

# Module 10

## Compression Members

# Lesson 22

## Short Axially Loaded Compression Members

## Instructional Objectives:

At the end of this lesson, the student should be able to:

- state additional assumptions regarding the strengths of concrete and steel for the design of short axially loaded columns,
- specify the values of design strengths of concrete and steel,
- derive the governing equation for the design of short and axially loaded tied columns,
- derive the governing equation for the design of short and axially loaded spiral columns,
- derive the equation to determine the pitch of helix in spiral columns,
- apply the respective equations to design the two types of columns by direct computation,
- use the charts of SP-16 to design these two types of columns subjected to axial loads as per IS code.

### 10.22.1 Introduction

Tied and helically bound are the two types of columns mentioned in sec.10.21.3 of Lesson 21. These two types of columns are taken up in this lesson when they are short and subjected to axially loads. Out of several types of plan forms, only rectangular and square cross-sections are covered in this lesson for the tied columns and circular cross-section for the helically bound columns. Axially loaded columns also need to be designed keeping the provision of resisting some moments which normally is the situation in most of the practical columns. This is ensured by checking the minimum eccentricity of loads applied on these columns as stipulated in IS 456. Moreover, the design strengths of concrete and steel are further reduced in the design of such columns. The governing equations of the two types of columns and the equation for determining the pitch of the helix in continuously tied column are derived and explained. The design can be done by employing the derived equation i.e., by direct computation or by using the charts of SP-16. Several numerical examples are solved to explain the design of the two types of columns by direct computation and using the charts of SP-16.

## 10.22.2 Further Assumptions Regarding the Strengths of Concrete and Steel

All the assumptions required for the derivation of the governing equations are given in sec.10.21.11 of Lesson 21. The stress-strain diagrams of mild steel (Fe 250) and cold worked deformed bars (Fe 415 and Fe 500) are given in Figs.1.2.3 and 4, respectively of Lesson 2. The stress block of compressive part of concrete is given in Fig.3.4.1.9 of Lesson 4, which is used in the design of beam by limit state of collapse. The maximum design strength of concrete is shown as constant at  $0.446 f_{ck}$  when the strain ranges from 0.002 to 0.0035. The maximum design stress of steel is  $0.87 f_y$ .

Sections 10.21.4 and 12 of Lesson 21 explain that all columns including the short axially loaded columns shall be designed with a minimum eccentricity (cls. 25.4 and 39.2 of IS 456). Moreover, the design strengths of concrete and steel are further reduced to  $0.4 f_{ck}$  and  $0.67 f_y$ , respectively, to take care of the minimum eccentricity of 0.05 times the lateral dimension, as stipulated in cl.39.3 of IS 456. It is noticed that there is not attempt at strain compatibility. Also the phenomenon of creep has not been directly considered.

$$e_{x\min} \geq \text{greater of } (l/500 + D/30) \text{ or } 20 \text{ mm}$$

(10.3)

$$e_{y\min} \geq \text{greater of } (l/500 + b/30) \text{ or } 20 \text{ mm}$$

The maximum values of  $l_{ex}/D$  and  $l_{ey}/b$  should not exceed 12 in a short column as per cl.25.1.2 of IS 456. For a short column, when the unsupported length  $l = l_{ex}$  (for the purpose of illustration), we can assume  $l = 12 D$  (or  $12b$  when  $b$  is considered). Thus, we can write the minimum eccentricity  $= 12D/500 + D/30 = 0.057D$ , which has been taken as  $0.05D$  or  $0.05b$  as the maximum amount of eccentricity of a short column.

It is, therefore, necessary to keep provision so that the short columns can resist the accidental moments due to the allowable minimum eccentricity by lowering the design strength of concrete by ten per cent from the value of  $0.446 f_{ck}$ , used for the design of flexural members. Thus, we have the design strength of concrete in the design of short column as  $(0.9)(0.446 f_{ck}) = 0.4014 f_{ck}$ , say  $0.40 f_{ck}$ . The reduction of the design strength of steel is explained below.

