LECTURE 1 INTRODUCTION AND REVIEW

Preamble: Engineering science is usually subdivided into number of topics such as 1. Solid Mechanics 2. Fluid Mechanics 3. Heat Transfer 4. Properties of materials and soon Although there are close links between them in terms of the physical principles involved and methods of analysis employed.

The solid mechanics as a subject may be defined as a branch of applied mechanics that deals with behaviours of solid bodies subjected to various types of loadings. This is usually subdivided into further two streams i.e Mechanics of rigid bodies or simply Mechanics and Mechanics of deformable solids.

The mechanics of deformable solids which is branch of applied mechanics is known by several names i.e. strength of materials, mechanics of materials etc.

Mechanics of rigid bodies: The mechanics of rigid bodies is primarily concerned with the static and dynamic behaviour under external forces of engineering components and systems which are treated as infinitely strong and undeformable Primarily we deal here with the forces and motions associated with particles and rigid bodies.

Mechanics of deformable solids :

Mechanics of solids:

The mechanics of deformable solids is more concerned with the internal forces and associated changes in the geometry of the components involved. Of particular importance are the properties of the materials used, the strength of which will determine whether the components fail by breaking in service, and the stiffness of which will determine whether the amount of deformation they suffer is acceptable. Therefore, the subject of mechanics of materials or strength of materials is central to the whole activity of engineering design. Usually the objectives in analysis here will be the determination of the stresses, strains, and deflections produced by loads. Theoretical analyses and experimental results have an equal roles in this field.

Analysis of stress and strain :

Concept of stress : Let us introduce the concept of stress as we know that the main problem of engineering mechanics of material is the investigation of the internal resistance of the body, i.e. the nature of forces set up within a body to balance the effect of the externally applied forces. The externally applied forces are termed as loads. These externally applied forces may be due to any one of the reason.

(i) due to service conditions (ii) due to environment in which the component works (iii) through contact with other members

(iv) due to fluid pressures (v) due to gravity or inertia forces.

As we know that in mechanics of deformable solids, externally applied forces acts on a body and body suffers a deformation. From equilibrium point of view, this action should be opposed or reacted by internal forces which are set up within the particles of material due to cohesion.

These internal forces give rise to a concept of stress. Therefore, let us define a stress Therefore, let us define a term stress

Stress: stress is defined as the force intensity or force per unit area.

Where A is the area of the section

Here we are using an assumption that the total force or total load carried by any material is uniformly distributed over its cross – section.

But the stress distributions may be for from uniform, with local regions of high stress known as stress concentrations.

If the force carried by a component is not uniformly distributed over its cross – sectional area, A, we must consider a small area, 'dA' which carries a small load dP, of the total force 'P', Then definition of stress is

As a particular stress generally holds true only at a point, therefore it is defined mathematically as

Units : The basic units of stress in S.I units i.e. (International system) are N / m2 (or Pa)

MPa = 106 Pa ; GPa = 109 Pa; KPa = 103 Pa

Some times N / mm2 units are also used, because this is an equivalent to MPa. While US customary unit is pound per square inch psi.

TYPES OF STRESSES :
only two basic stresses exists : (1) normal stress and (2) shear shear stress. Other stresses either are similar to these basic stresses or are a combination of these e.g. bending stress is a combination tensile, compressive and shear stresses. Torsional stress, as encountered in twisting of a shaft is a shearing stress.
Let us define the normal stresses and shear stresses in the following sections.
Normal stresses : We have defined stress as force per unit area. If the stresses are normal to the areas concerned, then these are termed as normal stresses. The normal stresses are generally denoted by a Greek letter (s)

This is also known as uniaxial state of stress, because the stresses acts only in one direction however, such a state rarely exists, therefore we have biaxial and triaxial state of stresses where either the two mutually perpendicular normal stresses acts or three mutually perpendicular normal stresses acts as shown in the figures below :

Tensile or compressive stresses : The normal stresses can be either tensile or compressive whether the stresses acts out of the area or into the area

Bearing Stress : When one object presses against another, it is referred to a bearing stress (They are in fact the compressive stresses).

Shear stresses :
Let us consider now the situation, where the cross – sectional area of a block of material is subject to a distribution of forces which are parallel, rather than normal, to the area concerned. Such forces are associated with a shearing of the material, and are referred to as shear forces. The resulting force interistes are known as shear stresses.

The resulting force intensities are known as shear stresses, the mean shear stress being equal to

Where P is the total force and A the area over which it acts.   As we know that the particular stress generally holds good only at a point therefore we can define shear stress at a point as

The greek symbol t (tau) (suggesting tangential) is used to denote shear stress. However, it must be borne in mind that the stress (resultant stress) at any point in a body is basically resolved into two components s and t one acts perpendicular and other parallel to the area concerned, as it is clearly defined in the following figure.

The single shear takes place on the single plane and the shear area is the cross - sectional of the rivett, whereas the double shear takes place in the case of Butt joints of rivetts and the shear area is the twice of the X - sectional area of the rivett.
LECTURE 2
ANALYSIS OF STERSSES
General State of stress at a point : Stress at a point in a material body has been defined as a force per unit area. But this definition is some what ambiguous since it depends upon what area we consider at that point. Let us, consider a point ‘q' in the interior of the body

Let us pass a cutting plane through a pont 'q' perpendicular to the x - axis as shown below

The corresponding force components can be shown like this   dFx = sxx. dax dFy = txy. dax   dFz = txz. dax
where dax is the area surrounding the point 'q' when the cutting plane ^ r is to x - axis.
In a similar way it can be assummed that the cutting plane is passed through the point 'q' perpendicular to the y - axis. The corresponding force components are shown below

