xx}$ = 2 x 314 = 628mm^2.

<u>Step – 1 </u>Shear strength of concrete

Pt = $\frac{100 \times 628}{350 \times 550} = 0.32\%$ $\quad From\ Table - 19 \quad \tau_c = 0.4Mpa.$

$\tau_c = \frac{Vu}{bd} = \frac{400 \times 10^3}{350 \times 550}$

= 2.07 Mpa

$\tau_c < \tau_v < \tau_{cMax.}$

∴ Design of shear reinforcement is required.

Step -2 Shear strength of concrete

Vcu = $\tau_c bd = \frac{0.4 \times 350 \times 550}{1000} = 77KN.$

$V_{us} = V_u - V_{cu}$ = 323KN

Step -3 Shear strength of bent up bar.

$(A_{sv})_{bent}$ = 2 x 314 = 628mm^2.

$(V_{us})_{bent} = 0.87 f_y (A_{sv})_{bent} \sin\alpha$.

$= \frac{0.87 \times 415 \times 628 \times sin45}{1000}$

= 160.3KN $\left[< \frac{V_{us}}{2} = 161.5\right]$

NOTE: If $(V_{us})_{bent} > \frac{V_{us}}{2}$ then $(V_{us})_{bent} = \frac{V_{us}}{2}$

Step – 4 Design of vertical stirrups

$(V_{us})_{st} = V_{us} - (V_{us})_{bent}$ =323-160.3=162.7KN.

Assuming 2L-#8 stirrups

$(A_{sv})_{st} = 2 \times \frac{\pi}{4} \times 8^2 = 100mm^2$

$S_v = \frac{0.87 fy A_{sv} d}{(v_{us})st} = \frac{0.87 \times 415 \times 100 \times 550}{162.7 \times 1000} = 122mm\frac{c}{c} = 120mm$

Provide 2L-#8@120 c/c

$S_{vmax} = \frac{0.87 fy Asv}{0.4b} = 257.89mm.$

0.75d=412.5mm, 300mm.

S_{vmax} = 258mm

Shear strength of solid slab

Generally slab do not require stirrups except in bridges. The design shear stress in slab given isn table 19 should b taken as ‘$k\tau_c$’ where ‘k’ is a constant given in clause 90.2.1.1

→ The shear stress $\tau_c < k\tau_c$ hence stirrups are not provided.

→ Shear stress is not required Broz thickness of slab is very less.

Self study: Design of beams of varying depth Page: 72, clause 40.1.1

Use of SP-16 for shear design

SP – 16 provides the shear strength of concrete in table 61 Pg 178 table 62 (179) provides $(V_{us})_{st}$ for different spacing of 2 legged stirrups of dia 6,8,10 & 12mm. Here it gives the value of $\frac{v_{us}}{d}$ in kN/cm where 'd' is in cm. Table 63, Pg 179 provides shear strength of 1bent up bar of different dia.

Procedure

Step- 1; Calculate $= \tau_c = \frac{V_u}{bd}$ & obtain τ_c from table 61 & also obtain τ_{cmax} from table 20, Pg 73 –IS956 If $\tau_c < \tau_c < \tau_{cmax}$ then design of shear reinforcement is necessary.

Step - 2 $V_{cu} = \tau_c bd$

$V_{us} = V_u - V_{cu}$

Assuming suitable stirrup determine the distance for $\frac{v_{us}}{b}$ in kN/cm.

H.W Design all the problems using SP-16 solved earlier.

Bond & Anchorages

$$F_b = \underbrace{(\pi\phi)}_{\text{Permitter length}} \quad (l_d) \quad (\tau_{bd})$$

stress

$T = \frac{\pi}{4} \times \emptyset^2 \times \sigma_s$

$F_b = T$

$l_d\ \tau_{bd} = \frac{\pi}{4} \times \emptyset^2 \times \sigma_s$

$l_d = \frac{\sigma_s \emptyset}{4\tau_{bd}}$

τ_{bd} = Anchorage bond stress

τ_{bf} =flexural bond stress

For CTDs HYSD bars, flexural bond stress is ignored bozo of undulations on surface of steel.

$\tau_{bf} = \frac{V}{\sum ojd}$ $\qquad \sum o = summation\ of\ permeter\ of\ bars$

Z= lever arm

Codal requirement

$$\frac{M_1}{V} + lo \geq ld \rightarrow pg - 44 \quad \rightarrow clause\ 26.2.3.3, P - 42$$

Where $l_d = \frac{\sigma_s \emptyset}{4\tau_{bd}}$; σ_s = tensile stress in steel

τ_{bd} = design bond stress.

