Module
10
Compression Members

Lesson 27

Slender Columns

Instructional Objectives:

At the end of this lesson, the student should be able to:

- define a slender column,
- give three reasons for its increasing importance and popularity,
- explain the behaviour of slender columns loaded concentrically,
- explain the behaviour of braced and unbraced single column or a part of rigid frame, bent in single or double curvatures,
- roles and importance of additional moments due to P- Δ effect and moments due to minimum eccentricities in slender columns,
- identify a column if sway or nonsway type,
- understand the additional moment method for the design of slender columns,
- apply the equations or use the appropriate tables or charts of SP-16 for the complete design of slender columns as recommended by IS 456.

11.27.1 Introduction

Slender and short are the two types of columns classified on the basis of slenderness ratios as mentioned in sec.10.21.5 of Lesson 21. Columns having both l_{ex}/D and l_{ey}/b less than twelve are designated as short and otherwise, they are slender, where l_{ex} and l_{ey} are the effective lengths with respect to major and minor axes, respectively; and D and b are the depth and width of rectangular columns, respectively. Short columns are frequently used in concrete structures, the design of such columns has been explained in Lessons 22 to 26, loaded concentrically or eccentrically about one or both axes. However, slender columns are also becoming increasingly important and popular because of the following reasons:

(i) the development of high strength materials (concrete and steel),

(ii) improved methods of dimensioning and designing with rational and reliable design procedures,

(iii) innovative structural concepts – specially, the architect's expectations for creative structures.

Accordingly, this lesson explains first, the behaviour of slender elastic columns loaded concentrically. Thereafter, reinforced concrete slender columns loaded concentrically or eccentrically about one or both axes are taken up. The design of slender columns has been explained and illustrated with numerical examples for easy understanding.

10.27.2 Concentrically Loaded Columns

It has been explained in Lessons 22 to 26 that short columns fail by reaching the respective stresses indicating their maximum carrying capacities. On the other hand, the slender or long columns may fail at a much lower value of the load when sudden lateral displacement of the member takes place between the ends. Thus, short columns undergo material failure, while long columns may fail by buckling (geometric failure) at a critical load or Euler's load, which is much less in comparison to that of short columns having equal area of cross-section. The buckling load is termed as Euler's load as Euler in 1744 first obtained the value of critical load for various support conditions. For more information, please refer to Additamentum, "De Curvis elasticis", in the "Methodus inveiendi Lineas Curvas maximi minimive proprietate gaudentes" Lausanne and Geneva, 1744. An English translation of this work is given in Isis No.58, Vol.20, p.1, November 1933.

The general expression of the critical load P_{cr} at which a member will fail by buckling is as follows:

$$P_{cr} = \pi^2 EI/(kl)^2$$

where E is the Young's modulus I is the moment of inertia about the axis of bending, l is the unsupported length of the column and k is the coefficient whose value depends on the degree of restraints at the supports. Expressing moment of inertia $I = Ar^2$, where A is the area of cross-section of the column and r is the radius of gyration, the above equations can be written as,

$$P_{cr} = \pi^2 EA/(kl/r)^2 \qquad (10.62)$$

Thus, P_{cr} of a particular column depends upon kl/r or slenderness ratio. It is worth mentioning that kl is termed as effective length l_e of the column.

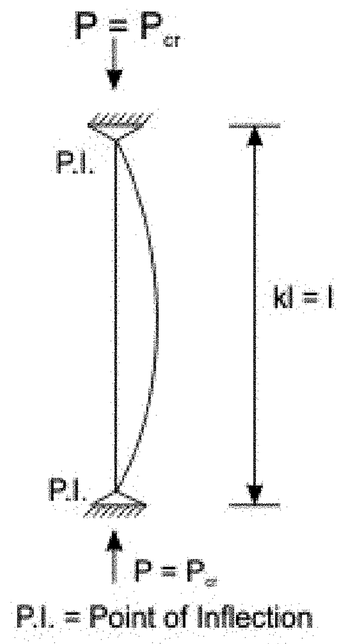


Fig. 10.27.1: Column - hinged at both ends, (k = 1)

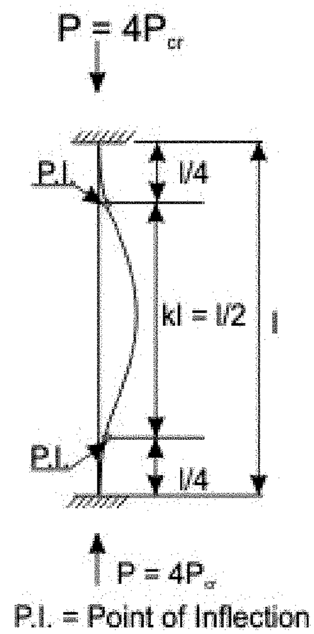


Fig. 10.27.2: Column - fixed at both ends, (k = 0.5)

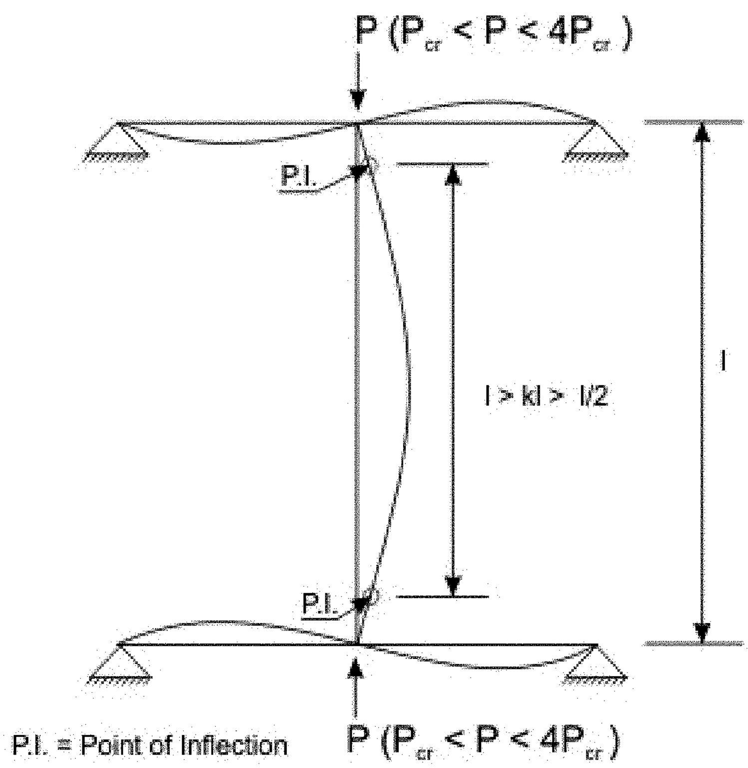


Fig.10.27.3: Column supported on cross-beams ($0.5 < k < 1$)

Figures 10.27.1 and 2 show two elastic slender columns having hinge supports at both ends and fixed supports against rotation at both ends, respectively. Figure 10.27.3 presents a column of real structure whose end supports are not either hinged or fixed. It has supports partially restrained against rotation by the top and bottom beams. Each of the three figures shows the respective buckled shape, points of inflection *PI*s (points of zero moment), the distance between the *PI*s and the value of *k*. All the three columns, having supports at both ends, have the *k* values less than one or at most one. By providing supports at both ends, one end of the column is prevented from undergoing lateral movement or sidesway with respect to the other end.

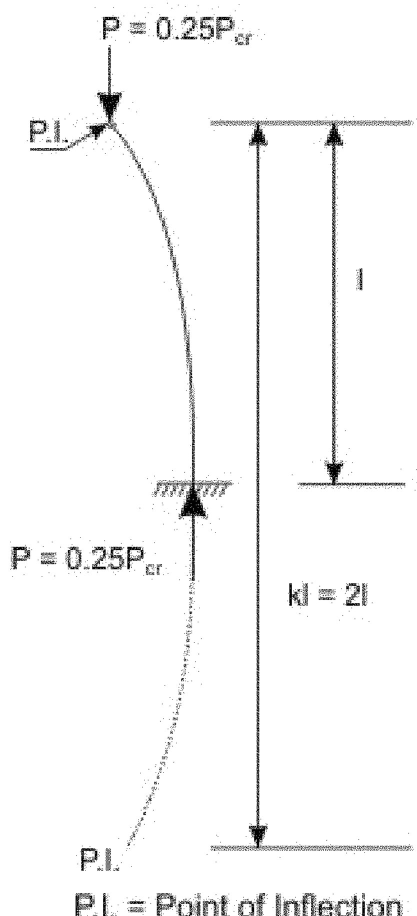


Fig. 10.27.4: Column - fixed at one end and free at other end, (k = 2)

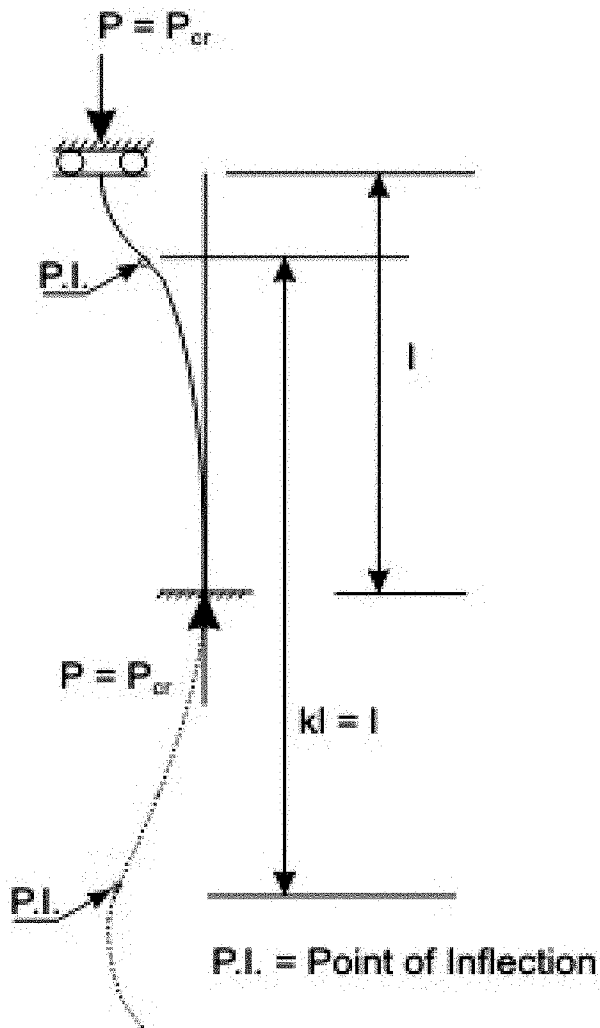


Fig. 10.27.5: Column rotationally fixed at both end, (k = 1)

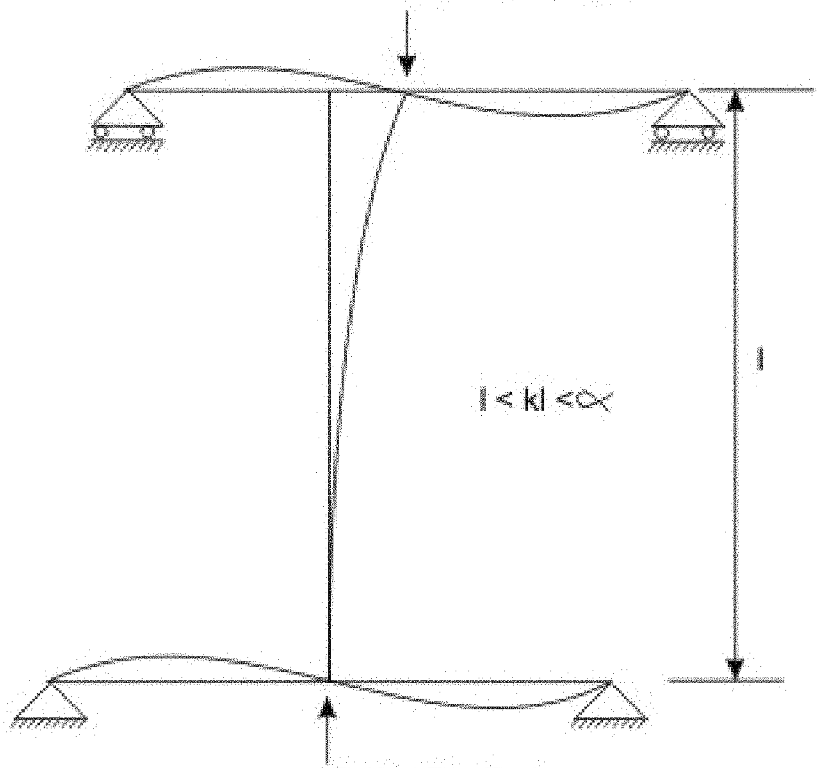


Fig.10.27.6: Column partially restrained at both ends
($1 < k < \infty$)

However, cantilever columns are entirely free at one end, as shown in Fig.10.27.4. Figure 10.27.5 shows another type of column, rotationally fixed at both ends but one end can move laterally with respect to the other. Like that of Fig.10.27.3, a real column, not hinged, fixed or entirely free but restrained by top and bottom beams, where sideway can also take place. Each of these three figures, like those of Figs.10.27.1 to 3, presents the respective buckled shape, points of inflection (*PI*s), if any, the distance between the *PI*s and the value of *k*. All these columns have the respective *k* values greater than one or at least one.