The corresponding force components may be written as dFx = tyx. day dFy = syy. day dFz = tyz. day
where day is the area surrounding the point 'q' when the cutting plane ^ r is to y - axis.
In the last it can be considered that the cutting plane is passed through the point 'q' perpendicular to the z - axis.
The corresponding force components may be written as dFx = tzx. daz
dFy = tzy. daz   dFz = szz. daz
where daz is the area surrounding the point 'q' when the cutting plane ^ r is to z - axis.
Thus, from the foregoing discussion it is amply clear that there is nothing like stress at a point 'q' rather we have a situation where it is a combination of state of stress at a point q. Thus, it becomes imperative to understand the term state of stress at a point 'q'. Therefore, it becomes easy to express astate of stress by the scheme as discussed earlier, where the stresses on the three mutually perpendiclar planes are labelled in the manner as shown earlier. the state of stress as depicted earlier is called the general or a triaxial state of stress that can exist at any interior point of a loaded body.
Before defining the general state of stress at a point. Let us make overselves conversant with the notations for the stresses.
We have already chosen to distinguish between normal and shear stress with the help of symbols s and t .
Cartesian - co-ordinate system
In the Cartesian co-ordinates system, we make use of the axes, X, Y and Z Let us consider the small element of the material and show the various normal stresses acting the faces

Thus, in the Cartesian co-ordinates system the normal stresses have been represented by sx, syand sz.
Cylindrical - co-ordinate system
In the Cylindrical - co-ordinate system we make use of co-ordinates r, q and Z.

Thus, in the Cylindrical co-ordinates system, the normal stresses i.e components acting over a element is being denoted by sr, sqand sz.
Sign convention : The tensile forces are termed as (+ve) while the compressive forces are termed as negative (-ve).
First sub – script : it indicates the direction of the normal to the surface.
Second subscript : it indicates the direction of the stress.
It may be noted that in the case of normal stresses the double script notation may be dispensed with as the direction of the normal stress and the direction of normal to the surface of the element on which it acts is the same. Therefore, a single subscript notation as used is sufficient to define the normal stresses.
Shear Stresses : With shear stress components, the single subscript notation is not practical, because such stresses are in direction parallel to the surfaces on which they act. We therefore have two directions to specify, that of normal to the surface and the stress itself. To do this, we stress itself. To do this, we attach two subscripts to the symbol ' t' , for shear stresses.
In cartesian and polar co-ordinates, we have the stress components as shown in the figures.
txy , tyx , tyz , tzy , tzx , txz   trq , tqr , tqz , tzq ,tzr , trz
So as shown above, the normal stresses and shear stress components indicated on a small element of material seperately has been combined and depicted on a single element. Similarly for a cylindrical co-ordinate system let us shown the normal and shear stresses components separately.

Now let us combine the normal and shear stress components as shown below :

Now let us define the state of stress at a point formally.
State of stress at a point :
By state of stress at a point, we mean an information which is required at that point such that it remains under equilibrium. or simply a general state of stress at a point involves all the normal stress components, together with all the shear stress components as shown in earlier figures.

Therefore, we need nine components, to define the state of stress at a point
sx txy txz   sy tyx tyz sz tzx tzy
If we apply the conditions of equilibrium which are as follows: å Fx = 0 ; å M x = 0 å Fy = 0 ; å M y = 0 å Fz = 0 ; å M z = 0
Then we get   txy = tyx   tyz = tzy   tzx = txy
Then we will need only six components to specify the state of stress at a point i.e   sx , sy, sz , txy , tyz , tzx
Now let us define the concept of complementary shear stresses.
Complementary shear stresses: The existence of shear stresses on any two sides of the element induces complementary shear stresses on the other two sides of the element to maintain equilibrium.

on planes AB and CD, the shear stress t acts. To maintain the static equilibrium of this element, on planes AD and BC, t' should act, we shall see that t' which is known as the complementary shear stress would come out to equal and opposite to the t . Let us prove this thing for a general case as discussed below:

The figure shows a small rectangular element with sides of length Dx, Dy parallel to x and y directions. Its thickness normal to the plane of paper is Dz in z – direction. All nine normal and shear stress components may act on the element, only those in x and y directions are shown.
Sign convections for shear stresses: Direct stresses or normal stresses   - tensile +ve   - compressive –ve
Shear stresses: - tending to turn the element C.W +ve.   - tending to turn the element C.C.W – ve.
The resulting forces applied to the element are in equilibrium in x and y direction. (Although other normal and shear stress components are not shown, their presence does not affect the final conclusion).
Assumption : The weight of the element is neglected.
Since the element is a static piece of solid body, the moments applied to it must also be in equilibrium. Let ‘O' be the centre of the element. Let us consider the axis through the point ‘O'. the resultant force associated with normal stresses sx and sy acting on the sides of the element each pass through this axis, and therefore, have no moment.
Now forces on top and bottom surfaces produce a couple which must be balanced by the forces on left and right hand faces   Thus,   tyx . D x . D z . D y = txy . D x . D z . D y
In other word, the complementary shear stresses are equal in magnitude. The same form of relationship can be obtained for the other two pair of shear stress components to arrive at the relations

LECTURE 3 Analysis of Stresses:
Consider a point ‘q' in some sort of structural member like as shown in figure below. Assuming that at point exist. ‘q' a plane state of stress exist. i.e. the state of state stress is to describe by a parameters sx, sy and txy These stresses could be indicate a on the two dimensional diagram as shown below:

This is a commen way of representing the stresses. It must be realize a that the material is unaware of what we have called the x and y axes. i.e. the material has to resist the loads irrespective less of how we wish to name them or whether they are horizontal, vertical or otherwise further more, the material will fail when the stresses exceed beyond a permissible value. Thus, a fundamental problem in engineering design is to determine the maximum normal stress or maximum shear stress at any particular point in a body. There is no reason to believe apriori that sx, sy and txy are the maximum value. Rather the maximum stresses may associates themselves with some other planes located at ‘q'. Thus, it becomes imperative to determine the values of sq and tq. In order tto achieve this let us consider the following.
Shear stress:
If the applied load P consists of two equal and opposite parallel forces not in the same line, than there is a tendency for one part of the body to slide over or shear from the other part across any section LM. If the cross section at LM measured parallel to the load is A, then the average value of shear stress t =