The value of this stress for different grades of steel is given in clause 26.2.1.1 →Pg- 93 of code for mild steel bar. These values are to be multiplied by 1.6 for deformed bars. In case of bar under compression the above value should be increased by 25% σ_s = 0.87fy for limit state design. If lo is insufficient to satisfy (1), then hooks or bents are provided. In MS bars Hooks are essential for anchorage

Min = 4ø K= 2 for MS bars

=4 for CTD bars

Hook for Ms bars

(K+1)ø

Standard 90^0 bond

Pg – 183 fully stressed =0.87fig

1. Check the adequacy of develop. Length for the simply supported length with the following data.
   (IV) c/s = 25 x 50cm (ii) span=5m (iii) factored load excluding self wt =160KN/m. iv) Concrete M20 grade, steel Fe 415 grade. (v) Steel provided on tension zone. 8 bars of 20mm dia.

Solve: C_e = 50mm, h=500mm, d=450mm

q_{self} = 0.25 x 0.5 x 1 x 25

= 3.125KN/m

q_{uself} = 1.5 x 3.125 = 4.6875KN/m.

Total load = 60 + 4.6875

= 64.6875KN/m

$V_u = \frac{64.6875 \times 5}{2} = 161.72KN$

X_{ulim} = 0.48 d=0.48 x 450=216mm

$M_{ulim} = 0.36 f_{ck}\, x_u b(d-0.42x_u)10^6$

= 139.69kN-m

Let Ws = 300, l_o = 150mm.

$$\frac{M_1}{V} + l_o = \frac{139.69}{161.72} + 0.15 = 1.01\text{m}$$

$$\text{Ld} = \frac{\emptyset\sigma_2}{4J_{bd}} \qquad \sigma_2 = 0.87fy$$

$\tau_{bd} = 1.6 \times 1.2 = 1.92$ Table R43.

$$l_d = \frac{20 \times 0.87 \times 415}{4 \times 1.92 \times 1000} = 0.94m.$$

$$\frac{M_1}{V} + l_o > ld\ ;\textit{hence safe}$$

2. A cantilever beam having a width of 200mm & effective depth 300mm, supports a VDL hug total intensity 80KN(factored) 4nos of 16mm dia bars are provided on tension side, check the adequacy of development length (l_d), M20 & Fe 415.

Design for torsion

$$\frac{I}{I_p} = \frac{Go}{l} = \frac{fs}{R} \Rightarrow fs = \frac{TR}{IP}$$

$$\tau_{tmax} = \frac{T}{kb^2D} \text{ or } \frac{T}{kb^2h}$$

For the material like steel

Principal stresses all 4 faces reinforcement

Longitudinal bars stirrups

Torsion

Secondary torsion Primary torsion

Nature of connection
Of constructions

In analysis the effect of
torsion rigidity is taken

to account

Eg. Chejja or sunshade L-window

Cross – cantilever
Eg- for Secondary torsion

Eg. (1) Plan of Framed Structure
Primary torsion

(3) Ring beam in elevated tank

(2) Arc of a circle

RCC Beams → BM(I)
SF (Shear force)(II)
Torsion (III)

Combination of (I) – (II)& (I) –(II)

IS-456 →Pg – 79 Procedure

(1) Flexure & Torsion

$M_{e1}=M_u + M_T$

$M_T = T_u\left(\frac{1+d/b}{1.7}\right) \quad D = h$

D= Overall depth, b=breadth of beam
Provide reinforcement for Mt in tension side
If $M_T > M_u$ provide compression reinforcement for

(2) Shear & Torsion

$V_e = V_u + \frac{1.6Tu}{b}$

$\tau_v = \frac{v_e}{bd}$ $; For\ safe\ design\ \tau_v > \tau_c\ design\ shear\ reinforcement.$

Shear reinforcement

$A_{sv} = \frac{T_u s_v}{bd,0.87fy} + \frac{v_u s_v}{25d_1(0.87fy)}$

$= \frac{(\tau_{ve} - \tau_c)bsv}{0.87fy}$

Pg.48 →clause 26.5.1.7
S_{Vmax} is least of

i) x_1
ii) $\frac{x1+y1}{4}$
iii) 300mm
$A_{sw} > \frac{0.1}{100} \times\ b\ \times d$

1. Design a rectangular reinforced concrete beam to carry a factored BM of 200KN-m, factored shear force of 120kN & factored torsion moment of 75 KN-m Assume M-20 concrete & Fe 415 steel

Sol: M_u = 20KN-m, T_u=75KN-m, V_u=120K.N.