For mild steel (Fe 250), the design strength at which the strain is 0.002 is  $f_y/1.15 = 0.87 f_y$ . However, the design strengths of cold worked deformed bars (Fe 415 and Fe 500) are obtained from Fig.1.2.4 of Lesson 2 or Fig.23A of IS 456. Table A of SP-16 presents the stresses and corresponding strains of Fe 415 and Fe 500. Use of Table A of SP-16 is desirable as it avoids error while reading from figures (Fig.1.2.4 or Fig.23A, as mentioned above). From Table A of SP-16, the

corresponding design strengths are obtained by making linear interpolation. These values of design strengths for which the strain is 0.002 are as follows:

(i) Fe 415:  $\{0.9f_{yd} + 0.05f_{yd}(0.002 - 0.00192)/(0.00241 - 0.00192)\} = 0.908f_{yd} = 0.789f_y$

(ii) Fe 500:  $\{0.85f_{yd} + 0.05f_{yd}(0.002 - 0.00195)/(0.00226 - 0.00195)\} = 0.859f_{yd} = 0.746f_y$

A further reduction in each of three values is made to take care of the minimum eccentricity as explained for the design strength of concrete. Thus, the acceptable design strength of steel for the three grades after reducing 10 per cent from the above mentioned values are  $0.783f_y$ ,  $0.710f_y$  and  $0.671f_y$  for Fe 250, Fe 415 and Fe 500, respectively. Accordingly, cl. 39.3 of IS 456 stipulates  $0.67f_y$  as the design strength for all grades of steel while designing the short columns. Therefore, the assumed design strengths of concrete and steel are  $0.4f_{ck}$  and  $0.67f_y$ , respectively, for the design of short axially loaded columns.

### 10.22.3 Governing Equation for Short Axially Loaded Tied Columns

Factored concentric load applied on short tied columns is resisted by concrete of area  $A_c$  and longitudinal steel of areas  $A_{sc}$  effectively held by lateral ties at intervals (Fig.10.21.2a of Lesson 21). Assuming the design strengths of concrete and steel are  $0.4f_{ck}$  and  $0.67f_y$ , respectively, as explained in sec. 10.22.2, we can write

$$P_u = 0.4f_{ck} A_c + 0.67f_y A_{sc} \quad (10.4)$$

where  $P_u$  = factored axial load on the member,

$f_{ck}$  = characteristic compressive strength of the concrete,

$A_c$  = area of concrete,

$f_y$  = characteristic strength of the compression reinforcement, and

$A_{sc}$  = area of longitudinal reinforcement for columns.

The above equation, given in cl. 39.3 of IS 456, has two unknowns  $A_c$  and  $A_{sc}$  to be determined from one equation. The equation is recast in terms of  $A_g$ , the gross area of concrete and  $\rho$ , the percentage of compression reinforcement employing

$$(10.5) \quad A_{sc} = pA_g/100$$

$$(10.6) \quad A_c = A_g(1 - p/100)$$

Accordingly, we can write

$$(10.7) \quad P_u/A_g = 0.4f_{ck} + (p/100) (0.67f_y - 0.4f_{ck})$$

Equation 10.7 can be used for direct computation of  $A_g$  when  $P_u$ ,  $f_{ck}$  and  $f_y$  are known by assuming  $p$  ranging from 0.8 to 4 as the minimum and maximum percentages of longitudinal reinforcement. Equation 10.4 also can be employed to determine  $A_g$  and  $p$  in a similar manner by assuming  $p$ . This method has been illustrated with numerical examples and is designated as Direct Computation Method.