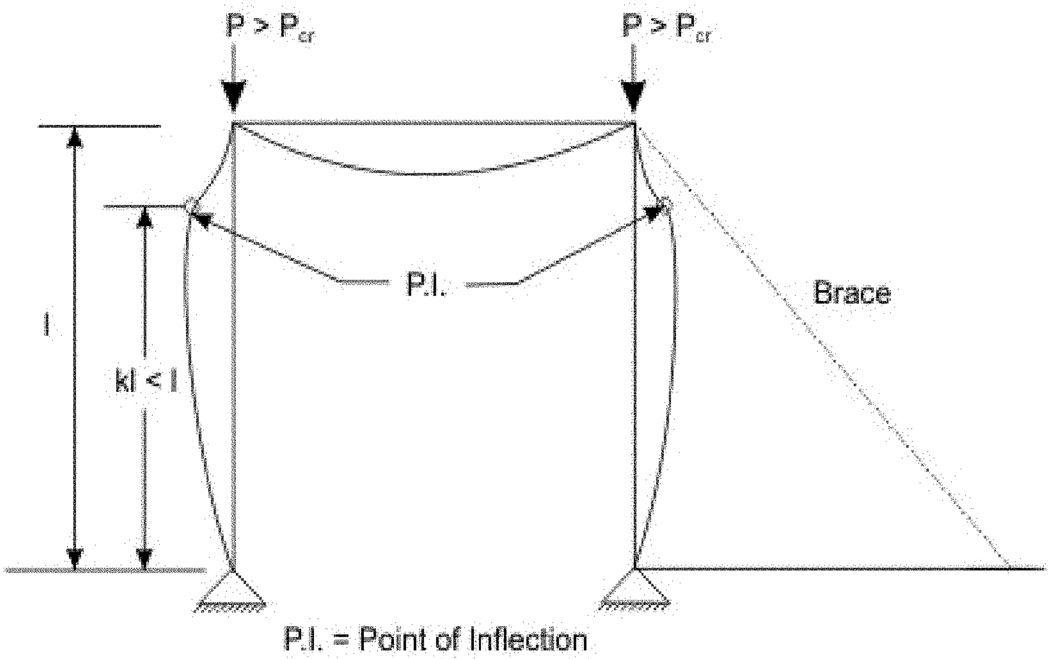


Fig. 10.27.7: Braced portal frame (k < 1)

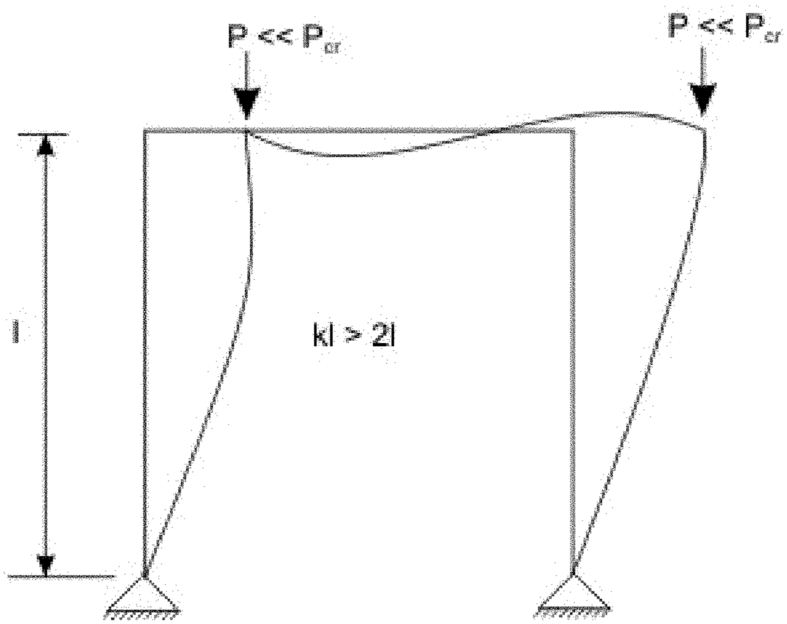


Fig. 10.27.8: Unbraced portal frame (k > 2)

Figures 10.27.7 and 8 present two reinforced concrete portal frames, a typical reinforced concrete rigid frame. Columns of Fig.10.27.7 are prevented from sidesway and those of Fig.10.27.8 are not prevented from sidesway, respectively, when subjected to concentric loadings. The buckled configuration of the frame, prevented from sidesway (Fig.10.27.7) is similar to that of Fig.10.27.3, except that the lower ends of the portal frame are hinged. One of the two points of inflection (*PI*s) is at the lower end of the column, while the other *PI* is slightly below the upper end of the column, depending on the degree of restraint. The value of *k* for such a frame is thus less than 1. The critical load is, therefore, slightly more than P_{cr} of the hinge-hinge column of Fig.10.27.1. The buckled configuration of the other portal frame of Fig.10.27.8, where sidesway is not prevented, is similar to the column of Fig.10.27.4 when it is made upside down, except that the upper end is not fixed but partially restrained by the supporting beam. In this case, the value of *k* exceeds 2, depending on the degree of restraint. One of the two *PI*s is at the bottom of the column. The critical load of the column of Fig.10.27.8 is much less than that of the column of Fig.10.27.1.

Table 10.14: Critical loads in terms of P_{cr} of hinge-hinge column and effective lengths $l_e = kl$ of elastic and reinforced concrete columns with different boundary conditions and for a constant unsupported length *l*

Sl. No.	Support conditions	Critical load P_{cr}	Effective length $l_e = kl$	Fig. No.
(A) Elastic single columns				
1.	Hinged at both ends, no sidesway	P_{cr}	l	10.27.1
2.	Fixed against rotation at both ends – no sidesway	$4P_{cr}$	0.5 l	10.27.2
3.	Partially restrained against rotation by top and bottom cross-beams, no sidesway	Between P_{cr} and $4P_{cr}$	$l > kl > l/2$	10.27.3
4.	Fixed at one end and entirely free at other end – sidesway not prevented	0.25 P_{cr}	2 l, one *PI* is on imaginary extension	10.27.4
5.	Rotationally fixed at both ends – sidesway not prevented	P_{cr}	l, one *PI* is on imaginary extension	10.27.5
6.	Partially restrained against rotation at both ends – sidesway not prevented	Between zero and slightly less than P_{cr} *	$l < kl < \alpha$	10.27.6
(B) Reinforced concrete columns				
7.	Hinged portal frame – no sidesway	$> P_{cr}$	$kl < l$	10.27.7
8.	Hinged portal frame – sidesway not prevented	$<< P_{cr}$	$kl > 2\,l$	10.27.8

Notes: 1. Buckled shapes are half sine wave between two points of inflection (*PI*s).

2. * The critical load is slightly less than P_{cr} of hinge-hinge column (Sl.No.1), when cross-beams are very rigid compared to columns, i.e., the case under Sl.No.6 approaches the case under Sl.No.1.

   The critical load is zero when cross-beams are very much flexible compared to columns, i.e., the case under Sl.No.6 approaches to hinge-hinge column of Sl.No.1, allowing sidesway. In that case, it becomes unstable and hence, carries zero load.

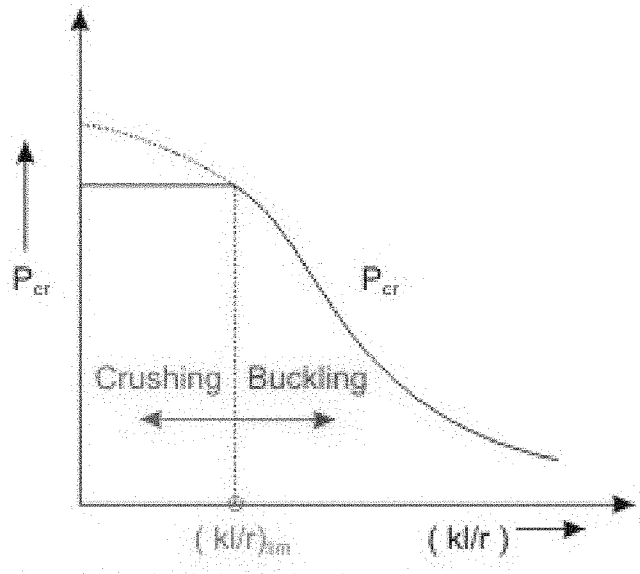


Fig. 10.27.9: Effect of slenderness on strength

Table 10.14 presents the critical load in terms of that of hinge-hinge column P_{cr} and effective lengths l_e (equal to the distance between two points of inflection *PI*s = *kl*) of elastic and reinforced concrete columns for a constant value of the unsupported length *l*.

The stress-strain curve of concrete, as shown in Fig.1.2.1 of Lesson 2, reveals that the initial tangent modulus of concrete E_c is much higher than E_t (tangent modulus at higher stress level). Taking this into account in Eq.10.62, Fig.10.27.9 presents a plot of buckling load P_{cr} versus *kl/r*. It is evident from the plot that the critical load is reducing with increasing slenderness ratio. For very short columns, the limiting factored concentric load estimated from Eq.10.39 of Lesson 24 will be found to be less than the critical load, determined from Eq.10.62. The column, therefore, will fail by direct crushing and not by buckling. We can also find out the limiting value of *kl/r* when the crushing load and the buckling load are the same. The $(kl/r)_{lim}$ is shown in Fig.10.27.9. The limiting value of *kl/r* also indicates that a column having *kl/r* more than $(kl/r)_{lim}$ will fail by buckling, while columns having any value of kl/r less than $(kl/r)_{lim}$ will fail by crushing of concrete.

The following are the observations of the discussions about the concentrically loaded columns:

1. As the slenderness ratio kl/r increases, the strength of concentrically loaded column decreases.

2. The effective length of columns either in single members or parts of rigid frames is between 0.5*l* and *l*, if the columns are prevented from sidesway by bracing or otherwise. The actual value depends on the degree of end restraints.

3. The effective length of columns either in single members or parts of rigid frames is always greater than one, if the columns are not prevented from sidesway. The actual value depends on the degree of end restraints.