Step -1= Design of BM & Torsion

Assume the ratio $\frac{D}{b}$ =2

$M_T = \frac{T_u(1+\frac{D}{b})}{l_o 7} = \frac{75(1+2)}{1.7} = 132.35KN - m.$

No compression reinforcement design is necessary

$M_T < M_u$

$M_{e1} = M_T + M_u = 200 + 132.35 = 332.35$KN-m.

$d_{bal} = \sqrt{\frac{M_{e1}}{\theta_{um} b}} = \sqrt{\frac{332.35 \times 10^6}{2.76 \times 300}} = 633.5$mm.

Assume b=300mm, θ_{lim}=2.76

Assuming overall depth as 700mm & width as 350mm & effective cover =50mm.

D Provided =700-50

=650mm

Area of steel required for under reinforced section,

$P_t = \frac{50 f_{ck}}{fy}\left[1 - \sqrt{1 \frac{4.6 M_{c1}}{fckbd^2}}\right]$

$= \frac{50 \times 20}{415}\left[1 - \sqrt{1 \frac{4.6 \times 332.35 \times 10^6}{20 \times 350 \times 650^2}}\right]$

$= 0.73 < 0.96$

(p_t lim)

$\therefore A_{st} = \frac{0.73 \times 350 \times 650}{100} = 1660.75 mm^2$

Assume 25mm dia bars = $\frac{1660.75}{4.91}$=3.38≈4

Provide 4 bars -#25 dia

(A_{st}) provided = 1963mm^2

Step – 2 Design for shear force & torsion

$V_e = V_u + \frac{1.6T_u}{b}$

$V_e = 120 + \frac{1.6 \times 75}{0.35} = 462.86KN$

$\tau_{ve} = \frac{462.86 \times 10^3}{350 \times 650} = 2.03 < 2.8$ (T_{cmax}) P-73

$Pt = \frac{4 \times 491}{350 \times 650} \times 100 = 0.86 \qquad \tau_{ve} > \tau_c$

Table – 19 $\tau c = 0.56 + \frac{(0.62-0.56)}{(1.00-0.75)}(0.86-0.56) = 0.58N/mm^2$

Assuming 2 – legged =12mm dia;

$A_{su} = 2 \times \frac{\pi}{4} \times 12^2 = 226mm^2$

From IS-456;Pg-75

$Sv = \frac{0.87fyA_{sv}}{\frac{T_u}{b_1d_1}+\frac{v_u}{2.5d_1}}$

b_1=275mm, d_1 = 600mm

y_1 = 600+25+2 x 6 = 637mm

x_1 = 275 + 25 + 2 x 6 = 312mm.

x_1 y_1 → dist of centre of stirrups.

Provide 2 - #12@ top as hanger bars

$Sv = \frac{0.87 \times 415 \times 226}{\frac{75 \times 10^6}{275 \times 600}+\frac{120 \times 10^3}{2.5 \times 600}} = 152 \approx 150c/c$

Check

1. $A_{sv} > \frac{(\tau_{ve}-\tau_c)svb}{0.87fy} = \frac{(2.03-0.58)150 \times 350}{0.87 \times 415}$
   226> 210.84
2. S_{vmax} a) x_1 = 312mm b) $\frac{x_1+y_1}{4} = \frac{312+637}{4} = 237.25$ c) 300mm
   d) 0.75d = 487.5

As D=h>450 , provide side face reinforcement

$A_{sw} = \frac{0.1}{100}$ x 350 x 650 = 227mm^2

Provide 2-#16 bars (A_{st} = 400mm^2) as side face

2. Repeat the same problem with Tu=150KN-m & other data remain same

Solve M_u = 200KN-m,T_u=150KN-m, V_u=120KN

Step – 1 Design of EM & torsion

Assume the ratio D/b = 2.

$M_T = \frac{T_u(1+\frac{d}{b})}{1.7} = \frac{150(1+2)}{1.7}$ = 264.70KN-m

$M_T > M_u$ Compression reinforcement is required.