On the other hand, SP-16 presents design charts based on Eq.10.7. Each chart of charts 24 to 26 of SP-16 has lower and upper sections. In the lower section,  $P_u/A_g$  is plotted against the reinforcement percentage  $p(= 100A_s/A_g)$  for different grades of concrete and for a particular grade of steel. Thus, charts 24 to 26 cover the three grades of steel with a wide range of grades of concrete. When the areas of cross-section of the columns are known from the computed value of  $P_u/A_g$ , the percentage of reinforcement can be obtained directly from the lower section of the chart. The upper section of the chart is a plot of  $P_u/A_g$  versus  $P_u$  for different values of  $A_g$ . For a known value of  $P_u$ , a horizontal line can be drawn in the upper section to have several possible  $A_g$  values and the corresponding  $P_u/A_g$  values. Proceeding vertically down for any of the selected  $P_u/A_g$  value, the corresponding percentage of reinforcement can be obtained. Thus, the combined use of upper and lower sections of the chart would give several possible sizes of the member and the corresponding  $A_{sc}$  without performing any calculation. It is worth mentioning that there may be some parallax error while using the charts. However, use of chart is very helpful while deciding the sizes of columns at the preliminary design stage with several possible alternatives.

Another advantage of the chart is that, the amount of compression reinforcement obtained from the chart are always within the minimum and maximum percentages i.e., from 0.8 to 4 per cent. Hence, it is not needed to examine if the computed area of steel reinforcement is within the allowable range as is needed while using Direct Computation Method. This method is termed as SP-16 method while illustrating numerical examples.

## 10.22.4 Governing Equation of Short Axially Loaded Columns with Helical Ties

Columns with helical reinforcement take more load than that of tied columns due to additional strength of spirals in contributing to the strength of columns. Accordingly, cl. 39.4 recommends a multiplying factor of 1.05 regarding the strength of such columns. The code further recommends that the ratio of volume of helical reinforcement to the volume of core shall not be less than 0.36 ( $A_g/A_c - 1$ ) ( $f_{ck}/f_y$ ), in order to apply the additional strength factor of 1.05 (cl. 39.4.1). Accordingly, the governing equation of the spiral columns may be written as

$$P_u = 1.05(0.4 f_{ck} A_c + 0.67 f_y A_{sc}) \quad (10.8)$$

All the terms have been explained in sec.10.22.3.

Earlier observations of several investigators reveal that the effect of containing holds good in the elastic stage only and it gets lost when spirals reach the yield point. Again, spirals become fully effective after spalling off the concrete cover over the spirals due to excessive deformation. Accordingly, the above two points should be considered in the design of such columns. The first point is regarding the enhanced load carrying capacity taken into account by the multiplying factor of 1.05. The second point is maintaining specified ratio of volume of helical reinforcement to the volume of core, as specified in cl.39.4.1 and mentioned earlier.

The second point, in fact, determines the pitch  $p$  of the helical reinforcement, as explained below with reference to Fig.10.21.2b of Lesson 21.

$$\text{Volume of helical reinforcement in one loop} = \pi(D_c - \phi_{sp}) a_{sp} \quad (10.9)$$

$$\text{Volume of core} = (\pi/4) D_c^2 p \quad (10.10)$$

where  $D_c$  = diameter of the core (Fig.10.21.2b)

$\phi_{sp}$  = diameter of the spiral reinforcement (Fig.10.21.2b)

$a_{sp}$  = area of cross-section of spiral reinforcement

$p$  = pitch of spiral reinforcement (Fig.10.21.2b)

To satisfy the condition of cl.39.4.1 of IS 456, we have

$$\{\pi(D_c - \phi_{sp}) a_{sp}\} / \{(\pi/4) D_c^2 p\} \geq 0.36(A_g / A_c - 1)(f_{ck} / f_y)$$

which finally gives

$$p \leq 11.1(D_c - \phi_{sp}) a_{sp} f_y / (D_c^2 - D_c^2) f_{ck} \quad (10.11)$$

Thus, Eqs.10.8 and 11 are the governing equations to determine the diameter of column, pitch of spiral and area of longitudinal reinforcement. It is worth mentioning that the pitch  $p$  of the spiral reinforcement, if determined from Eq.10.11, automatically satisfies the stipulation of cl.39.4.1 of IS 456. However, the pitch and diameter of the spiral reinforcement should also satisfy cl. 26.5.3.2 of IS 456:2000.