4. The critical load of braced frame against sidesway is always significantly larger than that of the unbraced frame.

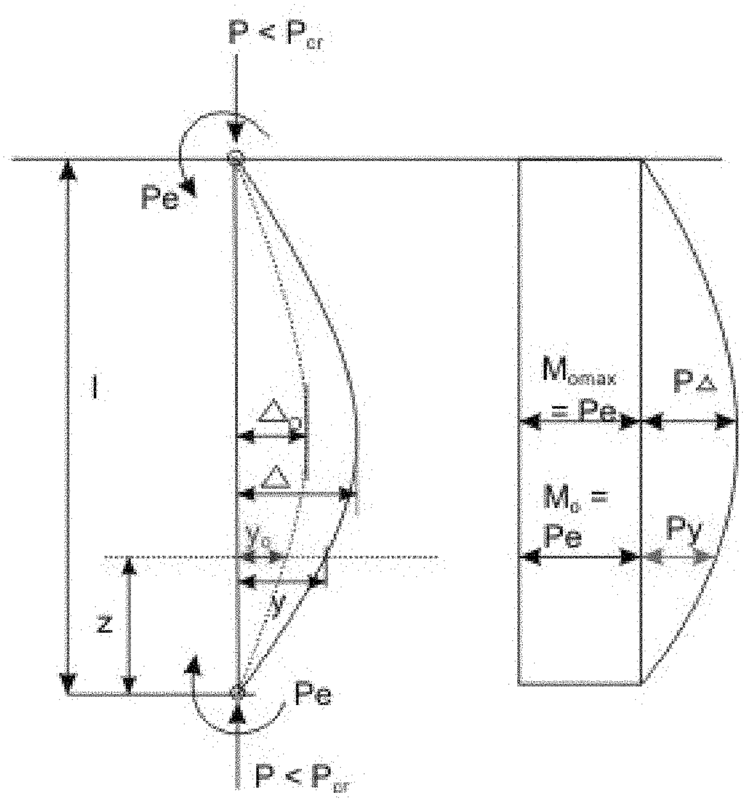


Fig. 10.27.10(a): Deflections Fig. 10.27.10(b): Moments

Fig. 10.27.10: Column bent in single curvature, (H = 0)

10.27.3 Slender Columns under Axial Load and Uniaxial Moment

(A) Columns bent in single curvature

Figure 10.27.10a shows a column bent in single curvature under axial load P less than its critical load P_{cr} with constant moment Pe. The deflection profile marked by dotted line is due to the constant moment. However, there will be additional moment of Py at a distance z from the origin (at the bottom of column) which will deflect the column further, as shown by the solid line. The constant moment Pe and additional moment Py are shown in Fig.10.27.10b. Thus, the total moment becomes

$$M = M_o + Py = P(e + y)$$
(10.63)

The maximum moment is $P(e + \Delta)$ at the mid-height of the column. This, we can write

$$M_{max} = M_o + P\Delta = P(e + \Delta)$$
(10.64)

This is known as $P - \Delta$ effect.

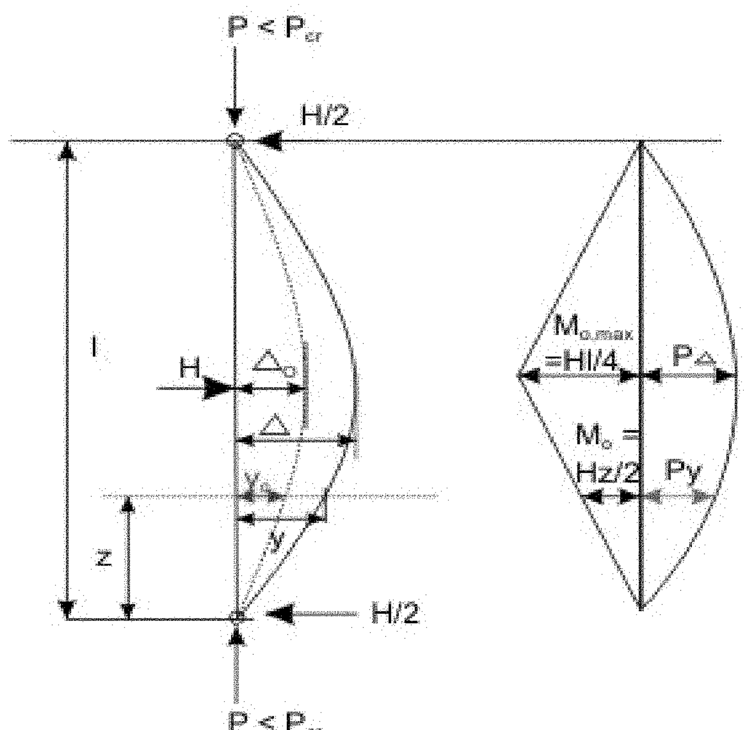


Fig. 10.27.11(a): Deflections　　Fig. 10.27.11(b): Moments

Fig. 10.27.11: Column bent in single curvature, (H = H)

Figure 10.27.11a shows another column whose bending is caused by a transverse load H. The bending moment at a distance z from the origin (bottom of the column) is $Hz/2$ causing deflection of the column marked by dotted line in the figure. The axial load P, less than its critical load P_{cr}, causes additional moment resulting in further deflection, marked by solid line in the figure. This additional deflection produces additional moment of Py at a section z from the origin. The two bending moment diagrams are shown in Fig.10.27.11b. Here again, the total moment is

$$M = M_o + Py = Hz/2 + Py \qquad (10.65)$$

The maximum moment at the mid-height of the column is

$$M_{max} = M_{o,max} + P\Delta = Hl/4 + P\Delta \qquad (10.66)$$

The total moment in Eqs.10.63 and 10.65 consists of the moment M_o that acts in the presence of P and the additional moment caused by P (= Py). The deflections y can be computed from y_o, the deflections without the axial load from the expression

$$y = y_o[1/\{1 - (P/P_{cr})\}] \qquad (10.67)$$

From Eq.10.64, we have

$$M_{max} = M_o + P\Delta = M_o + P\Delta_o[1/\{1 - (P/P_{cr})\}] \qquad (10.68)$$

Equation 10.68 can be written as

$$M_{max} = M_o \frac{1 + \psi(P/P_{cr})}{1 - (P/P_{cr})} \qquad (10.69)$$

where ψ depends on the type of loading and generally varies between ± 0.20. Since P/P_{cr} is always less than one, we can ignore $\psi(P/P_{cr})$ term of Eq.10.69, to have

$$M_{max} = M_o/\{1 - (P/P_{cr})\} \qquad (10.70)$$

where $1/\{1 - (P/P_{cr})\}$ is the moment magnification factor. In both the cases above (Figs.10.27.10 and 11), a direct addition of the maximum moment caused by transverse load or otherwise, to the maximum moment caused by P gives the total maximum moment as that is the most unfavourable situation. However, this is not the case for situation taken up in the following.

(B) Columns bent in double curvature

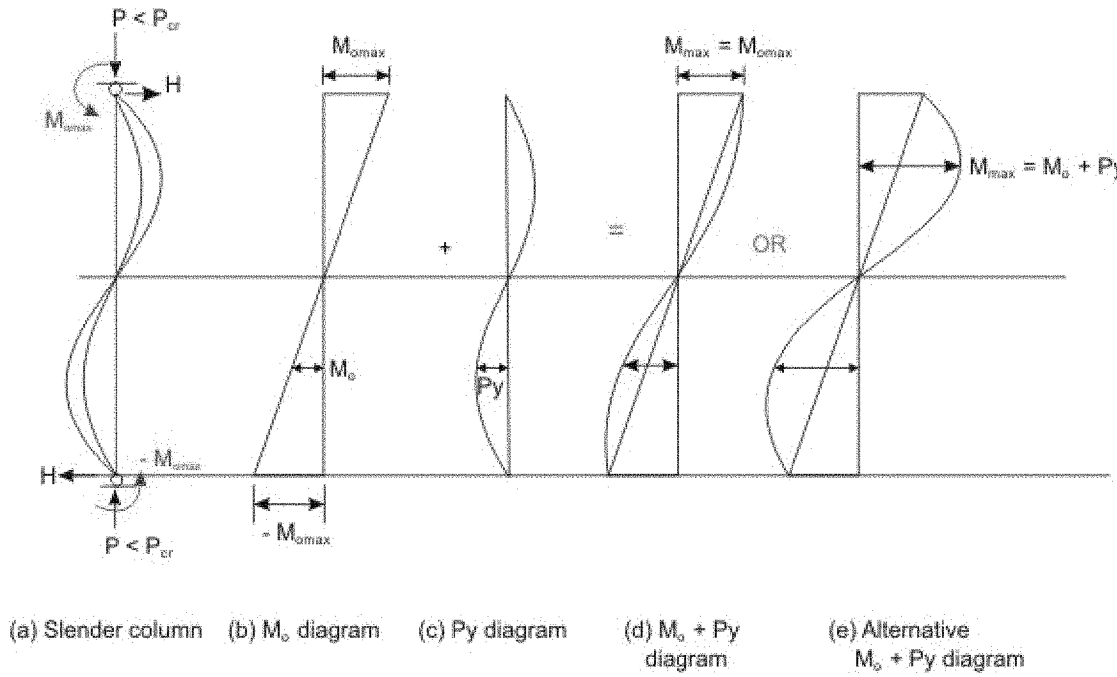


Fig. 10.27.12: Slender column under axial load and bending, bent in double curvature

Figure 10.27.12a shows a column subjected to equal end moment of opposite signs. From the moment diagrams M_o and Py (Figs.10.27.12b and c), it is clear that though M_o moments are maximum at the ends, the Py moments are maximum at some distance from the ends. The total moment can be either as shown in d or in e of Fig.10.27.12. In case of Fig.10.27.12d, the maximum moment remains at the ends and in Fig.10.27.12e, the maximum moment is at some distance from the ends, where M_o is comparatively smaller than $M_{o\ max}$ at the ends. Accordingly, the total maximum moment is moderately higher than $M_{o\ max}$.

From the above, it is evident that the moment M_o will be magnified most strongly if the section of $M_{o\ max}$ coincides with the section of maximum value of y, as in the case of column bent in single curvature of Figs.10.27.10 and 11. Similarly, if the two moments are unequal but of same sign as in Fig.10.27.10, the moment M_o will be magnified but not so much as in Fig.10.27.10. On the other hand, if the unequal end moments are of opposite signs and cause bending in double curvature, there will be little or no magnification of M_o moment.

This dependence of moment magnification on the relative magnitudes of the two moments can be expressed by modifying the earlier Eq.10.70 as

$$M_{max} = M_o\, C_m/\{1 - (P/P_{cr})\} \quad (10.71)$$

where $C_m = 0.6 + 0.4(M_1/M_2) \geq 0.4 \quad (10.72)$

The moment M_1 is smaller than M_2 and M_1/M_2 is positive if the moments produce single curvature and negative if they produce double curvature. It is further seen from Eq.10.72 that $C_m = 1$, when $M_1 = M_2$ and in that case, Eq.10.71 becomes the same as Eq.10.70.