P/A . The shear stress is tangential to the area over which it acts.
If the shear stress varies then at a point then t may be defined as
Complementary shear stress:
Let ABCD be a small rectangular element of sides x, y and z perpendicular to the plane of paper let there be shear stress acting on planes AB and CD . It is obvious that these stresses will from a couple (t . xz)y which can only be balanced by tangential forces on planes AD and BC. These are known as complementary shear stresses. i.e. the existence of shear stresses on sides AB and CD of the element implies that there must also be complementary shear stresses on to maintain equilibrium.
Let t' be the complementary shear stress induced on planes AD and BC. Then for the equilibrium (t . xz)y = t' (yz)x t = t'
Thus, every shear stress is accompanied by an equal complementary shear stress.
Stresses on oblique plane: Till now we have dealt with either pure normal direct stress or pure shear stress. In many instances, however both direct and shear stresses acts and the resultant stress across any section will be neither normal nor tangential to the plane.
A plane stse of stress is a 2 dimensional stae of stress in a sense that the stress components in one direction are all zero i.e sz = tyz = tzx = 0
examples of plane state of stress includes plates and shells.
Consider the general case of a bar under direct load F giving rise to a stress sy vertically
The stress acting at a point is represented by the stresses acting on the faces of the element enclosing the point.
The stresses change with the inclination of the planes passing through that point i.e. the stress on the faces of the element vary as the angular position of the element changes.
Let the block be of unit depth now considering the equilibrium of forces on the triangle portion ABC Resolving forces perpendicular to BC, gives
sq.BC.1 = sysinq . AB . 1 but AB/BC = sinq or AB = BCsinq
Substituting this value in the above equation, we get sq.BC.1 = sysinq . BCsinq . 1 or (1)
Now resolving the forces parallel to BC tq.BC.1 = sy cosq . ABsinq . 1
again AB = BCcosq tq.BC.1 = sycosq . BCsinq . 1 or tq = sysinqcosq (2)
If q = 900 the BC will be parallel to AB and tq = 0, i.e. there will be only direct stress or normal stress.
By examining the equations (1) and (2), the following conclusions may be drawn
(i) The value of direct stress sq is maximum and is equal to sy when q = 900.
(ii) The shear stress tq has a maximum value of 0.5 sy when q = 450
(iii) The stresses sq and sq are not simply the resolution of sy
Material subjected to pure shear:
Consider the element shown to which shear stresses have been applied to the sides AB and DC
Complementary shear stresses of equal value but of opposite effect are then set up on the sides AD and BC in order to prevent the rotation of the element. Since the applied and complementary shear stresses are of equal value on the x and y planes. Therefore, they are both represented by the symbol txy.
Now consider the equilibrium of portion of PBC
Assuming unit depth and resolving normal to PC or in the direction of sq
sq.PC.1 = txy.PB.cosq.1+ txy.BC.sinq.1 = txy.PB.cosq + txy.BC.sinq
Now writing PB and BC in terms of PC so that it cancels out from the two sides
PB/PC = sinq BC/PC = cosq sq.PC.1 = txy.cosqsinqPC+ txy.cosq.sinqPC
sq = 2txysinqcosq sq = txy.2.sinqcosq
(1)
Now resolving forces parallel to PC or in the direction tq.then txyPC . 1 = txy . PBsinq - txy . BCcosq
-ve sign has been put because this component is in the same direction as that of tq.
again converting the various quantities in terms of PC we have txyPC . 1 = txy . PB.sin2q - txy . PCcos2q = -[txy (cos2q - sin2q)]
= -txycos2q or (2)
the negative sign means that the sense of tq is opposite to that of assumed one.
Let us examine the equations (1) and (2) respectively

From equation (1) i.e,   sq = txy sin2q
The equation (1) represents that the maximum value of sq is txy when q = 450.
Let us take into consideration the equation (2) which states that   tq = - txy cos2q
It indicates that the maximum value of tq is txy when q = 00 or 900. it has a value zero when q = 450.
From equation (1) it may be noticed that the normal component sq has maximum and minimum values of +txy (tension) and -txy (compression) on plane at ± 450 to the applied shear and on these planes the tangential component tq is zero.

Hence the system of pure shear stresses produces and equivalent direct stress system, one set compressive and one tensile each located at 450 to the original shear directions as depicted in the figure below:

Material subjected to two mutually perpendicular direct stresses:
Now consider a rectangular element of unit depth, subjected to a system of two direct stresses both tensile, sx and syacting right angles to each other.

for equilibrium of the portion ABC, resolving perpendicular to AC   sq . AC.1 = sy sin q . AB.1 + sx cos q . BC.1
converting AB and BC in terms of AC so that AC cancels out from the sides   sq = sy sin2q + sxcos2q
Futher, recalling that cos2q - sin2q = cos2q or (1 - cos2q)/2 = sin2q
Similarly (1 + cos2q)/2 = cos2q
Hence by these transformations the expression for sq reduces to = 1/2sy (1 - cos2q) + 1/2sx (1 + cos2q)
On rearranging the various terms we get
(3)
Now resolving parallal to AC   sq.AC.1= -txy..cosq.AB.1+ txy.BC.sinq.1
The – ve sign appears because this component is in the same direction as that of AC.   Again converting the various quantities in terms of AC so that the AC cancels out from the two sides.
(4)
Conclusions :
The following conclusions may be drawn from equation (3) and (4)
(i)   The maximum direct stress would be equal to sx or sy which ever is the greater, when q = 00 or 900   (ii) The maximum shear stress in the plane of the applied stresses occurs when q = 450

LECTURE 4 Material subjected to combined direct and shear stresses: Now consider a complex stress system shown below, acting on an element of material. The stresses sx and sy may be compressive or tensile and may be the result of direct forces or as a result of bending.The shear stresses may be as shown or completely reversed and occur as a result of either shear force or torsion as shown in the figure below:

As per the double subscript notation the shear stress on the face BC should be notified as tyx , however, we have already seen that for a pair of shear stresses there is a set of complementary shear stresses generated such that tyx = txy
By looking at this state of stress, it may be observed that this state of stress is combination of two different cases:
(i) Material subjected to pure stae of stress shear. In this case the various formulas deserved are as follows
sq = tyx sin2 q   tq = - tyx cos 2 q
(ii) Material subjected to two mutually perpendicular direct stresses. In this case the various formula's derived are as follows.