## 10.22.5 Illustrative Examples

### Problem 1:

Design the reinforcement in a column of size 400 mm x 600 mm subjected to an axial load of 2000 kN under service dead load and live load. The column has an unsupported length of 4.0 m and effectively held in position and restrained against rotation in both ends. Use M 25 concrete and Fe 415 steel.

### Solution 1:

#### Step 1: To check if the column is short or slender

Given  $l = 4000$  mm,  $b = 400$  mm and  $D = 600$  mm. Table 28 of IS 456 =  $l_{ex} = l_{ey} = 0.65(l) = 2600$  mm. So, we have

$$l_{ex}/D = 2600/600 = 4.33 < 12$$

$$l_{ey}/b = 2600/400 = 6.5 < 12$$

Hence, it is a short column.

#### Step 2: Minimum eccentricity

$$e_{x \min} = \text{Greater of } (l_{ex}/500 + D/30) \text{ and } 20 \text{ mm} = 25.2 \text{ mm}$$

$$e_{y \min} = \text{Greater of } (l_{ey}/500 + b/30) \text{ and } 20 \text{ mm} = 20 \text{ mm}$$

$$0.05 D = 0.05(600) = 30 \text{ mm} > 25.2 \text{ mm} (= e_{x \min})$$



$$0.05 b = 0.05(400) = 20 \text{ mm} = 20 \text{ mm} (= e_{y \min})$$

Hence, the equation given in cl.39.3 of IS 456 (Eq.10.4) is applicable for the design here.

### Step 3: Area of steel

Fro Eq.10.4, we have

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc} \quad \dots (10.4)$$

$$3000(10^3) = 0.4(25)\{(400)(600) - A_{sc}\} + 0.67(415) A_{sc}$$

which gives,

$$A_{sc} = 2238.39 \text{ mm}^2$$

Provide 6-20 mm diameter and 2-16 mm diameter rods giving  $2287 \text{ mm}^2$  ( $> 2238.39 \text{ mm}^2$ ) and  $\rho = 0.953$  per cent, which is more than minimum percentage of 0.8 and less than maximum percentage of 4.0. Hence, o.k.

### Step 4: Lateral ties

The diameter of transverse reinforcement (lateral ties) is determined from cl.26.5.3.2 C-2 of IS 456 as not less than (i)  $\phi/4$  and (ii) 6 mm. Here,  $\phi$  = largest bar diameter used as longitudinal reinforcement = 20 mm. So, the diameter of bars used as lateral ties = 6 mm.

The pitch of lateral ties, as per cl.26.5.3.2 C-1 of IS 456, should be not more than the least of

- (i) the least lateral dimension of the column = 400 mm
- (ii) sixteen times the smallest diameter of longitudinal reinforcement bar to be tied =  $16(16) = 256 \text{ mm}$
- (iii) 300 mm

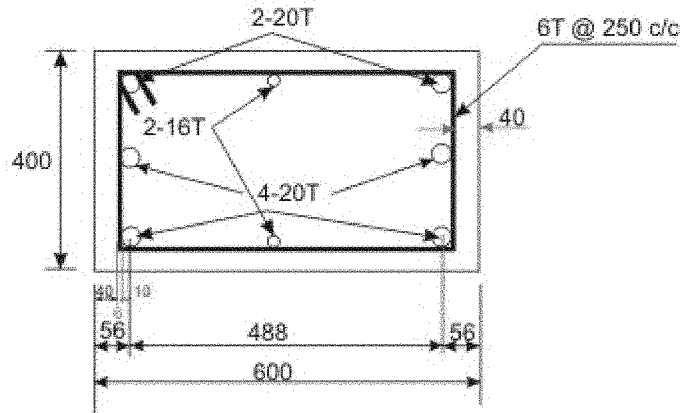


Fig. 10.22.1: Problem 1

Let us use  $p$  = pitch of lateral ties = 250 mm. The arrangement of longitudinal and transverse reinforcement of the column is shown in Fig. 10.22.1.

### Problem 2:

Design the column of Problem 1 employing the chart of SP-16.