For the column of Fig.10.27.12a, the deflections caused by M_o are magnified when axial load P is applied. The deflection can be obtained from

$$y = y_o\,[1/\{1 - (P/4P_{cr})\}] \quad (10.73)$$

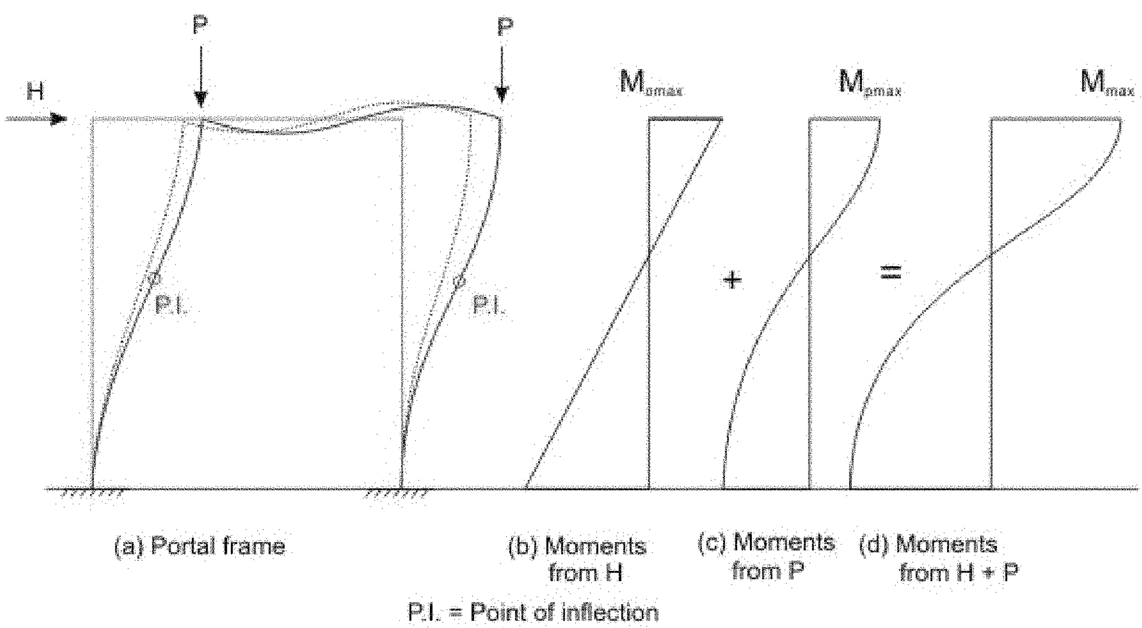


Fig. 10.27.13: Fixed portal frame - laterally unbraced

(C) Portal frame laterally unbraced and braced

Here, the sidesway can occur only for the entire frame simultaneously. A fixed portal frame, shown in Fig.10.27.13a, is under horizontal load H and compression force P. The moments due to H and P and the total moment diagrams are shown in Fig.10.27.13b, c and d, respectively. The deformations of the frame due to H are shown in Fig.10.27.13a by dotted curves, while the solid curves are the magnified deformations. It is observed that the maximum values of positive and negative M_o are at the ends of the column where the maximum values of positive and negative moments due to *P* also occur. Thus, the total moment shall be at the ends as the two effects are fully additive.

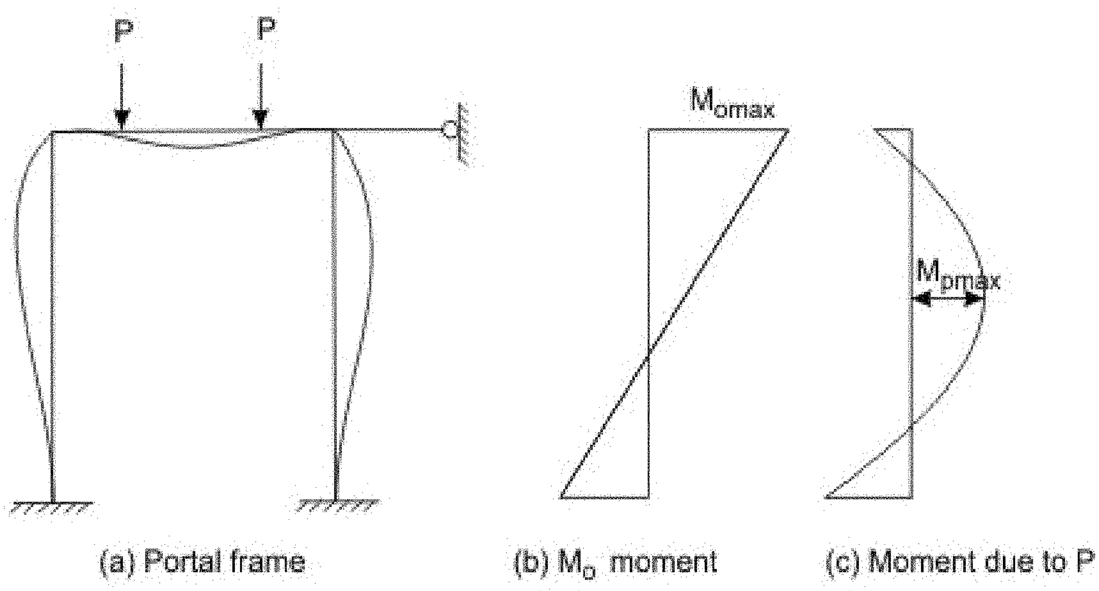
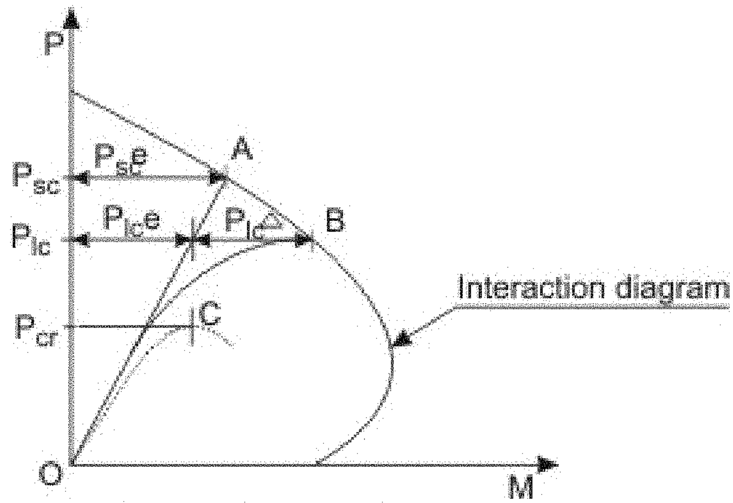


Fig. 10.27.14: Fixed portal frame - laterally braced

Figure 10.27.14a shows a fixed portal frame, laterally braced so that no sidesway can occur. Figures 10.27.14b and c show the moments M_o and due to P. It is seen that the maximum values of the two different moments do not occur at the same location. As a result, the magnification of the moment either may not be true or shall be small.

(D) Columns with different slenderness ratios



OA = Short column (material failure),
OB = Long column (material failure),
OC = Very long column (buckling failure)

Fig. 10.27.15: Behaviour of slender column

Figure 10.27.15 shows the interaction diagram of P and M at the mid-height section of the column shown in Fig.10.27.10. Three loading paths OA, OB and OC are also shown in the figure for three columns having the same cross-sectional area and the eccentricity of loads but with different slenderness ratios. The three columns are loaded with increasing P and M (at constant eccentricity) up to failure. The loading path OA is linear indicating $\Delta = 0$, i.e., for a very short column. It should be noted that Δ should be theoretically zero only when either the effective length or the eccentricity is zero. In a practical short column, however, some lateral deflection shall be there, which, in turn will cause additional moment not more than five per cent of the primary moment and may be neglected. The loading path OA terminates at point A of the interaction diagram, which shows the failure load P_{sc} of the short column with moment $M_{sc} = P_{sc}\, e$. The short column fails by crushing of concrete at the mid-height section. This type of failure is designated as material failure, either a tension failure or a compression failure depending on the location of the point A on the interaction curve.

The load path OB is for a long column, where the deflection Δ caused by increasing value of P is significant. Finally, the long column fails at load P_{lc} and moment $M_{lc} = P_{lc}(e + \Delta)$. The loading path OB further reveals that the secondary moment $P_{lc}\Delta$ is comparable to the primary moment $P_{lc}\, e$. Moreover, the failure load and the primary moment of the long column P_{lc} and $P_{lc}\, e$, respectively, are less than those of the short column (P_{sc} and $P_{sc}\, e$, respectively), though both the columns have the same cross-sectional areas and eccentricities but different slenderness ratios. Here also, the mid-height section of the column undergoes material failure, either a compression failure or a tension failure, depending on the location of the point B on the interaction diagram.

The loading path OC, on the other hand, is for a very long column when the lateral deflection Δ is so high that the slope of the path *dP/dM* at C is zero. The column is so slender that the failure is due to buckling (instability) at a comparatively much low value of the load P_{cr}, though this column has the same cross-sectional area and the eccentricity of load as of the other two columns. Such instability failure occurs for very slender columns, specially when they are not braced.

The following points are summarised from the discussion made in sec.10.27.3.

1. Additional deflections and moments are caused by the axial compression force P in columns. The additional moments increase with the increase of *kl/r*, when other parameters are equal.

2. Laterally braced compression members and bent in single curvature have the same or nearby locations of the maxima of both M_o and Py. Thus, being fully additive, they have large moment magnification.

3. Laterally braced compression members and bent in double curvature have different locations of the maxima of both M_o and Py. As a result, the moment magnification is either less or zero.

4. Members of frames not braced laterally, the maxima of M_o and Py mostly occur at the ends of column and cause the maximum total moment at the ends of columns only. Additional moments and additional deflections increase with the increase of *kl/r*.

10.27.4 Effective Length of Columns

Annex E of IS 456 presents two figures (Figs.26 and 27) and a table (Table 26) to estimate the effective length of columns in frame structures based on a research paper, "Effective length of column in multistoreyed building" by R.H. Wood in The Structural Engineer Journal, No.7, Vol.52, July 1974. Figure 26 is for columns in a frame with no sway, while Fig.27 is for columns in a frame with sway. These two figures give the values of k (i.e., l_e/l) from two parameters β_1 and β_2 which are obtained from the following expression:

$$\beta = \sum K_c / \sum K_c + \sum K_b \qquad (10.74)$$

where K_c and K_b are flexural stiffnesses of columns and beams, respectively. The quantities β_1 and β_2 at the top and bottom joints A and B, respectively, are determined by summing up the K values of members framing into a joint at top and bottom, respectively. Thus β_1 and β_2 for the frame shown in Fig.10.27.16 are as follows:

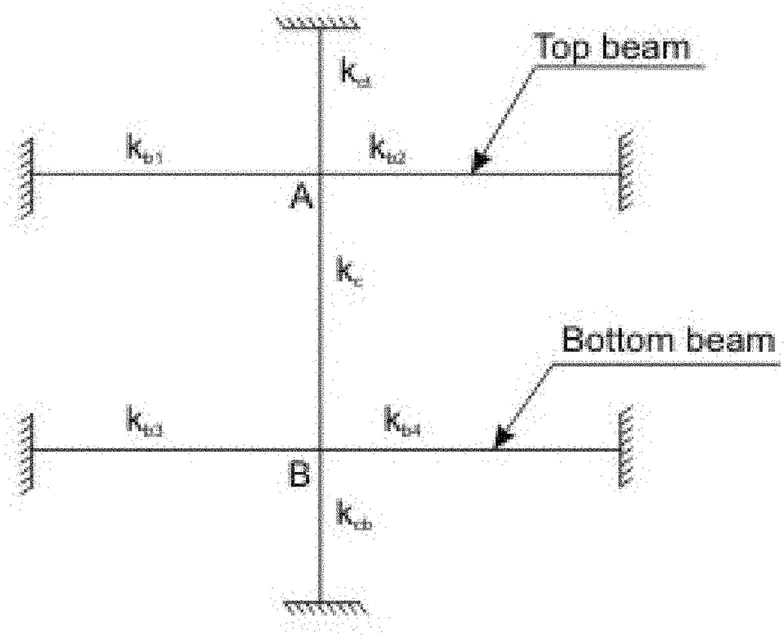


Fig. 10.27.16: Stiffness of columns in Wood's chart

$\beta_1 = (K_c + K_{ct})/(K_c + K_{ct} + K_{b1} + K_{b2})$
(10.75)

$\beta_2 = (K_c + K_{cb})/(K_c + K_{cb} + K_{b3} + K_{b4})$
(10.76)

However, assuming idealised conditions, the effective length in a given plane may be assessed from Table 28 in Annex E of IS 456, for normal use.