To get the required equations for the case under consideration,let us add the respective equations for the above two cases such that

These are the equilibrium equations for stresses at a point. They do not depend on material proportions and are equally valid for elastic and inelastic behaviour
This eqn gives two values of 2q that differ by 1800 .Hence the planes on which maximum and minimum normal stresses occurate 900 apart.

From the triangle it may be determined

Substituting the values of cos2 q and sin2 q in equation (5) we get

This shows that the values oshear stress is zero on the principal planes.
Hence the maximum and minimum values of normal stresses occur on planes of zero shearing stress. The maximum and minimum normal stresses are called the principal stresses, and the planes on which they act are called principal plane the solution of equation

will yield two values of 2q separated by 1800 i.e. two values of q separated by 900 .Thus the two principal stresses occur on mutually perpendicular planes termed principal planes.
Therefore the two – dimensional complex stress system can now be reduced to the equivalent system of principal stresses.

Let us recall that for the case of a material subjected to direct stresses the value of maximum shear stresses

Therefore,it can be concluded that the equation (2) is a negative reciprocal of equation (1) hence the roots for the double angle of equation (2) are 900 away from the corresponding angle of equation (1).
This means that the angles that angles that locate the plane of maximum or minimum shearing stresses form angles of 450 with the planes of principal stresses.
Futher, by making the triangle we get

Because of root the difference in sign convention arises from the point of view of locating the planes on which shear stress act. From physical point of view these sign have no meaning.
The largest stress regard less of sign is always know as maximum shear stress.
Principal plane inclination in terms of associated principal stress:
We know that the equation yields two values of q i.e. the inclination of the two principal planes on which the principal stresses s1 and s2 act. It is uncertain,however, which stress acts on which plane unless equation. is used and observing which one of the two principal stresses is obtained. Alternatively we can also find the answer to this problem in the following manner

Consider once again the equilibrium of a triangular block of material of unit depth, Assuming AC to be a principal plane on which principal stresses sp acts, and the shear stress is zero.
Resolving the forces horizontally we get:
sx .BC . 1 + txy .AB . 1 = sp . cosq . AC dividing the above equation through by BC we get

LECTURE 5 GRAPHICAL SOLUTION – MOHR'S STRESS CIRCLE
The transformation equations for plane stress can be represented in a graphical form known as Mohr's circle. This grapical representation is very useful in depending the relationships between normal and shear stresses acting on any inclined plane at a point in a stresses body. To draw a Mohr's stress circle consider a complex stress system as shown in the figure

The above system represents a complete stress system for any condition of applied load in two dimensions
The Mohr's stress circle is used to find out graphically the direct stress s and sheer stress t on any plane inclined at q to the plane on which sx acts.The direction of q here is taken in anticlockwise direction from the BC.
STEPS:
In order to do achieve the desired objective we proceed in the following manner
(i) Label the Block ABCD. (ii) Set up axes for the direct stress (as abscissa) and shear stress (as ordinate) (iii) Plot the stresses on two adjacent faces e.g. AB and BC, using the following sign convention.

Direct stresses - tensile positive; compressive, negative Shear stresses – tending to turn block clockwise, positive – tending to turn block counter clockwise, negative
[i.e shearing stresses are +ve when its movement about the centre of the element is clockwise]
This gives two points on the graph which may than be labeled as respectively to denote stresses on these planes.
(iv) Join . (v) The point P where this line cuts the s axis is than the centre of Mohr's stress circle and the line joining is diameter. Therefore the circle can now be drawn. Now every point on the circle then represents a state of stress on some plane through C.

Proof: Consider any point Q on the circumference of the circle, such that PQ makes an angle 2q with BC, and drop a perpendicular from Q to meet the s axis at N.Then OQ represents the resultant stress on the plane an angle q to BC. Here we have assumed that sx > sy
Now let us find out the coordinates of point Q. These are ON and QN.
From the figure drawn earlier
ON = OP + PN OP = OK + KP OP = sy + 1/2 (sx-sy) = sy / 2 + sy / 2 + sx / 2 + sy / 2
= (sx + sy) / 2
PN = Rcos(2q - b)
hence ON = OP + PN = (sx + sy) / 2 + Rcos(2q - b)
= (sx + sy) / 2 + Rcos2q cosb + Rsin2qsinb
now make the substitutions for Rcosb and Rsinb.
Thus, ON = 1/2 (sx + sy) + 1/2 (sx - sy)cos2q + txysin2q (1) Similarly QM = Rsin(2q - b)
= Rsin2qcosb - Rcos2qsinb
Thus, substituting the values of R cosb and Rsinb, we get QM = 1/2 (sx - sy)sin2q - txycos2q (2)
If we examine the equation (1) and (2), we see that this is the same equation which we have already derived analytically
Thus the co-ordinates of Q are the normal and shear stresses on the plane inclined at q to BC in the original stress system.
N.B: Since angle PQ is 2q on Mohr's circle and not q it becomes obvious that angles are doubled on Mohr's circle. This is the only difference, however, as They are measured in the same direction and from the same plane in both figures.
Further points to be noted are :
(1) The direct stress is maximum when Q is at M and at this point obviously the sheer stress is zero, hence by definition OM is the length representing the maximum principal stresses s1 and 2q1 gives the angle of the plane q1 from BC. Similar OL is the other principal stress and is represented by s2
(2) The maximum shear stress is given by the highest point on the circle and is represented by the radius of the circle.
This follows that since shear stresses and complimentary sheer stresses have the same value; therefore the centre of the circle will always lie on the s axis midway between sx and sy . [since +txy & -txy are shear stress & complimentary shear stress so they are same in magnitude but different in sign.]
(3) From the above point the maximum sheer stress i.e. the Radius of the Mohr's stress circle would be

While the direct stress on the plane of maximum shear must be mid – may between sx and sy i.e

(4) As already defined the principal planes are the planes on which the shear components are zero. Therefore are conclude that on principal plane the sheer stress is zero.
(5) Since the resultant of two stress at 900 can be found from the parallogram of vectors as shown in the diagram.Thus, the resultant stress on the plane at q to BC is given by OQ on Mohr's Circle.