### Solution 2:

Steps 1 and 2 are the same as those of Problem 1.

### Step 3: Area of steel

$$P_u/A_g = 3000(10^3)/(600)(400) = 12.5 \text{ N/mm}^2$$

From the lower section of Chart 25 of SP-16, we get  $p = 0.95\%$  when  $P_u/A_g = 12.5 \text{ N/mm}^2$  and concrete grade is M 25. This gives  $A_{sc} = 0.95(400)(600)/100 = 2288 \text{ mm}^2$ . The results of both the problems are in good agreement. Marginally higher value of  $A_{sc}$  while using the chart is due to parallax error while reading the value from the chart. Here also, 6-20 mm diameter bars + 2-16 mm diameter bars ( $A_{sc}$  provided =  $2287 \text{ mm}^2$ ) is o.k., though it is  $1 \text{ mm}^2$  less.

Step 4 is the same as that of Problem 1. Figure 10.22.1, thus, is also the figure showing the reinforcing bars (longitudinal and transverse reinforcement) of this problem (same column as that of Problem 1).

### Problem 3:

Design a circular column of 400 mm diameter with helical reinforcement subjected to an axial load of 1500 kN under service load and live load. The column has an unsupported length of 3 m effectively held in position at both ends but not restrained against rotation. Use M 25 concrete and Fe 415 steel.

### Solution 3:

#### Step 1: To check the slenderness ratio

Given data are: unsupported length  $l = 3000$  mm,  $D = 400$  mm. Table 28 of Annex E of IS 456 gives effective length  $l_e = l = 3000$  mm. Therefore,  $l_e/D = 7.5 < 12$  confirms that it is a short column.

#### Step 2: Minimum eccentricity

$$e_{min} = \text{Greater of } (l/500 + D/30) \text{ or } 20 \text{ mm} = 20 \text{ mm}$$

$$0.05 D = 0.05(400) = 20 \text{ mm}$$

As per cl.39.3 of IS 456,  $e_{min}$  should not exceed  $0.05D$  to employ the equation given in that clause for the design. Here, both the eccentricities are the same. So, we can use the equation given in that clause of IS 456 i.e., Eq.10.8 for the design.

#### Step 3: Area of steel

From Eq.10.8, we have

$$P_u = 1.05(0.4 f_{ck} A_c + 0.67 f_y A_{sc}) \quad \dots (10.8)$$

$$A_c = A_g - A_{sc} = 125714.29 - A_{sc}$$

Substituting the values of  $P_u$ ,  $f_{ck}$ ,  $A_g$  and  $f_y$  in Eq.10.8,

$$1.5(1500)(10^3) = 1.05\{0.4(25)(125714.29 - A_{sc}) + 0.67(415) A_{sc}\}$$

we get the value of  $A_{sc} = 3304.29 \text{ mm}^2$ . Provide 11 nos. of 20 mm diameter bars ( $= 3455 \text{ mm}^2$ ) as longitudinal reinforcement giving  $p = 2.75\%$ . This  $p$  is between 0.8 (minimum) and 4 (maximum) per cents. Hence o.k.

#### Step 4: Lateral ties

It has been mentioned in sec.10.22.4 that the pitch  $p$  of the helix determined from Eq.10.11 automatically takes care of the cl.39.4.1 of IS 456. Therefore, the pitch is calculated from Eq.10.11 selecting the diameter of helical reinforcement from cl.26.5.3.2 d-2 of IS 456. However, automatic satisfaction of cl.39.4.1 of IS 456 is also checked here for confirmation.

Diameter of helical reinforcement (cl.26.5.3.2 d-2) shall be not less than greater of (i) one-fourth of the diameter of largest longitudinal bar, and (ii) 6 mm.

Therefore, with 20 mm diameter bars as longitudinal reinforcement, the diameter of helical reinforcement = 6 mm.