$M_G = M_T + M_u$ = 264.7 + 200 = 4.64.7KN-m

$d_{bal} = \sqrt{\frac{M_{e1}}{\theta_{lim} b}} = \sqrt{\frac{464.7 \times 10^6}{2.76 \times 300} = 749.15mm}$

Assume b= 300mm, θ_{lim} = 2.76.

Assuming over al depth as 800mm & width as 400mm & effective cover = 50mm.

D provided = 800-50 = 750mm
Area of steel required for under reinforced section,

$$P_t = \frac{50fck}{fy}\left[1 - \sqrt{\frac{4.6M_{e1}}{fckbd^2}}\right]$$

$$= \frac{50 \times 20}{415}\left[1 - \sqrt{\frac{4.6 \times 464.7 \times 10^6}{20 \times 400 \times 750^2}}\right]$$

$P_t = 0.66 < 0.96(p_{tlim})$

Assume 25mm dia bars = $\frac{2280}{491}$= 5.86≈6.

Provide 6 bars - #25 dia

A_{st2}= 255.99

A_{st} = 3135.99 ⇒ 7 - #25.

M_{e2} = 264.7 – 200 = 64.7KN-m.

A_{sc} = $\frac{M_{e2}}{f_{sc}(d-d)} = 260.95$ $\qquad$ $fsc = \frac{d^1}{d} = 0.066\ from\ graph$

$fsc = 354.2$

A_{st1} = $\frac{p_t bd}{100}$, A_{st2} = $\frac{f_{sc}A_{sc}}{0.87fy}$; $A_{st1} + A_{st2} = A_{st}$

A_{st} = kb of bars

<u>Step – 2</u> Design for shear force torsion

$V_e = v_u + \frac{1.6T_u}{b}$

$= 120 + \frac{1.6 \times 150}{0.4} = 720KN.$

$\tau_{ve} = \frac{720 \times 10^3}{400 \times 750} = 2.4 < 2.8\ (\tau_{cmax})\quad P-73$

P_t = 0.98 $\qquad$ τ_c=0.61

Assuming 2 legged 12mm dia

$A_{sv} = 2\times \frac{\pi}{4} \times\ 12^2 = 226mm^2$

From IS-456, P_g – 75

$S_v = \frac{0.87fyAsv}{\frac{Tu}{b_1d_1}+\frac{Vu}{2.5d_1}}$

Limit state of service ability

Deflection $\qquad$ Crack –width $\qquad$ Vibration

(Un factored load) <u>Working load</u>

1. Deflection → Span to effective depth ratio
   → Calculation

$y_{max} = \frac{5}{384} \frac{wl^4}{EI}$

$y_{max} \leq \frac{l}{250}$ $'N\& \; mm'$

As control of deflection by codal provision for l/d ratio

Cause 23 .2.1 Pg – 37 of IS 456 – 2000

Type of beam	l/d ratio	
	Span, $l \leq 10m$	Span>10m
i) Cantilever beam	7	Should be calculated
ii) Simply supported beam	20	$\frac{20 \times 10}{span}$
iii) Continuous beam	26	$\frac{26 \times 10}{span}$

Effect on l/d ratio

1. Tension reinforcement : > 1%

   Pg – 38; fs = 0.58fy $\frac{(Ast)req}{(Ast)prov}$

   SP-24 →Explanatory hand look

   $M_{ft} = [\ 0.225 + 0.003225 f_s + 0.625 \log_{10}(p_t)]^{-1} \leq 2$

2. Compression reinforcement.

   $Mfc = \left[\frac{1.6pc}{pc+0.275}\right] \leq 1.5$

3. Flange action or effect

   $M_{fl} = 0.8$ for $\frac{bw}{bf} \leq 0.3$

   $= 0.8 + \frac{2}{7}\left[\frac{bw}{bf} - 0.3\right] for \frac{bw}{bf} > 0.3$

   $\frac{l}{d} = m_{ft} \times m_{fc} \times m_{fl} \times \left(\frac{l}{d}\right) basic$

Design

1. Flexure + torsion
2. Check for shear + Torsion, bond & Anchorage
3. Check for deflection

1. A simply supported R-C beam of effective span 6.5m has the C/S as 250mm wide by 400mm effective depth. The beam is reinforced with 4 bars of 20mm dia at the tension side & 2-bars of 16mm dia on compression face. Check the adequacy of the beam with respect to limit state deflection, if M20 grade concrete & mild steel bars have been used.
B=250mm, d=400mm, f_{ck}=20Mpa, f_y=250Mpa