10.27.5 Determination of Sway or No Sway Column

Clause E-2 of IS 456 recommends the stability index Q to determine if a column is a no sway or sway type. The stability index Q is expressed as:

$Q = \sum P_u \Delta_u / H_u h_z$
(10.77)

where $\sum P_u$ = sum of axial loads on all columns in the storey,
Δ_u = elastically computed first-order lateral deflection,
H_u = total lateral force acting within the storey, and

h_z = height of the storey.

The column may be taken as no sway type if the value of Q is $\leq$ 0.4, otherwise, the column is considered as sway type.

10.27.6 Design of Slender Columns

The design of slender columns, in principle, is to be done following the same procedure as those of short columns. However, it is essential to estimate the total moment i.e., primary and secondary moments considering P-Δ effects. These secondary moments and axial forces can be determined by second-order rigorous structural analysis – particularly for unbraced frames. Further, the problem becomes more involved and laborious as the principle of superposition is not applicable in second-order analysis.

However, cl.39.7 of IS 456 recommends an alternative simplified method of determining additional moments to avoid the laborious and involved second-order analysis. The basic principle of additional moment method for estimating the secondary moments is explained in the next section.

10.27.7 Additional Moment Method

In this method, slender columns should be designed for biaxial eccentricities which include secondary moments (Py of Eq.10.63 and 10.65) about major and minor axes. We first consider braced columns which are bent symmetrically in single curvature and cause balanced failure i.e., $P_u = P_{ub}$.

(A) Braced columns bent symmetrically in single curvature and undergoing balanced failure

For braced columns bent symmetrically in single curvature, we have from Eqs.10.63 and 10.65,

$$M = M_o + Py = M_o + P\,e_a = M_o + M_a$$
(10.78)

where P is the factored design load P_u, M are the total factored design moments M_{ux} and M_{uy} about the major and minor axes, respectively; M_o are the primary factored moments M_{oux} and M_{ouy} about the major and minor axes, respectively; M_a are the additional moments M_{ax} and M_{ay} about the major and minor axes, respectively and e_a are the additional eccentricities e_{ax} and e_{ay} along the minor and major axes, respectively. The quantities M_o and P of Eq.10.78 are known and hence, it is required to determine the respective values of e_a, the additional eccentricities only.

Let us consider the columns of Figs.10.27.10 and 11 showing Δ as the maximum deflection at the mid-height section of the columns. The column of Fig.10.27.10, having a constant primary moment M_o, causes constant curvature ϕ, while the column of Fig.10.27.11, having a linearly varying primary moment with a maximum value of $M_{o\ max}$ at the mid-height section of the column, has a linearly varying curvature with the maximum curvature of ϕ_{max} at the mid-height section the column. The two maximum curvatures can be expressed in terms of their respective maximum deflection Δ as follows:

The constant curvature (Fig.10.27.10) $\phi_{\max} = 8\Delta/l_e^2$
(10.79)

The linearly varying curvature (Fig.10.27.11) $\phi_{\max} = 12\Delta/l_e^2$
(10.80)

where l_e are the respective effective lengths kl of the columns. We, therefore, consider the maximum ϕ as the average value lying in between the two values of Eqs.10.79 and 80 as

$\phi_{\max} = 10\Delta/l_e^2$
(10.81)

Accordingly, the maximum additional eccentricities e_a, which are equal to the maximum deflections Δ, can be written as

$e_a = \Delta = \phi l_e^2 /10$
(10.82)

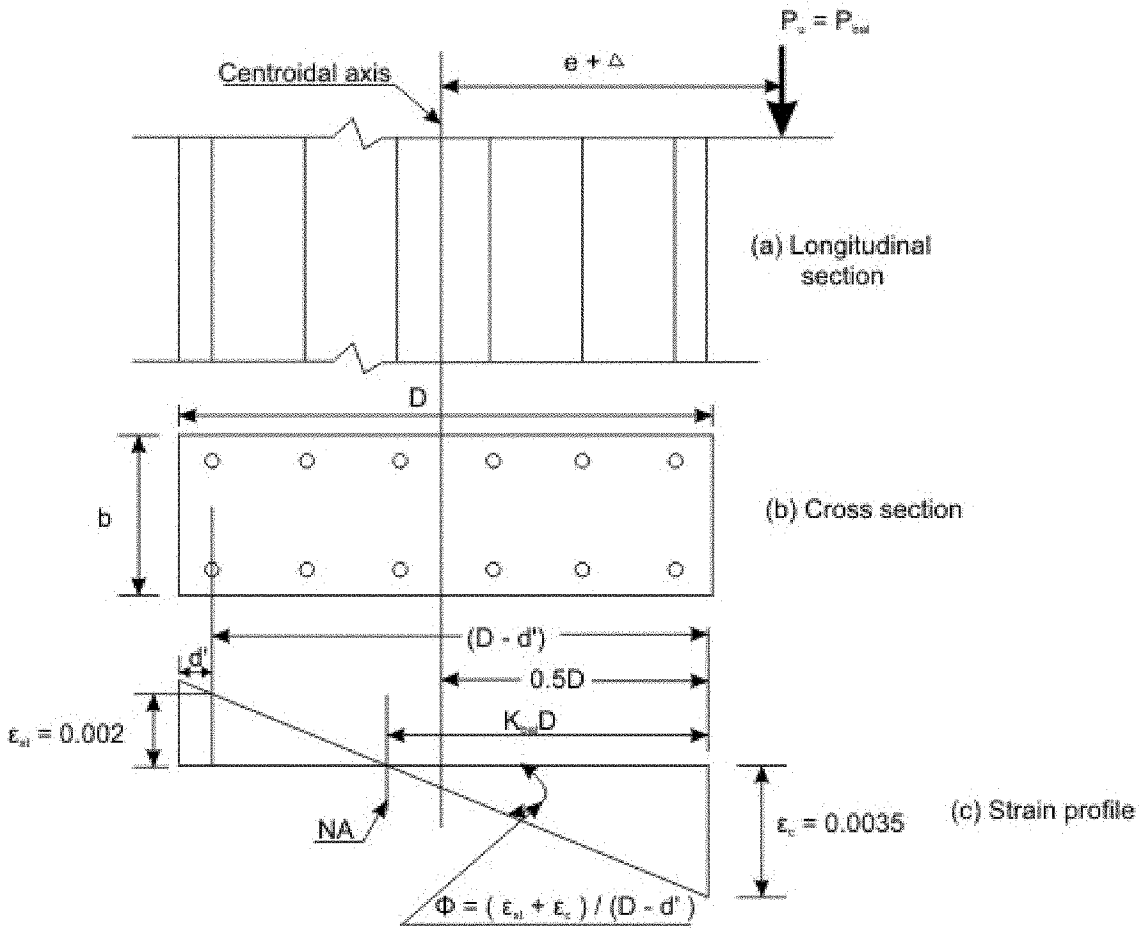


Fig. 10.27.17: Maximum curvature at mid-height section whenP_u = P_{bal}

Assuming the column undergoes a balanced failure when $P_u = P_{ub}$, the maximum curvature at the mid-height section of the column, shown in Figs.10.27.17a and b, can be expressed as given below, assuming (i) the values of ε_c = 0.0035, ε_{st} = 0.002 and d'/D = 0.1, and (ii) the additional moment capacities are about eighty per cent of the total moment.

ϕ = eighty per cent of {(0.0035 + 0.002)/0.9D} (see Fig.10.27.17c)

or $\phi = 1/200D$ (10.83)

Substituting the value of ϕ in Eq.10.82,

$e_a = D(l_e/D)^2/2000$ (10.84)

Therefore, the additional moment M_a can be written as,

$M_a = Py = P\Delta = Pe_a = (PD/2000)\,(l_e/D)^2$
(10.85)

Thus, the additional moments M_{ax} and M_{ay} about the major and minor axes, respectively, are:

$M_{ax} = (P_uD/2000)\,(l_{ex}/D)^2$
(10.86)

$M_{ay} = (P_ub/2000)\,(l_{ey}/b)^2$
(10.87)

where P_u = axial load on the member,

l_{ex} = effective length in respect of the major axis,

l_{ey} = effective length in respect of the minor axis,

D = depth of the cross-section at right angles to the major axis, and

b = width of the member.

Clause 39.7.1 of IS 456 recommends the expressions of Eqs.10.86 and 87 for estimating the additional moments M_{ax} and M_{ay} for the design. These two expressions of the additional moments are derived considering the columns to be braced and bent symmetrically undergoing balanced failure. Therefore, proper modifications are necessary for different situations like braced columns with unequal end moments with the same or different signs, unbraced columns and columns causing compression failure i.e., when $P_u > P_{ub}$.

(B) Braced columns subjected to unequal primary moments at the two ends

For braced columns without any transverse loads occurring in the height, the primary maximum moment ($M_{o\,max}$ of Eq.10.64), with which the additional moments of Eqs.10.86 and 87 are to be added, is to be taken as:

$M_{o\,max} = 0.4\,M_1 + 0.6\,M_2$
(10.88)

and further $M_{o\,max} \geq 0.4\,M_2$
(10.89)

where M_2 is the larger end moment and M_1 is the smaller end moment, assumed to be negative, if the column is bent in double curvature.

To eliminate the possibility of total moment $M_{u\,max}$ becoming less than M_2 for columns bent in double curvature (see Fig.10.27.12) with M_1 and M_2 having opposite signs, another condition has been imposed as

$M_{u\,max} \geq M_2$
(10.90)

The above recommendations are given in notes of cl.39.7.1 of IS 456.

(C) Unbraced columns

Unbraced frames undergo considerable deflection due to P-Δ effect. The additional moments determined from Eqs.10.86 and 87 are to be added with the maximum primary moment $M_{o\,max}$ at the ends of the column. Accordingly, we have

$M_{o\,max} = M_2 + M_a$
(10.91)

The above recommendation is given in the notes of cl.39.7.1 of IS 456.

(D) Columns undergoing compression failure ($P_u > P_{ub}$)

It has been mentioned in part A of this section that the expressions of additional moments given by Eqs.10.86 and 10.87 are for columns undergoing balanced failure (Fig.10.27.17). However, when the column causes compression failure, the *e/D* ratio is much less than that of balanced failure at relatively high axial loads. The entire section may be under compression causing much less curvatures. Accordingly, additional moments of Eqs.10.86 and 10.87 are to be modified by multiplying with the reduction factor k as given below:

(i) For $P_u > P_{ubx}$: $k_{ax} = (P_{uz} - P_u)/(P_{uz} - P_{ubx})$
(10.92)

(ii) For $P_u > P_{uby}$: $k_{ay} = (P_{uz} - P_u)/(P_{uz} - P_{uby})$
(10.93)

with a condition that k_{ax} and k_{ay} should be ≤ 1
(10.94)

where P_u = axial load on compression member

P_{uz} is given in Eq.10.59 of Lesson 26 and is,

$P_{uz} = 0.45\, f_{ck}\, A_c + 0.75\, f_y\, A_{st}$ ... (10.59)

P_{ubx}, P_{uby} = axial loads with respect to major and minor axes, respectively, corresponding to the condition of maximum compressive strain of 0.0035 in concrete and tensile strain of 0.002 in outermost layer of tension steel.

It is seen from Eqs.10.92 and 10.93 that the values of k (k_{ax} and k_{ay}) vary linearly from zero (when $P_u = P_{uz}$) to one (when $P_u = P_{ub}$). Since Eqs.10.92 and 10.93 are not applicable for $P_u < P_{ub}$, another condition has been imposed as given in Eq.10.94.