(6) The graphical method of solution for a complex stress problems using Mohr's circle is a very powerful technique, since all the information relating to any

plane within the stressed element is contained in the single construction. It thus, provides a convenient and rapid means of solution. Which is less prone to arithmetical errors and is highly recommended.

LECTURE 6 ILLUSRATIVE PROBLEMS: Let us discuss few representative problems dealing with complex state of stress to be solved either analytically or graphically.

PROB 1: A circular bar 40 mm diameter carries an axial tensile load of 105 kN. What is the Value of shear stress on the planes on which the normal stress has a value of 50 MN/m2 tensile.

PROB 2: For a given loading conditions the state of stress in the wall of a cylinder is expressed as follows: (a) 85 MN/m2 tensile (b) 25 MN/m2 tensile at right angles to (a) (c) Shear stresses of 60 MN/m2 on the planes on which the stresses (a) and (b) act; the sheer couple acting on planes carrying the 25 MN/m2 stress is clockwise in effect. Calculate the principal stresses and the planes on which they act. What would be the effect on these results if owing to a change of loading (a) becomes compressive while stresses (b) and (c) remain unchanged

Salient points of Mohr's stress circle: 1. complementary shear stresses (on planes 900 apart on the circle) are equal in magnitude 2. The principal planes are orthogonal: points L and M are 1800 apart on the circle (900 apart in material)

3. There are no shear stresses on principal planes: point L and M lie on normal stress axis. 4. The planes of maximum shear are 450 from the principal points D and E are 900 , measured round the circle from points L and M.

5. The maximum shear stresses are equal in magnitude and given by points D and E 6. The normal stresses on the planes of maximum shear stress are equal i.e. points D and E both have normal stress co-ordinate which is equal to the two principal stresses.

As we know that the circle represents all possible states of normal and shear stress on any plane through a stresses point in a material. Further we have seen that the co-ordinates of the point ‘Q' are seen to be the same as those derived from equilibrium of the element. i.e. the normal and shear stress components on any plane passing through the point can be found using Mohr's circle. Worthy of note:

1. The sides AB and BC of the element ABCD, which are 900 apart, are represented on the circle by and they are 1800 apart.

2. It has been shown that Mohr's circle represents all possible states at a point. Thus, it can be seen at a point. Thus, it, can be seen that two planes LP and PM, 1800 apart on the diagram and therefore 900 apart in the material, on which shear stress tq is zero. These planes are termed as principal planes and normal stresses acting on them are known as principal stresses. Thus , s1 = OL s2 = OM

3. The maximum shear stress in an element is given by the top and bottom points of the circle i.e by points J1 and J2 ,Thus the maximum shear stress would be equal to the radius of i.e. tmax= 1/2(s1- s2),the corresponding normal stress is obviously the distance OP = 1/2 (sx+ sy) , Further it can also be seen that the planes on which the shear stress is maximum are situated 900 from the principal planes (on circle), and 450 in the material.

4.The minimum normal stress is just as important as the maximum. The algebraic minimum stress could have a magnitude greater than that of the maximum principal stress if the state of stress were such that the centre of the circle is to the left of orgin. i.e. if s1 = 20 MN/m2 (say) s2 = -80 MN/m2 (say) Then tmaxm = (s1 - s2 / 2) = 50 MN/m2

If should be noted that the principal stresses are considered a maximum or minimum mathematically e.g. a compressive or negative stress is less than a positive stress, irrespective or numerical value.

5. Since the stresses on perpendular faces of any element are given by the co-ordinates of two diametrically opposite points on the circle, thus, the sum of the two normal stresses for any and all orientations of the element is constant, i.e. Thus sum is an invariant for any particular state of stress.

Sum of the two normal stress components acting on mutually perpendicular planes at a point in a state of plane stress is not affected by the orientation of these planes.

This can be also understand from the circle Since AB and BC are diametrically opposite thus, what ever may be their orientation, they will always lie on the diametre or we can say that their sum won't change, it can also be seen from analytical relations We know on plane BC; q = 0 sn1 = sx
on plane AB; q = 2700 sn2 = sy Thus sn1 + sn2= sx+ sy
6. If s1 = s2, the Mohr's stress circle degenerates into a point and no shearing stresses are developed on xy plane.
7. If sx+ sy= 0, then the center of Mohr's circle coincides with the origin of s - t co-ordinates.

LECTURE 7-ANALYSIS OF STRAINS
CONCEPT OF STRAIN: if a bar is subjected to a direct load, and hence a stress the bar will change in length. If the bar has an original length L and changes by an amount dL, the strain produce is defined as follows:
Strain is a measure of the deformation of the material and is a nondimensional Quantity i.e. it has no units. Since in practice, the extensions of materials under load are very very small, it is often convenient to measure the strain in the form of strain x 10-6 i.e. micro strain, when the symbol used becomes m Î.
Sign convention for strain: Tensile strains are positive whereas compressive strains are negative. The strain defined earlier was known as linear strain or normal strain.
Shear strain: The tangent of the angle through which two adjacent sides of an element rotates relative to their initial position is termed shear strain. In many cases the angle is very small and the angle it self is used, (in radians).
Shear strain: shear stresses acts along the surface.
This Shear strain is measured in radians & hence is non – dimensional i.e. it has no unit.So we have two types of strain i.e. normal stress & shear stresses.
Hook's Law : A material is said to be elastic if it returns to its original unloaded dimensions when load is removed.
Hook's law therefore states that Stress ( ) a strain( )

Modulus of elasticity : Within the elastic limits of materials i.e. within the limits in which Hook's law applies, it has been shown that
Stress / strain = constant. This constant is given by the symbol E and is termed as the modulus of elasticity or Young's modulus of elasticity
Thus The value of Young's modulus E is generally assumed to be the same in tension or compression and for most engineering material has high, numerical value of the order of 200 GPa
Poisson's ratio: If a bar is subjected to a longitudinal stress there will be a strain in this direction equal to s / E . There will also be a strain in all directions at right angles to s . The final shape being shown by the dotted lines.