$A_{st} = 4 \times \frac{\pi}{4} \times 20^2 = 1256\ mm^2$

$A_{sc} = 2 \times \frac{\pi}{4} \times 16^2 = 402\ mm^2$

$P_t = \frac{1256 \times 100}{250 \times 400} = 1.256$

$P_l = \frac{402 \times 100}{250 \times 400} = 0.402$

From Pg – 37, $(l/d)_{basic} = 20$

$f_s = 0.58 \times 250 \times 1 = 145$

$m_{ft} = [0.225 + 0.00\ 3225 \times 145 + 0.625\log_{10}(1.256)]^{-1} = 1.325$

$m_{fc} = \left[\frac{1.6 \times 0.402}{0.402+0.275}\right] = 1.12\ (graph)$

$m_{fl} = 1$ (rectangular section)

$(\frac{l}{d})$0Renquired $= 1.325\ x\ 1.12\ x\ 1\ x\ 20 = 29.79$

Check

$(\frac{l}{d})\ provided = \frac{6500}{400} = 16.25 < 29.79; safe$

2) Check the adequacy of a T-beam with following details (i) Web width (wb) = 300mm, (ii) Effective depth (d) = 700mm (iii) flange width (bf) = 2200mm (iv) effective span of simply supported beam(l) =8m (v) reinforcement a) tension reinforcement – 6bars of 25 dia b) compression reinforcement – 3 bars of 20 dia (vi) Material M25 concrete & Fe 500 steel.

<u>Deflection calculation</u>
- Short term deflection
- Long term deflection (shrinkage, creep)

1. Short term deflection

Ec= $5000\sqrt{fck}\,; pg-16, cluse$ 6.2.3.1.

Slope of tangent drawn @ origin →Tangent modulus

Slope of tangent drawn @ Specified point →secant Modulus 50% of Material

$I_{gr}=\frac{bh^3}{12}$ For elastic; Ief → cracked section

Pg. 88 $I_{eff}=\frac{Ir}{1.2\frac{Mr}{M}\frac{Z}{d}\left(1-\frac{x}{d}\right)\frac{bw}{b}}$; $I_r \leq I_{eff} \leq I_{gross}$

I_r = Moment of inertia of cracked section

M_r = cracking Moment =

- NA →stress is zero
- CG→ It is point where the wt. of body is concentrated
- $Y_t \neq x$
- M= Max. BM under service load: Z=lever arm

X= depth of NA : bw = width of web: b= width of compression face

(For flanged section b=bf)

For continuous beam, a modification factor xe given in the code should be used for Ir, Igr, & Mr. The depth of NA 'x' & lever arm Z has to be calculated by elastic analysis is working stress method explained briefly below.

Introduction to WSM

$M = \frac{E_s}{E_c} = \frac{x-d^1}{3\sigma_{cbc}}$; $\sigma_{cbc} =$ Permissible stress P g.80

From property of similar triangles,

$$\epsilon_s^1 = \frac{x-d^1}{x} \times \epsilon \;\&\; \epsilon_s = \frac{d-x}{x} \times \epsilon_c \rightarrow 1$$

$f_c = E_c$ x $\epsilon_c \rightarrow 2$ $\quad f_s^1 = E_s \times \in_s^1 = mE_c \in_s^1 \rightarrow 3$

$f_s = E_s \in_s = mE_c \in_s \rightarrow$ 3a.

C = T.

$A_{sc}\ f_s^1 + \frac{1}{2} f_c bx = A_{st} f_s\ sub\ 1, 2\ \&\ 3$

$\frac{1}{2} bx^2 + x[(m(A_{st} + A_{sc})] - m[(A_{st}d + A_{sc}d^1)] = 0 \rightarrow 4$

Eq (4) can also be written in the form of

$$\frac{bx^2}{2} + (1.5m - 1)A_{sc}(x - d^1) = mA_{st}(d - x) \rightarrow 5$$

In eq.5, the modular ratio for compression steel is taken as (1.5m)

<u>Use of SP-16 for calculating I_{ef}</u>

1. Using Table – 91: Pg – 225-228, we can find NA depth for simply reinforced, Pc =0

2. Using Table – 87-90, find out cracked moment of inertia Ir

3. I_{eff} chart -89, Pg.216

Cracked moment of inertia can be found by the following equation.