The above recommendations are given in cl.39.7.1.1 of IS 456.

The following discussion is very important for the design of slender columns.

Additional moment method is one of the methods of designing slender columns as discussed in A to D of this section. This method is recommended in cl.39.7 of IS 456 also. The basic concept here is to enhance the primary moments by adding the respective additional moments estimated in a simple way avoiding laborious and involved calculations of second-order structural analysis. However, these primary moments under eccentric loadings should not be less than the moments corresponding to the respective minimum eccentricity, as stipulated in the code. Hence, the primary moments in such cases are to be replaced by the minimum eccentricity moments. Moreover, all slender columns, including those under axial concentric loadings, are also to be designed for biaxial bending, where the primary moments are zero. In such cases, the total moment consisting of the additional moment multiplied with the modification factor, if any, in each direction should be equal to or greater than the respective moments under minimum eccentricity conditions. As mentioned earlier, the minimum eccentricity consideration is given in cl.25.4 of IS 456.

10.27.8 Illustrative Example

The following illustrative example is taken up to explain the design of slender columns. The example has been solved in step by step using (i) the equations of Lessons 21 to 27 and (ii) employing design charts and tables of SP-16, to compare the results.

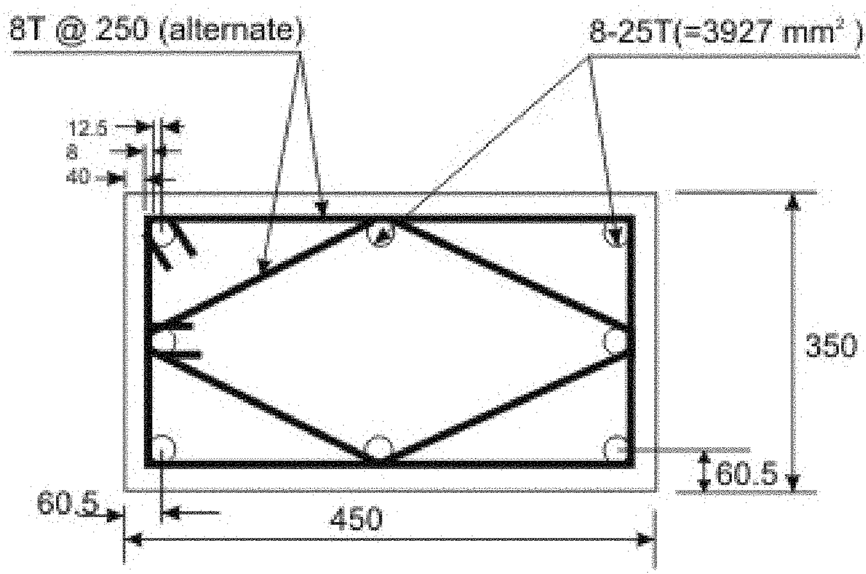


Fig. 10.27.18: Problem 1

Problem 1:

Determine the reinforcement required for a braced column against sidesway with the following data: size of the column = 350 x 450 mm (Fig.10.27.18); concrete and steel grades = M 30 and Fe 415, respectively; effective lengths l_{ex} and l_{ey} = 7.0 and 6.0 m, respectively; unsupported length l = 8 m; factored load P_u = 1700 kN; factored moments in the direction of larger dimension = 70 kNm at top and 30 kNm at bottom; factored moments in the direction of shorter dimension = 60 kNm at top and 30 kNm at bottom. The column is bent in double curvature. Reinforcement will be distributed equally on four sides.

Solution 1:

Step 1: Checking of slenderness ratios

l_{ex}/D = 7000/450 = 15.56 > 12,

l_{ey}/b = 6000/350 = 17.14 > 12.

Hence, the column is slender with respect to both the axes.

Step 2: Minimum eccentricities and moments due to minimum eccentricities (Eq.10.3 of Lesson21)

$e_{x\,min} = l/500 + D/30$ = 8000/500 + 450/30 = 31.0 > 20 mm

$e_{y\,min} = l/500 + b/30$ = 8000/500 + 350/30 = 27.67 > 20 mm

M_{ox} (Min. ecc.) = $P_u(e_{x\,min})$ = (1700) (31) (10^{-3}) = 52.7 kNm

M_{oy} (Min. ecc.) = $P_u(e_{y\,min})$ = (1700) (27.67) (10^{-3}) = 47.04 kNm

Step 3: Additional eccentricities and additional moments

Method 1: Using Eq. 10.84

$e_{ax} = D(l_{ex}/D)^2/2000 = (450)(7000/450)^2/2000 = 54.44$ mm

$e_{ay} = b(l_{ex}/b)^2/2000 = (350)(6000/350)^2/2000 = 51.43$ mm

$M_{ax} = P_u(e_{ax}) = (1700)(54.44)(10^{-3}) = 92.548$ kNm

$M_{ay} = P_u(e_{ay}) = (1700)(51.43)(10^{-3}) = 87.43$ kNm

Method 2: Table I of SP-16

For l_{ex}/D = 15.56, Table I of SP-16 gives:

e_{ax}/D = 0.1214, which gives e_{ax} = (0.1214) (450) = 54.63 mm

For l_{ey}/D = 17.14, Table I of SP-16 gives:

e_{ay}/b = 0.14738, which gives e_{ay} = (0.14738) (350) = 51.583 mm

It is seen that values obtained from Table I of SP-16 are comparable with those obtained by Eq. 10.84 in Method 1.

Step 4: Primary moments and primary eccentricities (Eqs.10.88 and 89)

$M_{ox} = 0.6M_2 - 0.4M_1 = 0.6(70) - 0.4(30) = 30$ kNm, which should be $\geq$ 0.4 M_2 (= 28 kNm). Hence, o.k.

$M_{oy} = 0.6M_2 - 0.4M_1 = 0.6(60) - 0.4(30) = 24$ kNm, which should be $\geq$ 0.4 M_2 (= 24 kNm). Hence, o.k.

Primary eccentricities:

$e_x = M_{ox}/P_u = (30/1700)(10^3) = 17.65$ mm

$e_y = M_{oy}/P_u = (24/1700)(10^3) = 14.12$ mm

Since, both primary eccentricities are less than the respective minimum eccentricities (see Step 2), the primary moments are revised to those of Step 2. So, M_{ox} = 52.7 kNm and M_{oy} = 47.04 kNm.

Step 5: Modification factors

To determine the actual modification factors, the percentage of longitudinal reinforcement should be known. So, either the percentage of longitudinal reinforcement may be assumed or the modification factor may be assumed which should be verified subsequently. So, we assume the modification factors of 0.55 in both directions.

Step 6: Total factored moments

$$M_{ux} = M_{ox} + \text{(Modification factor)} (M_{ax}) = 52.7 + (0.55)(92.548)$$

$$= 52.7 + 50.9 = 103.6 \text{ kNm}$$

$$M_{uy} = M_{oy} + \text{(Modification factor)} (M_{ay}) = 47.04 + (0.55)(87.43)$$

$$= 47.04 + 48.09 = 95.13 \text{ kNm}$$

Step 7: Trial section (Eq.10.61 of Lesson 26)

The trial section is determined from the design of uniaxial bending with P_u = 1700 kN and M_u = 1.15 $(M_{ux}^2 + M_{uy}^2)^{1/2}$. So, we have M_u = (1.15){(103.6)^2 + (95.13)^2}$^{1/2}$ = 161.75 kNm. With these values of P_u (= 1700 kN) and M_u (= 161.75 kNm), we use chart of SP-16 for the d'/D = 0.134. We assume the diameters of longitudinal bar as 25 mm, diameter of lateral tie = 8 mm and cover = 40 mm, to get d' = 40 + 8 + 12.5 = 60.5 mm. Accordingly, d'/D = 60.5/450 = 0.134 and d'/b = 60.5/350 = 0.173. We have:

$$P_u/f_{ck}\, bD = 1700(10^3)/(30)(350)(450) = 0.3598$$

$$M_u/f_{ck}\, bD^2 = 161.75(10^6)/(30)(350)(450)(450) = 0.076$$

We have to interpolate the values of p/f_{ck} for d'/D = 0.134 obtained from Charts 44 (for d'/D = 0.1) and 45 (d'/D = 0.15). The values of p/f_{ck} are 0.05 and 0.06 from Charts 44 and 45, respectively. The corresponding values of p are 1.5 and 1.8 per cent, respectively. The interpolated value of p for d'/D = 0.134 is 1.704 per cent, which gives A_{sc} = (1.704)(350)(450)/100 = 2683.8 mm^2. We use 4-25 + 4-20 (1963 + 1256 = 3219 mm^2), to have p provided = 2.044 per cent giving p/f_{ck} = 0.068.

Step 8: Calculation of balanced loads P_b

The values of P_{bx} and P_{by} are determined using Table 60 of SP-16. For this purpose, two parameters k_1 and k_2 are to be determined first from the table. We have p/f_{ck} = 0.068, d'/D = 0.134 and d'/b = 0.173. From Table 60, k_1 = 0.19952 and k_2 = 0.243 (interpolated for d'/D = 0.134) for P_{bx}. So, we have: $P_{bx}/f_{ck}bD = k_1 + k_2\,(p/f_{ck})$ = 0.19952 + 0.243(0.068) = 0.216044, which gives P_{bx} = 0.216044(30)(350)(450)(10^{-3}) = 1020.81 kN.

Similarly, for P_{by}: d'/b = 0.173, p/f_{ck} = 0.068. From Table 60 of SP-16, k_1 = 0.19048 and k_2 = 0.1225 (interpolated for d'/b = 0.173). This gives $P_{by}/f_{ck}bD$ = 0.19048 + 0.1225(0.068) = 0.19881, which gives P_{by} = (0.19881)(30)(350)(450)(10^{-3}) = 939.38 kN.

Since, the values of P_{bx} and P_{by} are less than P_u, the modification factors are to be used.

Step 9: Determination of P_{uz}

Method 1: From Eq.10.59 of Lesson 26

$$P_{uz} = 0.45\, f_{ck}\, A_g + (0.75\, f_y - 0.45\, f_{ck})\, A_{sc}$$

$$= 0.45(30)(350)(450) + \{0.75(415) - 0.45(30)\}(3219) = 3084.71 \text{ kN}$$

Method 2: Using Chart 63 of SP-16

We get P_{uz}/A_g = 19.4 N/mm^2 from Chart 63 of SP-16 using p = 2.044 per cent. Therefore, P_{uz} = (19.4)(350)(450)(10^{-3}) = 3055.5 kN, which is in good agreement with that of Method 1.

Step 10: Determination of modification factors

Method 1: From Eqs.10.92 and 10.93

$$k_{ax} = (P_{uz} - P_u)/(P_{uz} - P_{ubx}) \qquad \ldots (10.92)$$

or $\quad k_{ax}$ = (3084.71 – 1700)/(3084.71 – 1020.81) = 0.671 and

$$k_{ay} = (P_{uz} - P_u)/(P_{uz} - P_{uby}) \qquad \ldots (10.93)$$

or $\quad k_{ay}$ = (3084.71 – 1700)/(3084.71 – 939.39) = 0.645

The values of the two modification factors are different from the assumed value of 0.55 in Step 5. However, the moments are changed and the section is checked for safety.

Method 2: From Chart 65 of SP-16

From Chart 65 of SP-16, for the two parameters, P_{bx}/P_{uz} = 1020.81/3084.71 = 0.331 and P_u/P_{uz} = 1700/3084.71 = 0.551, we get k_{ax} = 0.66. Similarly, for the two parameters, P_{by}/P_{uz} = 939.38/3084.71 = 0.3045 and P_u/P_{uz} = 0.551, we have k_{ay} = 0.65. Values of k_{ax} and k_{ay} are comparable with those of Method 1.