From Eq.10.11, we have

$$\text{Pitch of helix } p \leq 11.1(D_c - \phi_{sp}) a_{sp} f_y / (D^2 - D_c^2) f_{ck} \quad \dots (10.11)$$

where  $D_c = 400 - 40 - 40 = 320$  mm,  $\phi_{sp} = 6$  mm,  $a_{sp} = 28$  mm<sup>2</sup>,  $f_y = 415$  N/mm<sup>2</sup>,  $D = 400$  mm and  $f_{ck} = 25$  N/mm<sup>2</sup>.

$$\text{So, } p \leq 11.1(320 - 6) (28) (415) / (400^2 - 320^2) (25) \leq 28.125 \text{ mm}$$

As per cl.26.5.3.2 d-1, the maximum pitch is the lesser of 75 mm and  $320/6 = 53.34$  mm and the minimum pitch is lesser of 25 mm and  $3(6) = 18$  mm. We adopt pitch = 25 mm which is within the range of 18 mm and 53.34 mm. So, provide 6 mm bars @ 25 mm pitch forming the helix.

#### Checking of cl. 39.4.1 of IS 456

The values of helical reinforcement and core in one loop are obtained from Eqs.10.8 and 9, respectively. Substituting the values of  $D_c$ ,  $\phi_{sp}$ ,  $a_{sp}$  and pitch  $p$  in the above two equations, we have

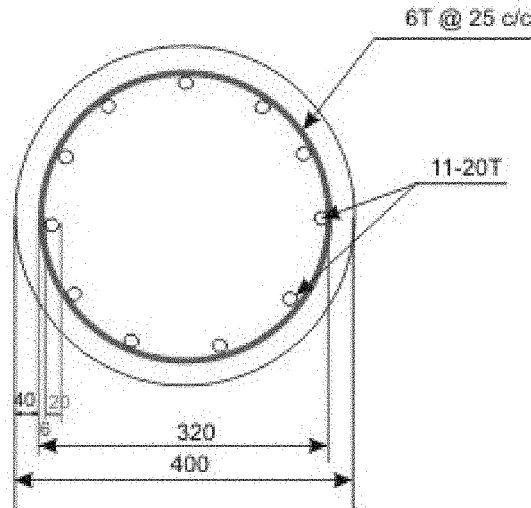
$$\text{Volume of helical reinforcement in one loop} = 27632 \text{ mm}^3 \text{ and}$$

$$\text{Volume of core in one loop} = 2011428.571 \text{ mm}^3.$$

$$\text{Their ratio} = 27632 / 2011428.571 = 0.0137375$$

$$0.36(A_g/A_c - 1) (f_{ck}/f_y) = 0.012198795$$

It is, thus, seen that the above ratio (0.0137375) is not less than  $0.36(A_g/A_c - 1) (f_{ck}/f_y)$ .



**Fig. 10.22.2: Problem 3**

Hence, the circular column of diameter 400 mm has eleven longitudinal bars of 20 mm diameter and 6 mm diameter helix with pitch  $p = 25$  mm. The reinforcing bars are shown in Fig.10.22.2.

### 10.22.6 Practice Questions and Problems with Answers

**Q.1:** State and explain the values of design strengths of concrete and steel to be considered in the design of axially loaded short columns.

**A.1:** See sec. 10.22.2.

**Q.2:** Derive the governing equation for determining the dimensions of the column and areas of longitudinal bars of an axially loaded short tied column.

**A.2:** See sec. 10.22.2.

**Q.3:** Derive the governing equation for determining the diameter and areas of longitudinal bars of an axially loaded circular spiral short column.

**A.3:** First and second paragraph of sec. 10.22.4.

**Q.4:** Derive the expression of determining the pitch of helix in a short axially loaded spiral column which satisfies the requirement of IS 456.

**A.4:** See third paragraph onwards up to the end of sec. 10.22.4.

**Q.5:** Design a short rectangular tied column of  $b = 300$  mm having the maximum amount of longitudinal reinforcement employing the equation given in

cl.39.3 of IS 456, to carry an axial load of 1200 kN under service dead load and live load using M 25 and Fe 415. The column is effectively held in position at both ends and restrained against rotation at one end. Determine the unsupported length of the column.