For singly reinforced section

$I_r = \frac{bx^3}{3} + mA_{st}(d - x)^2$

For doubly reinforced section,

$I_r = \frac{bx^3}{3} + (m - 1)A_{sc}(x - d^1)^2$

$+ mA_{st}(d\text{-}x)^2$

Shrinkage deflection

2. <u>Long Term deflection</u>

——Creep effect deflection

a) shrinkage deflection
   reduces stiffness (EI)
   ε_{sh}=0.004 to 0.0007 for plain concrete

   = 0.0002 to 0.0003 for RCC

   Ysh = $k_2 \Psi_{cs} l^2$

   K_3 = cantilever – 0.5

   Simply supported member – 0.125.

   Continuous at one end – 0.086 Pg-88

   Full Continued – 0.063

$$\Psi \in_s = \frac{k_4 \in_{cs}}{h};\ k_4 = \frac{0.72(p_t - pc)}{\sqrt{pt}} \leq 1.0\ for\ (p_t - pc)0.25 - 1.0$$

$$k_4 = \frac{0.65(p_t - pc)}{\sqrt{pt}} \leq 1.0\ for\ (p_t - pc) - 1.0$$

b) creep deflection → permanent

y_{scp} =Initial deflection + creep deflection using E_{cc} in plane of E_c due to permanent $E_{sc} = \frac{Ec}{1+c_c}$

   C_c= Creep co-efficient

   1.2 for 7 days loading
   1.6 for 28 days loading
   1.1 for 1 year loading

   Y_{sp} = Short per deflection using E_c

   $Y_{cp} = Y_{scp} - Y_{sp}$

A reinforced concrete <u>cantilever beam</u> 4m span has a rectangular section of size 300 X 600mm overall. It is reinforced with 6 bars of 20mm dia on tension side & 2 bars of 22mm dia on comp. side at an effective cover of 37.5mm. Compute the total deflection at the free end when it is subjected to UDL at service load of 25KN/m, 60% of this load is permanent in nature. Adopt M20 concrete & Fe 415 steel.

Sol:

$A_{st} = 6 \text{ x } \frac{\pi}{4} \times 20^2 = 1885mm^2 \quad l = 4m, w = \frac{25KN}{m}.$

$A_{sc} = 2 \text{ x } \frac{\pi}{4} \times 22^2 = 760mm^2$

$f_{ck} = 200Mpa, f_y = 415Mpa, E_s = 2 \text{ x } 10^5Mpa.$

$E_c = 5000\sqrt{20} = 2.236 \times 10^4 Mpa$

$f_{cr} = 0.07\sqrt{fck} = 3.13Mpa$

$m = \frac{Es}{Ec} = \frac{2 \times 10^5}{2.236 \times 10^4} = 8.94$

$Ig = \frac{300 \times 600^3}{12} = 5.4 \times 10^9 mm^4 = \frac{b03}{12}$

$Yt = \frac{D=h}{2} = 300mm \; (\frac{600}{2})$

Step : 1 Short term deflection

$y_{short} = \frac{wl^4}{8E_c I_{eff}}$

$I_{eff} = \frac{Ir}{102 - \frac{M_{cr}Z}{M\;d}(1 - \frac{x}{d})\frac{bw}{b}}$

$M_{cr} = \frac{f_{cr}I_g}{y_t} = \frac{3.13 \times 5.4 \times 10^9}{300} = \frac{56.34 \times 10^6 N-mm}{106} = 56.34KN - M$

$M = \frac{wl^2}{2} = \frac{25 \times 4^2}{2} = 200KN - m.$

$\frac{M_{cr}}{M} = \frac{56.34}{200} = 0.282$

From equilibrium condition

$\frac{bx^2}{2} + (m-1)A_{sc}(x - d^1) = mA_{st}(d - x)$

$$\frac{300x^2}{2} + (8.94 - 1)760(x - 37.5) = 8.94 \times 1885(600 - x)$$

x^2 #52.58x-64705.5=0

x = 189.28mm

$Z \approx d - \frac{x}{3} = 562.5 - \frac{189.28}{3} = 499.41mm.$

$I_r = \frac{b \times x^3}{3} + (m-1)A_{sc}(x - d^1)^2 + mA_{st}(d-x)^2$

$= 3.1645 \times 10^9 mm^4$

$I_{eff} = \frac{3.1645 \times 10^9}{102 - 0.282 \times \frac{499.41}{562.5}\left(1 - \frac{189.28}{562.5}\right)\frac{300}{300}}$