Step 11: Total moments incorporating modification factors

M_{ux} = M_{ox} (from Step 4) + (k_{ax}) M_{ax} (from Step 3)

= 52.7 + 0.671(92.548) = 114.8 kNm

M_{uy} = M_{oy} (from Step 4) + k_{ay} (M_{ay}) (from Step 3)

= 47.04 + (0.645)(87.43) = 103.43 kNm.

Step 12: Uniaxial moment capacities

The two uniaxial moment capacities M_{ux1} and M_{uy1} are determined as stated: (i) For M_{ux1}, by interpolating the values obtained from Charts 44 and 45, knowing the values of $P_u/f_{ck}bD$ = 0.3598 (see Step 7), p/f_{ck} = 0.068 (see Step 7), d'/D = 0.134 (see Step 7), (ii) for M_{uy1}, by interpolating the values obtained from Charts 45 and 46, knowing the same values of $P_u/f_{ck}bD$ and p/f_{ck} as those of (i) and d'/D = 0.173 (see Step 7). The results are given below:

(i) $M_{ux1}/f_{ck}bD^2$ = 0.0882 (interpolated between 0.095 and 0.085)

(ii) $M_{uy1}/f_{ck}bb^2$ = 0.0827 (interpolated between 0.085 and 0.08)

So, we have, M_{ux1} = 187.54 kNm and M_{uy1} = 136.76 kNm.

Step 13: Value of α_n

Method 1: From Eq.10.60 of Lesson 26

We have P_u/P_{uz} = 1700/3084.71 = 0.5511. From Eq.10.60 of Lesson 26, we have α_n = 0.67 + 1.67 (P_u/P_{uz}) = 1.59.

Method 2: Interpolating the values between (P_u/P_{uz}) = 0.2 and 0.6

The interpolated value of α_n = 1.0 + (0.5511 – 0.2)/0.6 = 1.5852. Both the values are comparable. We use α_n = 1.5852.

Step 14: Checking of column for safety

Method 1: From Eq.10.58 of Lesson 26

We have in Lesson 26:

$$(M_{ux}/M_{ux1})^{\alpha_n} + (M_{uy}/M_{uy1})^{\alpha_n} \le 1 \qquad \dots (10.58)$$

Here, putting the values of M_{ux}, M_{ux1}, M_{uy}, M_{uy1} and α_n, we get: $(114.8/187.54)^{1.5452}$ + $(103.43/136.76)^{1.5852}$ = 0.4593 + 0.6422 = 1.1015. Hence, the section or the reinforcement has to be revised.

Method 2: Chart 64 of SP-16

The point having the values of (M_{ux}/M_{ux1}) = 114.8/187.54 = 0.612 and (M_{uy}/M_{uy1}) = 103.43/136.76 = 0.756 gives the value of P_u/P_z more than 0.7. The value of P_u/P_{uz} here is 0.5511 (see Step 13). So, the section needs revision.

We revise from Step 7 by providing 8-25 mm diameter bars (= 3927 mm^2, p = 2.493 per cent and p/f_{ck} = 0.0831) as the longitudinal reinforcement keeping the values of b and D unchanged. The revised section is checked furnishing the repeated calculations from Step 8 onwards. The letter R is used before the number of step to indicate this step as revised one.

Step R8: Calculation of balanced loads P_b

Table 60 of SP-16 gives k_1 = 0.19952, and k_2 = 0.243. We have p/f_{ck} = 0.0831 now. So, P_{bx} = {0.19952 + (0.243)(0.0831)} (30)(350)(450)(10^{-3}) = 1038.145 kN. Similarly, k_1 = 0.19048, k_2 = 0.1225 and p/f_{ck} = 0.0831 give P_{by} = {0.19048 + (0.1225)(0.0831)} (30)(350)(450)(10^{-3}) = 948.12 kN.

The values of P_{bx} and P_{by} are less than P_u (= 1700 kN). So, modification factors are to be incorporated.

Step R9: Determination of P_{uz} (Eq. 10.59 of Lesson 26)

P_{uz} = 0.45(30)(350)(450) + {0.75(415) – 0.45(30)}(3927) = 3295.514 kN.

Step R10: Determination of modification factors (Eqs.10.92 and 10.93)

k_{ax} = (3295.514 – 1700)/(3295.514 – 1038.145) = 0.707

k_{ay} = (3295.514 – 1700)/(3295.514 – 948.12) = 0.68

Step R11: Total moments incorporating modification factors

M_{ux} = 52.70 + 0.707(92.548) = 118.13 kNm

M_{uy} = 47.04 + 0.68(87.43) = 106.49 kNm

Step R12: Uniaxial moment capacities

Using Charts 44 and 45 for M_{ux1} and Charts 45 and 46 for M_{uy1}, we get (i) the coefficient 0.1032 (interpolating 0.11 and 0.10) and (ii) the coefficient 0.0954 (interpolating 0.1 and 0.09) for M_{ux1} and M_{uy1}, respectively.

M_{ux1} = (0.1032)(30)(350)(450)(450)(10^{-6}) = 219.429 kNm

M_{uy1} = (0.0954)(30)(450)(350)(350)(10^{-6}) = 157.77 kNm

Step R13: Value of α_n (Eq.10.60 of Lesson 26)

P_u/P_{uz} = 1700/3295.514 = 0.5158 which gives

α_n = 1 + (0.5158 – 0.2)/0.6 = 1.5263

Step R14: Checking of column for safety (Eq.10.58 of Lesson 26)

$(118.13/219.424)^{1.5263} + (106.49/157.77)^{1.5263} = 0.3886 + 0.5488 = 0.9374 < 1.0$

Hence, the revised reinforcement is safe. The section is shown in Fig.10.27.18.

10.27.9 Practice Questions and Problems with Answers

Q.1: Define a slender column. Give three reasons for its increasing importance and popularity.

A.1: See sec. 10.27.1.

Q.2: Explain the behaviour of a slender column subjected to concentric loading. Explain Euler's load.

A.2: See sec.10.27.3.

Q.3: Choose the correct answer.

(A) As the slenderness ratio increases, the strength of concentrically loaded column:
(i) increases (ii) decreases

(B) For braced columns, the effective length is between

(i) *l* and 2*l* (ii) 0.5*l* and 2*l* (iii) 0.5*l* and *l*

(C) The critical load of a braced frame is

(i) always larger than that of an unbraced column

(ii) always smaller than that of an unbraced column

(iii) sometimes larger and sometimes smaller than that of an unbraced column

A.3: A. (ii), B. (iii), C. (i)

Q.4: Explain the behaviour of slender columns under axial load and uniaxial bending, bent in single curvature.

A.4: Part (A) of sec. 10.27.3.

Q.5: Explain the behaviour of slender columns under axial load and uniaxial bending, bent in double curvature.

A.5: Part (B) of sec. 10.27.3.

Q.6: Explain the behaviour of columns in portal frame both braced and unbraced.

A.6: Part (C) of sec. 10.27.3.

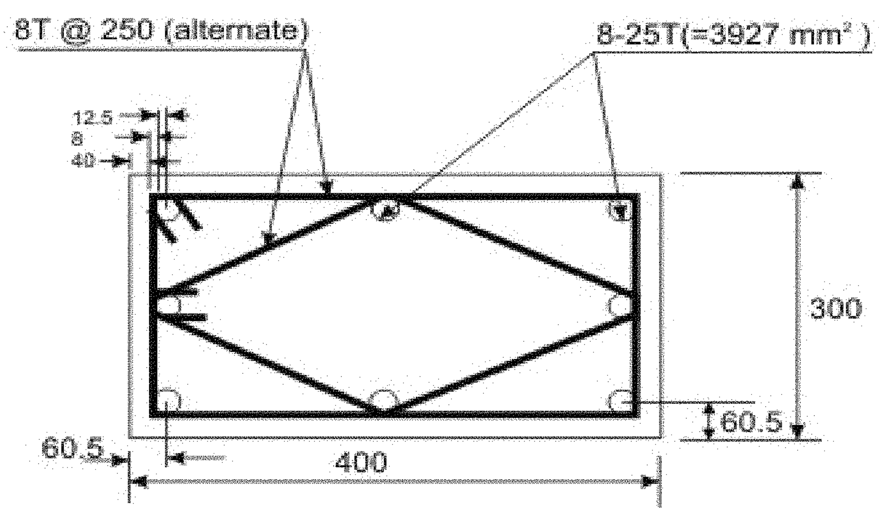


Fig. 10.27.19: Q.7

Q.7: Check the column of Fig.10.27.19, if subjected to an axial factored load of P_u = 1500 kN only when the unsupported length of the column = l = 8.0 m, l_{ex} = l_{ey} = 6.0 m, D = 400 mm, b = 300 mm, using concrete of M 20 and steel grade in Fe 415.

A.7: Solution:

Step 1: Slenderness ratios

L_{ex}/D = 6000/400 = 15 > 12

L_{ey}/b = 6000/300 = 20 > 12

The column is slender about both the axes.

Step 2: Minimum eccentricities and moments due to minimum eccentricities (Eq.10.3 of Lesson 21)

$e_{x\,min}$ = $l/500 + D/30$ = 8000/500 + 400/30 = 29.33 mm > 20 mm

$e_{y\,min}$ = 8000/500 + 300/30 = 26 mm > 20 mm

M_x due to min. ecc. = P_u ($e_{x\,min}$) = 1500(29.33) = 43.995 kNm

M_y due to min. ecc. = P_u ($e_{y\,min}$) = 1500(26.0) = 39.0 kNm

Step 3: Primary moments

Since the column is concentrically loaded, the primary moments are zero. Therefore, the additional moments must be greater than the respective moments due to minimum eccentricity.

Step 4: Additional eccentricities and moments (Eq.10.84)

e_{ax} = $D(l_{ex}/D)^2/2000$ = $400(6000/400)^2/2000$ = 45 mm > $e_{x\,min}$ (= 29.23 mm)

e_{ay} = $b(l_{ey}/b)^2/2000$ = $300(6000/300)^2/2000$ = 60 mm > $e_{y\,min}$ (= 26 mm)

Step 5: Calculation of balance loads P_{bx} and P_{by}

Given A_{sc} = 3927 mm^2 (8 bars of 25 mm diameter give p = 3.2725 per cent. So, p/f_{ck} = 0.1636. Using 8 mm diameter lateral tie, d' = 40 + 8 + 12.5 =

60.5 mm giving d'/D = 60.5/400 = 0.15125 ≅ 0.15 and d'/b = 60.5/300 = 0.2017 ≅ 0.20.

From Table 60 of SP-16, we get k_1 = 0.196 and k_2 = 0.061. Thus, we have:

$$P_{bx} = \{0.196 + (0.061)(0.1636)\}(20)(300)(400)(10^{-3}) = 494.35 \text{ kN}$$

Similarly, for P_{by}: k_1 = 0.184 and k_2 = -0.011, we get

$$P_{by} = \{0.184 - (0.011)(0.1636)\}(20)(300)(400)(10^{-3}) = 437.281 \text{ kN}$$

Since, P_{bx} and P_{by} are less than P_u (= 1500 kN), modification factors are to be incorporated.