It has been observed that for an elastic materials, the lateral strain is proportional to the longitudinal strain.
Poison's ratio (m) = - lateral strain / longitudinal strain
For most engineering materials the value of m his between 0.25 and 0.33.
Three – dimensional state of strain : Consider an element subjected to three mutually perpendicular tensile stresses sx , syand sz as shown in the figure below.

If sy and sz were not present the strain in the x direction from the basic definition of Young's modulus of Elasticity E would be equal to Îx= sx/ E
The effects of sy and sz in x direction are given by the definition of Poisson's ratio ‘ m ' to be equal as -m sy/ E and -m sz/ E
The negative sign indicating that if syand sz are positive i.e. tensile, these they tend to reduce the strain in x direction thus the total linear strain is x direction is given by

Principal strains in terms of stress: In the absence of shear stresses on the faces of the elements let us say that sx , sy , sz are in fact the principal stress. The resulting strain in the three directions would be the principal

strains.
i.e. We will have the following relation.

For Two dimensional strain: system, the stress in the third direction becomes zero i.e sz = 0 or s3 = 0
Although we will have a strain in this direction owing to stresses s1& s2 .
Hence the set of equation as described earlier reduces to
Hence a strain can exist without a stress in that direction

Hydrostatic stress : The term Hydrostatic stress is used to describe a state of tensile or compressive stress equal in all directions within or external to a body. Hydrostatic stress causes a change in volume of a material, which if expressed per unit of original volume gives a volumetric strain denoted by Îv.
So let us determine the expression for the volumetric strain.
Volumetric Strain: Consider a rectangle solid of sides x, y and z under the action of principal stresses s1 , s2 , s3 respectively.
Then Î1 , Î2 , and Î3 are the corresponding linear strains, than the dimensions of the rectangle becomes
(x + Î1 . x); (y + Î2 . y); (z + Î3 . z)
hence the
ALITER : Let a cuboid of material having initial sides of Length x, y and z. If under some load system, the sides changes in length by dx, dy, and dz then the new volume (x + dx) (y + dy) (z +dz)
New volume = xyz + yzdx + xzdy + xydz Original volume = xyz
Change in volume = yzdx +xzdy + xydz
Volumetric strain = (yzdx +xzdy + xydz) / xyz = Îx+ Îy+ Îz
Neglecting the products of epsilon's since the strains are sufficiently small.
Volumetric strains in terms of principal stresses:
As we know that

Strains on an oblique plane
(a) Linear strain
Consider a rectangular block of material OLMN as shown in the xy plane. The strains along ox and oy are Îx and Îy , and gxy is the shearing strain.
Then it is required to find an expression for Îq, i.e the linear strain in a direction inclined at q to OX, in terms of Îx ,Îy , gxy and q. Let the diagonal OM be of length 'a' then ON = a cos q and OL = a sin q , and the increase in length of those under strains are Îxacos q and Îya sin q (i.e. strain x original length) respectively. If M moves to M', then the movement of M parallel to x axis is Îxacos q + gxy sin q and the movement parallel to the y axis is Îyasin q
Thus the movement of M parallel to OM , which since the strains are small is practically coincident with MM'. and this would be the summation of portions (1) and (2) respectively and is equal to

This expression is identical in form with the equation defining the direct stress on any inclined plane q with Îx and Îy replacing sx and sy and ½ gxy replacing txy i.e. the shear stress is replaced by half the shear strain
Shear strain: To determine the shear stain in the direction OM consider the displacement of point P at the foot of the perpendicular from N to OM and the following expression can be derived as
In the above expression ½ is there so as to keep the consistency with the stress relations.
Futher -ve sign in the expression occurs so as to keep the consistency of sign convention, because OM' moves clockwise with respect to OM it is considered to be negative strain.
The other relevant expressions are the following :

Let us now define the plane strain condition
Plane Strain :
In xy plane three strain components may exist as can be seen from the following figures:

Therefore, a strain at any point in body can be characterized by two axial strains i.e Îx in x direction, Îy in y - direction and gxy the shear strain.

In the case of normal strains subscripts have been used to indicate the direction of the strain, and Îx , Îy are defined as the relative changes in length in the co-ordinate directions.

With shear strains, the single subscript notation is not practical, because such strains involves displacements and length which are not in same direction.The symbol and subscript gxy used for the shear strain referred to the x and y planes. The order of the subscript is unimportant. gxy and gyx refer to the same physical quantity. However, the sign convention is important.The shear strain gxy is considered to be positive if it represents a decrease the angle between the sides of an element of material lying parallel the positive x and y axes. Alternatively we can think of positive shear strains produced by the positive shear stresses and viceversa.

Plane strain :   An element of material subjected only to the strains as shown in Fig. 1, 2, and 3 respectively is termed as the plane strain state.

Thus, the plane strain condition is defined only by the components Îx , Îy , gxy : Îz = 0; gxz= 0; gyz= 0

It should be noted that the plane stress is not the stress system associated with plane strain. The plane strain condition is associated with three dimensional stress system and plane stress is associated with three dimensional strain system.