$= 3.01 \times 10^9 mm^4$

$Y_{short} = \frac{25 \times (4000)^4}{8 \times 2.236 \times 10^4 \times 3.061 \times 10^9}$

$= 11.31mm$; $I_{cr} \le I_{eff} \le I_g$

$\therefore I_{eff} = I_{cr} = 3.1645 \times 10^9 mm^4.$

Step – 2 long term deflection

a) Due to shrinkage

$Y_{cs} = k_3 \Psi_{cs} l^2$ $\quad p_t = \frac{1885 \times 100}{300 \times 562.5} = 1.117$

K3 = 0.5 for cantilever $\quad p_c = \frac{760 \times 100}{300 \times 562.5} = 0.45$

$P_t - P_c = 1.117 - 0.45 = 0.667 < 1.0$

$K_4 = \frac{0.72 (p_t - p_c)}{\sqrt{pt}} = 0.454$

Ecs = shrinkage strain = 0.0003 (Assumed value)

$\Psi cs = \frac{k_4 \in_{cs}}{h} = 2.27 \times 10^{-7} = \frac{0.454 \times 0.0003}{600}$

$Y_{cs} = 0.5 \text{ x } 2.2 + 10^{-7} \text{ x } 4000^2 = 1.82\text{mm}.$

b) Due to creep

$E_{cc} = \frac{Ec}{1+c_c}$; $c_c = 1.6$ [*from code for 28 days*]

$= \frac{2.236 \times 10^4}{1+1.6} = 8600 Mpa$

$Y_{scp} = \frac{w_p \times l^4}{8 E_{cc} I_{eff}}$

$= \frac{0.6 \times 25 \times 4000^4}{8 \times 8600 \times 3.1605 \times 10^9} = 17.64$

$Y_{sp} = 0.6\ y_{short} = 6.8\text{mm}.$

$Y_{cp} = 17.64 - 6.8 = 10.84$

$Y = y_{short} + y_{cs} + y_{cp} = 23.97$

$= 11.31 + 1.82 + 10.84$

Unsafe

Doubly reinforced section

$$wd = \frac{vol}{0.3 \times 0.75 \times 1 \times 25}$$

$M_d = \frac{w_d R^2}{8} = 17.58 KN - m.$

$M_l = \frac{wdl}{4} = \frac{80 \times 5}{4} = 100 KN - W$

$M_u = 1.5(M_D + M_c) = 1.5(17.58+100) = 176.37\text{KN-M}$

$X_{ulim} = 0.48 \text{ x } d = 324\text{mm}$

$M_{ulim} = Q_{lim} bd^2$

$= \frac{4.14 \times 300 \times 675^2}{10^6}$

= 565.88KN-m.

Singly reinforced s/n $A_{st} = \frac{pt_{lim}bd}{100}$

$P_{tlim} = 7.43\%$ $\quad = \frac{1.43 \times 300 \times 675}{100}$

6 -#25bar is taken.

$(A_{st})_{provided} = 6 \times 490 = 2940 > 2895$

2b) b=150mm, d=400mm, p_t=0.75%, v_v=150KN, f_{ck} = 25, f_y=415

$\tau_v = \frac{150 \times 10^3}{150 \times 400} = \frac{2.5N}{mm^2}$

$\tau_c = 0.57 N/mm^2$.

$\tau_{cmax} = 3.1 N/mm^2$.

$\tau_c < \tau_v < \tau_{cmac}$

$v_{cu} = \tau_c bd = \frac{0.57 \times 150 \times 400}{10^3} = 34.2KN.$

$v_{us} = v_u - v_{cu} = 115.8KN.$

Assume 2L-10# $A_{su} = 2 \times \frac{\pi}{4} \times 10^2$

$= 157.07 mm^2$.

$S_v = \frac{0.87 fy A_{sv} d}{v_{us}}$

$= \frac{0.87 \times 415 \times 157.07 \times 400}{115.8 \times 10^3}$

= 195.88mm

$S_{vmax} = \frac{0.87 f_y A_{sv}}{0.4b} = 945.16mm$

0.75d = 300mm

300mm

∴ provide 2l-#10@ 195.99 c/c.

2c) b=200mm, d=300mm, p_t=0.8%, v_v=180KN, f_{ck}=20, f_y=415.