Step 6: Determination of P_{uz} (Eq.10.59 of Lesson 26)

$$P_{uz} = 0.45(20)(300)(400) + \{0.75(415) - 0.45(20)\}(3927)(10^{-3}) = 2266.94 \text{ kN}$$

Step 7: Determination of modification factors

k_{ax} = (2266.94 – 1500)/(2266.94 – 494.35) = 0.433 and

k_{ay} = (2266.94 – 1500)/(2266.94 – 437.281) = 0.419

Step 8: Additional moments and total moments

$$M_{ax} = 1500(0.433)(45) = 29.2275 \text{ kNm}$$

$$M_{ay} = 1500(0.419)(60) = 37.71 \text{ kNm}$$

Since, primary moments are zero as the column is concentrically loaded, the total moment shall consist of the additional moments. But, as both the additional moments are less than the respective moment due to minimum eccentricity, the revised additional moments are: M_{ax} = 43.995 kNm and M_{ay} = 39.0 kNm, which are the total moments also.

Thus, we have:

M_{ux} = 43.995 kNm, M_{uy} = 39.0 kNm and P_u = 1500 kN.

Step 9: Uniaxial moment capacities

We have, $P_u/f_{ck}\,bD$ = {1500/(20)(300)(400)}(1000) = 0.625, p/f_{ck} = 0.1636 and d'/D = 0.15 for M_{ux1}; and d'/b = 0.2 for M_{uy1}. The coefficients are 0.11 (from Chart 45) and 0.1 (from Chart 46) for M_{ux1} and M_{uy1}, respectively. So, we get,

$$M_{ux1} = 0.11(20)(300)(400)(400)(10^{-6}) = 225.28 \text{ kNm, and}$$

$$M_{uy1} = 0.1(20)(300)(300)(400)(10^{-6}) = 72.0 \text{ kNm}$$

Step 10: Value of α_n (Eq.10.60 of Lesson 26)

Here, P_u/P_{uz} = 1500/2266.94 = 0.6617. So, we get

$$\alpha_n = 1.0 + (0.4617/0.6) = 1.7695$$

Step 11: Checking the column for safety (Eq.10.58 of Lesson 26)

$$(M_{ux}/M_{ux1})^{\alpha_n} + (M_{uy}/M_{uy1})^{\alpha_n} \le 1$$

Here, $(43.995/225.28)^{1.7695} + (39.0/72.0)^{1.7695} = 0.0556 + 0.3379 = 0.3935 < 1$

Hence, the column is safe to carry P_u = 1500 kN.

11.27.10 References

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11.27.11 Test 27 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions.

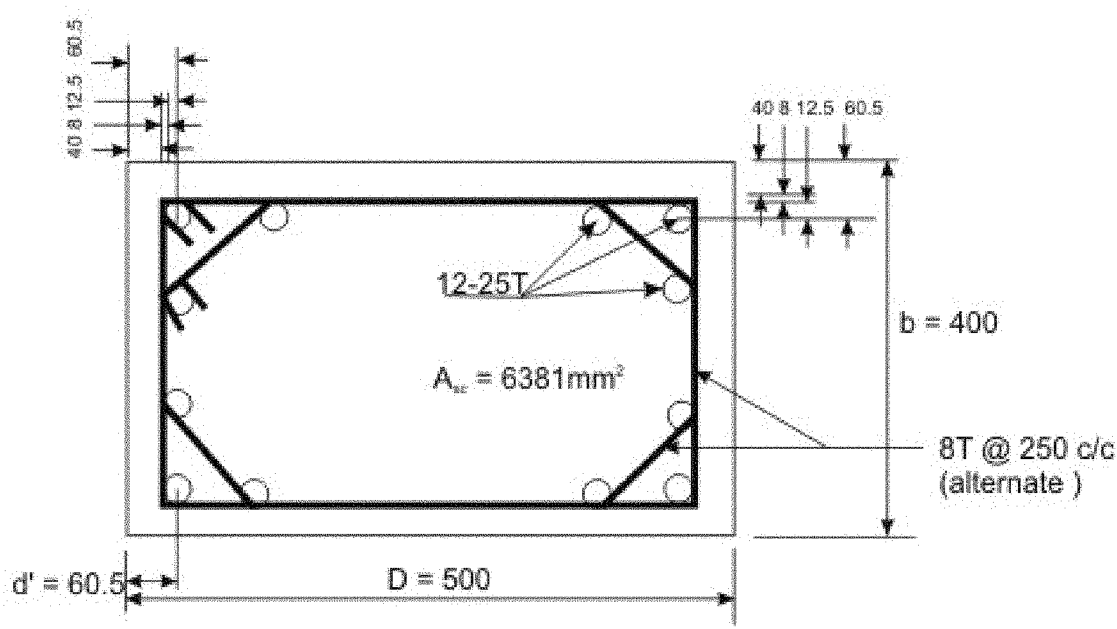

Fig. 10.27.20: TQ.1

TQ.1: Determine the primary, additional and total moments of the column shown in Fig.10.27.20 for the three different cases:

(i) When the column is braced against sidesway and is bent in single curvature.

(ii) When the column is braced against sidesway and is bent in double curvature.

(iii) When the column is unbraced.

Use the following data: P_u = 2000 kN, concrete grade = M 20, steel grade = Fe 415, unsupported length l = 8.0 m, l_{ex} = 7.0 m, l_{ey} = 6.0 m, A_{sc} = 6381 mm^2 (12-25 mm diameter bars), lateral tie = 8 mm diameter @ 250 mm c/c, d' = 60.5 mm, D = 500 mm and b = 400 mm. The factored moments are: 70 kNm at top and 40 kNm at bottom in the direction of larger dimension and 60 kNm at top and 30 kNm at bottom in the direction of shorter dimension.

A.TQ.1: Solution

The following are the common steps for all three cases.

Step 1: Slenderness ratios

l_{ex}/D = 7000/500 = 14 > 12 and l_{ey}/b = 6000/400 = 15 > 12

The column is slender about both axes.

Step 2: Minimum eccentricities and moments due to minimum eccentricities (Eq.10.3 of Lesson 21)

$e_{x\,min}$ = l/500 + D/30 = 8000/500 + 500/30 = 32.67 mm > 20 mm, and

$e_{y\,min}$ = l/500 + b/30 = 8000/500 + 400/30 = 29.34 mm > 20 mm

M_x (min. ecc.) = 2000(32.67)(10^{-3}) = 65.34 kNm, and

M_y (min. ecc.) = 2000(29.34)(10^{-3}) = 58.68 kNm

Step 3: Additional eccentricities and moments due to additional eccentricities (Eq.10.84)

e_{ax} = $D(l_{ex}/D)^2/2000$ = 500(7000/500)^2/2000 = 49 mm > $e_{x\,min}$ (= 32.67 mm)

e_{ay} = $b(l_{ey}/b)^2/2000$ = $400(6000/400)^2/2000$ = 45 mm > $e_{y\,min}$ (= 29.34 mm)

M_{ax} = $P_u(e_{ax})$ = $(2000)(49)(10^{-3})$ = 98 kNm, and

M_{ay} = $P_u(e_{ay})$ = $(2000)(45)(10^{-3})$ = 90 kNm

Step 4: Calculation of balanced loads

Using d'/D = 0.121 and p/f_{ck} = 3.1905/20 = 0.159525 in Table 60 of SP-16, we have k_1 = 0.20238 and k_2 = 0.2755 (by linear interpolation). This gives

P_{bx} = {0.20238 + 0.2755(0.159525)}(20)(400)(500)(10^{-3}) = 983.32 kN

Similarly, d'/b = 0.15125 and p/f_{ck} = 0.159525 in Table 60 of SP-16 gives k_1 = 0.1957 and k_2 = 0.198625 (by linear interpolation). So, we get

P_{by} = {0.1957 + 0.198625(0.159525)}(20)(400)(500)(10^{-3}) = 909.54 kN

Both P_{bx} and P_{by} are smaller than P_u (= 2000 kN). Hence, modification factors are to be incorporated.

Step 5: Calculation of P_{uz} (Eq.10.59 of Lesson 26)

P_{uz} = $0.45\, f_{ck}\, A_g + (0.75\, f_y - 0.45\, f_{ck})\, A_{sc}$

= 0.45(20)(400)(500) + {0.75(415) – 0.45(20)}(6381) = 3728.66 kN

Step 6: Modification factors and revised additional moments (Eqs.10.92 and 10.93)

k_{ax} = (3728.66 - 2000)/(3728.66 – 983.32) = 0.6297, and

k_{ay} = (3728.66 - 2000)/(3728.66 – 909.54) = 0.6132

The revised additional moments are:

M_{ax} = 98(0.6297) = 61.71 kNm, and

M_{ay} = 90(0.6132) = 55.19 kNm

Now, the different cases are explained.

Case (i): Braced column in single curvature

Primary moments = 0.4 M_1 + 0.6 M_2, but should be equal to or greater than 0.4 M_2 and moment due to minimum eccentricities. So, we get,

M_{ox} = largest of 58 kNm, 28 kNm and 65.34 kNm = 65.34 kNm

M_{oy} = largest of 48 kNm, 24 kNm and 58.68 kNm = 58.68 kNm

Additional moments are M_{ax} = 61.71 kNm and M_{ay} = 55.19 kNm (incorporating the respective modification factors).

Total moments = M_{ux} = M_{ox} + M_{ax} = 65.34 + 61.71 = 127.05 kNm > 65.34 kNm (moment due to minimum eccentricity), and

M_{uy} = M_{oy} + M_{ay} = 58.68 + 55.19 = 113.87 kNm > 58.68 kNm (moment due to minimum eccentricity).

Case (ii): Braced column in double curvature

Primary moments = - 0.4 M_1 + 0.6 M_2, but should be equal to or greater than 0.4M_2 and the moment due to minimum eccentricity. So, we get,

M_{ox} = largest of 26 kNm, 28 kNm and 65.34 kNm = 65.34 kNm

M_{oy} = largest of 24 kNm, 24 kNm and 58.68 kNm = 58.68 kNm

Additional moments are M_{ax} = 61.71 kNm and M_{ay} = 55.19 kNm

Final moments = M_{ux} = M_{ox} + M_{ax} = 65.34 + 61.71 = 127.05 kNm > 65.34 kNm (moment due to minimum eccentricity), and

M_{uy} = 58.68 + 55.19 = 113.87 kNm > 58.68 kNm (moment due to minimum eccentricity).

Case (iii): Unbraced column

Primary moments = M_2 and should be greater than or equal to moment due to minimum eccentricity.

M_{ox} = 70 kNm > 65.34 kNm (moment due to minimum eccentricity), and

M_{oy} = 60 kNm > 58.68 kNm (moment due to minimum eccentricity).

Additional moments are M_{ax} = 61.71 kNm and M_{ay} = 55.19 kNm

Final moments = M_{ux} = M_{ox} + M_{ax} = 70.0 + 61.71 = 131.71 kNm > 65.34 kNm (moment due to minimum eccentricity), and

M_{uy} = M_{oy} + M_{ax} = 60.0 + 55.19 = 115.19 kNm > 58.68 kNm (moment due to minimum eccentricity).

10.27.12 Summary of this Lesson

This lesson mentions the reasons of increasing importance and popularity of slender columns and explains the behaviour of slender columns loaded concentrically or eccentrically. The role of minimum eccentricity that cannot be avoided in any practical column is explained for slender columns. The moments due to minimum eccentricities in both directions should be taken into account for a slender column loaded concentrically as it should be designed under biaxial bending. On the other hand, the given primary moments are also to be checked so that they are equal to or greater than the respective moments due to minimum eccentricity for all slender columns.

Both braced and unbraced columns, bent in single or double curvatures, are explained. The importance of modification factors of the additional moments due to P-Δ effect is explained. Effective lengths and important parameter to determine the slenderness ratios are illustrated for different types of support conditions either in single column or when the column is a part of rigid frames. Additional moment method, a simple method for the design of slender columns, is explained, which is recommended in IS 456. Numerical problems in illustrative example, practice problem and test questions will help in understanding and applying the method for the design of slender columns, as stipulated in IS 456. Direct computations from the given equations as well as use of design charts and tables of SP-16 are illustrated for the design.