LECTURE 8   PRINCIPAL STRAIN

For the strains on an oblique plane we have an oblique we have two equations which are identical in form with the equation defining the direct stress on any inclined plane q .

Since the equations for stress and strains on oblique planes are identical in form, so it is evident that Mohr's stress circle construction can be used equally well to represent strain conditions using the horizontal axis for linear strains and the vertical axis for half the shear strain.

It should be noted, however that the angles given by Mohr's stress circle refer to the directions of the planes on which the stress act and not the direction of the stresses themselves.

The direction of the stresses and therefore associated strains are therefore normal (i.e. at 900) to the directions of the planes. Since angles are doubled in Mohr's stress circle construction it follows therefore that for a true similarity of working a relative rotation of axes of 2 x 900 = 1800 must be introduced. This is achieved by plotting positive sheer strains vertically downwards on the strain circle construction.

The sign convention adopted for the strains is as follows:

Linear Strains : extension - positive   compression - negative

{ Shear of strains are taken positive, when they increase the original right angle of an unstrained element. }

Shear strains : for Mohr's strains circle sheer strain gxy - is +ve referred to x - direction the convention for the shear strains are bit difficult. The first subscript in the symbol gxy usually denotes the shear strains associated with direction. e.g. in gxy– represents the shear strain in x - direction and for gyx– represents the shear strain in y - direction. If under strain the line associated with first subscript moves counter clockwise with respect to the other line, the shearing strain is said to be positive, and if it moves clockwise it is said to be negative.

N.B: The positive shear strain is always to be drown on the top of Îx .If the shear stain gxy is given]

Moh's strain circle

For the plane strain conditions can we derivate the following relations

A typical point P on the circle given the normal strain and half the sheer strain 1/2gxy associated with a particular plane. We note again that an angle subtended at the centre of Mohr's circle by an arc connecting two points on the circle is twice the physical angle in the material.

Mohr strain circle :

Since the transformation equations for plane strain are similar to those for plane stress, we can employ a similar form of pictorial representation. This is

known as Mohr's strain circle.
The main difference between Mohr's stress circle and stress circle is that a factor of half is attached to the shear strains.

Points X' and Y' represents the strains associated with x and y directions with Î and gxy /2 as co-ordiantes
Co-ordinates of X' and Y' points are located as follows :

In x – direction, the strains produced, the strains produced by sx,and - t xy are Îx and - gxy /2
where as in the Y - direction, the strains are produced by Îy and + gxy are produced by sy and + txy
These co-ordinated are consistent with our sign notation (i.e. + ve shear stresses produces produce +ve shear strain & vice versa)
on the face AB is txy+ve i.e strains are (Îy, +gxy /2) where as on the face BC, txy is negative hence the strains are (Îx, - gxy /2)

A typical point P on the circle gives the normal strains and half the shear strain, associated with a particular plane we must measure the angle from x – axis (taken as reference) as the required formulas for Îq , -1/2 gq have been derived with reference to x-axis with angle measuring in the c.c.W direction

CONSTRUCTION :
In this we would like to locate the points x' & y' instead of AB and BC as we have done in the case of Mohr's stress circle.
steps
1. Take normal or linear strains on x-axis, whereas half of shear strains are plotted on y-axis.
2. Locate the points x' and y'   3. Join x' and y' and draw the Mohr's strain circle   4. Measure the required parameter from this construction.

Note: positive shear strains are associated with planes carrying positive shear stresses and negative strains with planes carrying negative shear stresses.
ILLUSTRATIVE EXAMPLES :

Use of strain Gauges :
Although we can not measure stresses within a structural member, we can measure strains, and from them the stresses can be computed, Even so, we can only measure strains on the surface. For example, we can mark points and lines on the surface and measure changes in their spacing angles. In doing this we are of course only measuring average strains over the region concerned. Also in view of the very small changes in dimensions, it is difficult to archive accuracy in the measurements
In practice, electrical strain gage provide a more accurate and convenient method of measuring strains.
A typical strain gage is shown below.

The gage shown above can measure normal strain in the local plane of the surface in the direction of line PQ, which is parallel to the folds of paper. This strain is an average value of for the region covered by the gage, rather than a value at any particular point.
The strain gage is not sensitive to normal strain in the direction perpendicular to PQ, nor does it respond to shear strain. therefore, in order to determine the state of strain at a particular small region of the surface, we usually need more than one strain gage.
To define a general two dimensional state of strain, we need to have three pieces of information, such as Îx , Îy and gxy referred to any convenient orthogonal co-ordinates x and y in the plane of the surface. We therefore need to obtain measurements from three strain gages. These three gages must be arranged at different orientations on the surface to from a strain rossett. Typical examples have been shown, where the gages are arranged at either 450 or 600 to each other as shown below :

A group of three gages arranged in a particular fashion is called a strain rosette. Because the rosette is mounted on the surface of the body, where the material is in plane stress, therefore, the transformation equations for plane

strain to calculate the strains in various directions.
Knowing the orientation of the three gages forming a rosette, together with the in – plane normal strains they record, the state of strain at the region of the surface concerned can be found. Let us consider the general case shown in the figure below, where three strain gages numbered 1, 2, 3, where three strain gages numbered 1, 2, 3 are arranged at an angles of q1 , q2 , q3 measured c.c.w from reference direction, which we take as x – axis.
Now, although the conditions at a surface, on which there are no shear or normal stress components. Are these of plane stress rather than the plane strain, we can still use strain transformation equations to express the three measured normal strains in terms of strain components Îx , Îy , Îz and gxy referred to x and y co-ordiantes as

This is a set of three simultaneous linear algebraic equations for the three unknows Îx, Îy , gxy to solve these equation is a laborious one as far as manually is concerned, but with computer it can be readily done.Using these later on, the state of strain can be determined at any point.
Let us consider a 450 degree stain rosette consisting of three electrical – resistance strain gages arranged as shown in the figure below :

The gages A, B,C measure the normal strains Îa , Îb , Îc in the direction of lines OA, OB and OC.
Thus

Thus, substituting the relation (3) in the equation (2) we get gxy = 2Îb- (Îa + Îc) and other equation becomes Îx = Îa ; Îy= Îc
Since the gages A and C are aligned with the x and y axes, they give the strains Îx and Îy directly
Thus, Îx , Îy and gxy can easily be determined from the strain gage readings.
Knowing these strains, we can calculate the strains in any other directions by means of Mohr's circle or from the transformation equations.
The 600 Rossett: For the 600 strain rosette, using the same procedure we can obtain following relation.