$\tau_c = \frac{vu}{bd} = \frac{180 \times 10^3}{200 \times 300} = \frac{3N}{mm^2}$

τ_{cmax} = 2.8 pt 0.75 → τ_c 0.56

1.0 0.62

$Pf = 0.8; \tau = 0.56 + \frac{(0.62-0.56)}{\frac{(1-0.75)}{(0.8-0.75)}}$

τ_c = 0.572 N/mm^2.

$\tau_v > \tau_{cmax}$; unsafe, hence increased

Let 'd' be 350mm

τ_v = 2.57N/mm^2.

∴ $\tau_c < \tau_v < \tau_{cmax}$; safe sh.rei required.

Step 2 shear reinforcement.

$V_{cu} = \tau_c bd$ = 0.572 x 200 x 350 = 40KN.

$V_{us} = v_u - v_{cu}$ = 180-40 = 140KN.

Assume 2L-#10; A_{sv}=157mm^2.

Spaces of vertical stirrups

$S_v = \frac{0.87 \times 415 \times 157 \times 350}{140 \times 1000} = 141.71mm \approx 140mm$

Step 3: Check for max. spacing

1. $S_{vmax} = \frac{0.87 \times 157 \times 415}{0.4 \times 200} = 708.5mm.$
2. 0.75d = 0.75 x 350 = 262.5mm
3. 300mm $s_v < s_{umax}$ is 140 < 262.5mm

Hence provide 2L-#10@140 c/c.

4) le=10m

$W=q^{1}+q^{11}$ $q^{1} = 45$ KN/m.

$b_w = 300$mm, s=3m, f_{ck}=20Mpa, f_y=415Mpa

$bf = \frac{lo}{6} + bw + 60f \qquad Assume\ D_f = 100mm$

$= \frac{10000}{6} + 300 + 6(100)$

= 2566.67mm < 3000mm

$h = \frac{lo}{12}\ to\ \frac{lo}{15}$ $[8333.33\ to\ 666.67] Assume\ c_e = 50mm$

h=750mm, d=700mm

b x h x l density

self = 0.3 x 0.65 x 1 x 25

$q^{11} = 4.875$ KN/m

W= 45 + 4.875 = 49.875 KN/m

$M = \frac{50.625 \times 10^2}{8} + \frac{50 \times 10}{4} = \frac{748.8KN}{m}.$

$M_u = 1136.7$ KN/m

$(A_{st})_{oppr} = \frac{Mu}{0.87fy\left(d - \frac{Df}{2}\right)} = 4843.56mm^2 \quad 10 - \#25 = 9910mm^2$

Step2: $x_u = \frac{0.87fyAst}{0.36f_{ck}b_f} = 95.92mm < D_f < x_{ulim}$

$M_{ur} = 0.36_{fck}\ x_u b_f(d-0.42x_u) = 1169.4$KN-m.

2. Step 2 : $b_f = \frac{lo}{6} + bw + 6D_f$

$= \frac{6000}{6} + 300 + 6(120)$

= 2020mm.

$h = \frac{lo}{12}\ to\ \frac{lo}{15} [500\ to\ 400]\ self\ wt = 1\ x\ 1\ x\ 0.12\ x\ 25 = 34/m^2$

h=450mm   q=8 5KN/m^2

d=h-50=400mm   w-q x s x 1 = 8 x 3 x 1

$(A_{st})_{app} = \frac{Mu}{0.87fy\left(d - \frac{Df}{2}\right)}$ $self\ wt = \frac{24KN}{m}, 0.3\ x\ 0.33\ x\ 1\ x\ 25 = 24$

$= \frac{101.25 \times 10^6}{0.87 \times 415 \left[400 - \frac{120}{2}\right]}$ $Md = \frac{wl^2}{8} = W = 26.4,\ \ Md = 119.14\ \ Mu = 178.7$

$= 824.80mm^2$.

Provide 3-#20 $\therefore (A_{st})_{provided}$ 3 x 314 = 942 > 824.8

Step 2 : Assume NA to be on flange

$C_u = 0.36f_{ck}b_f x_u$

$T_u = 0.87f_y A_{st}$.

$x_u = \frac{0.87fyA_{st}}{0.36f_{ck}b_f} = 23.38mm < \ D_f < \ x_u$

$x_{ulim} = 192mm$.

$M_{ur} = 0.36f_{ck}b_f x_u\ (d-0.42x_u)$

= 0.36 x 20 x 2020 x 23.38 [400-0.42 x 23.38]

= 132.67KN/m < 101.25KN-m.