LECTURE 9 STRESS - STRAIN RELATIONS
Stress – Strain Relations: The Hook's law, states that within the elastic limits the stress is proportional to the strain since for most materials it is impossible to describe the entire stress – strain curve with simple mathematical expression, in any given problem the behavior of the materials is represented by an idealized stress – strain curve, which emphasizes those aspects of the behaviors which are most important is that particular problem.
(i) Linear elastic material: A linear elastic material is one in which the strain is proportional to stress as shown below:
There are also other types of idealized models of material behavior.
(ii) Rigid Materials: It is the one which donot experience any strain regardless of the applied stress.
(iii) Perfectly plastic(non-strain hardening): A perfectly plastic i.e non-strain hardening material is shown below:
(iv) Rigid Plastic material(strain hardening): A rigid plastic material i.e strain hardening is depicted in the figure below:

(v) Elastic Perfectly Plastic material: The elastic perfectly plastic material is having the characteristics as shown below:

(vi) Elastic – Plastic material: The elastic plastic material exhibits a stress Vs strain diagram as depicted in the figure below:

Elastic Stress – strain Relations :
Previously stress – strain relations were considered for the special case of a uniaxial loading i.e. only one component of stress i.e. the axial or normal component of stress was coming into picture. In this section we shall generalize the elastic behavior, so as to arrive at the relations which connect all the six components of stress with the six components of elastic stress. Futher, we would restrict overselves to linearly elastic material.

Before writing down the relations let us introduce a term ISOTROPY
ISOTROPIC: If the response of the material is independent of the orientation of the load axis of the sample, then we say that the material is isotropic or in other words we can say that isotropy of a material in a characteristics, which gives us the information that the properties are the same in the three orthogonal directions x y z, on the other hand if the response is dependent on orientation it is known as anisotropic.
Examples of anisotropic materials, whose properties are different in different directions are
(i) Wood (ii) Fibre reinforced plastic (iii) Reinforced concrete
HOMOGENIUS: A material is homogenous if it has the same composition through our body. Hence the elastic properties are the same at every point in the body. However, the properties need not to be the same in all the direction for the material to be homogenous. Isotropic materials have the same elastic properties in all the directions. Therefore, the material must be both homogenous and isotropic in order to have the lateral strains to be same at every point in a particular component.
Generalized Hook's Law: We know that for stresses not greater than the proportional limit.

These equation expresses the relationship between stress and strain (Hook's law) for uniaxial state of stress only when the stress is not greater than the proportional limit. In order to analyze the deformational effects produced by all the stresses, we shall consider the effects of one axial stress at a time. Since we presumably are dealing with strains of the order of one percent or less. These effects can be superimposed arbitrarily. The figure below shows the general triaxial state of stress.

Let us consider a case when sx alone is acting. It will cause an increase in dimension in X-direction whereas the dimensions in y and z direction will be decreased.

Therefore the resulting strains in three directions are Similarly let us consider that normal stress sy alone is acting and the resulting strains are

Now let us consider the stress sz acting alone, thus the strains produced are

In the following analysis shear stresses were not considered. It can be shown that for an isotropic material's a shear stress will produce only its corresponding shear strain and will not influence the axial strain. Thus, we can write Hook's law for the individual shear strains and shear stresses in the following manner.
The Equations (1) through (6) are known as Generalized Hook's law and are the constitutive equations for the linear elastic isotropic materials. When these equations isotropic materials. When these equations are used as written, the strains can be completely determined from known values of the stresses. To engineers the plane stress situation is of much relevance (i.e. sz = txz = tyz = 0), Thus then the above set of equations reduces to

Hook's law is probably the most well known and widely used constitutive equations for an engineering materials." However, we can not say that all the engineering materials are linear elastic isotropic ones. Because now in the present times, the new materials are being developed every day. Many useful materials exhibit nonlinear response and are not elastic too.
Plane Stress: In many instances the stress situation is less complicated for example if we pull one long thin wire of uniform section and examine – small parallepiped where x – axis coincides with the axis of the wire

So if we take the xy plane then sx , sy , txy will be the only stress components acting on the parrallepiped. This combination of stress components is called the plane stress situation
A plane stress may be defined as a stress condition in which all components associated with a given direction (i.e the z direction in this example) are

zero

Plane strain: If we focus our attention on a body whose particles all lie in the same plane and which deforms only in this plane. This deforms only in this plane. This type of deformation is called as the plane strain, so for such a situation.

Îz= gzx = gzy = 0 and the non – zero terms would be Îx, Îy & gxy

i.e. if strain components Îx, Îy and gxy and angle q are specified, the strain components Îx', Îy' and gxy' with respect to some other axes can be determined.

ELASTIC CONSTANTS

In considering the elastic behavior of an isotropic materials under, normal, shear and hydrostatic loading, we introduce a total of four elastic constants namely E, G, K, and g .

It turns out that not all of these are independent to the others. In fact, given any two of them, the other two can be foundout . Let us define these elastic constants

(i) E = Young's Modulus of Rigidity = Stress / strain

(ii) G = Shear Modulus or Modulus of rigidity
= Shear stress / Shear strain

(iii) g = Possion's ratio = - lateral strain / longitudinal strain

(iv) K = Bulk Modulus of elasticity = Volumetric stress / Volumetric strain

Where Volumetric strain = sum of linear stress in x, y and z direction.

Volumetric stress = stress which cause the change in volume.

Let us find the relations between them .