**A.5:**

**Step 1: Dimension  $D$  and area of steel  $A_{sc}$**

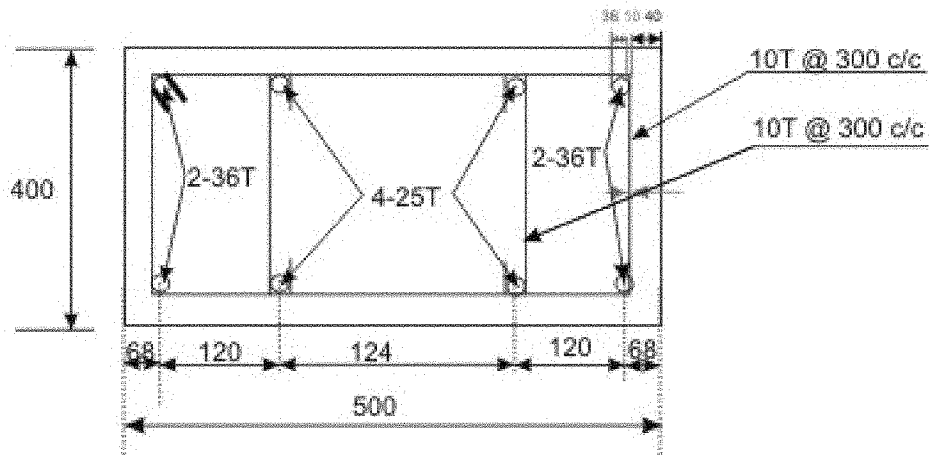
Substituting the values of  $P_u = 1.5(1200) = 1800$  kN and  $A_{sc} = 0.04(300)D$  in Eq.10.4, we have

$$1800(10^3) = 0.4(25)(300D)(1 - 0.04) + 0.67(415)(0.04)(300D)$$

we get  $D = 496.60$  mm. Use 300 mm x 500 mm column.

$A_{sc} = 0.04(300)(500) = 6000 \text{ mm}^2$ , provide 4-36 mm diameter + 4-25 mm diameter bars to give  $4071 + 1963 = 6034 \text{ mm}^2 > 6000 \text{ mm}^2$ .

**Step 2: Lateral ties**



**Fig. 10.22.3: Q.5**

Diameter of lateral ties shall not be less than the larger of (i)  $36/4 = 9$  mm and (ii) 6 mm. Use 10 mm diameter bars as lateral ties.

Pitch of the lateral ties  $p$  shall not be more than the least of (i) 300 mm, (ii)  $16(25) = 400$  mm and (iii) 300 mm.

So, provide 10 mm diameter bars @ 300 mm c/c. The reinforcement bars are shown in Fig.10.22.3.

The centre to centre distance between two corner longitudinal bars along 500 mm direction is  $500 - 2(4) + 10 + 18 = 364$  mm which is less than 48 (diameter of lateral tie). Hence, the arrangement is satisfying Fig.9 of cl. 26.5.3.2 b-2 of IS 456.

### Step 3: Unsupported length

As per the stipulation in cl. 25.1.2 of IS 456, the column shall be considered as short if  $l_{ex} = 12(D) = 6000$  mm and  $l_{ey} = 12(300) = 3600$  mm. For the type of support conditions given in the problem, Table 28 of IS 456 gives unsupported length is the least of (i)  $l = l_{ex}/0.8 = 6000/0.8 = 7500$  mm and (ii)  $l_{ey}/0.8 = 3600/0.8 = 4500$  mm. Hence, the unsupported length of the column is 4.5 m if the minimum eccentricity clause (cl. 39.3) is satisfied, which is checked in the next step.

### Step 4: Check for minimum eccentricity

According to cl. 39.4 of IS 456, the minimum eccentricity of  $0.05b$  or  $0.05D$  shall not exceed as given in cl. 25.4 of IS 456. Thus, we have

(i)  $0.05(500) = l/500 + 500/30$  giving  $l = 4165$  mm

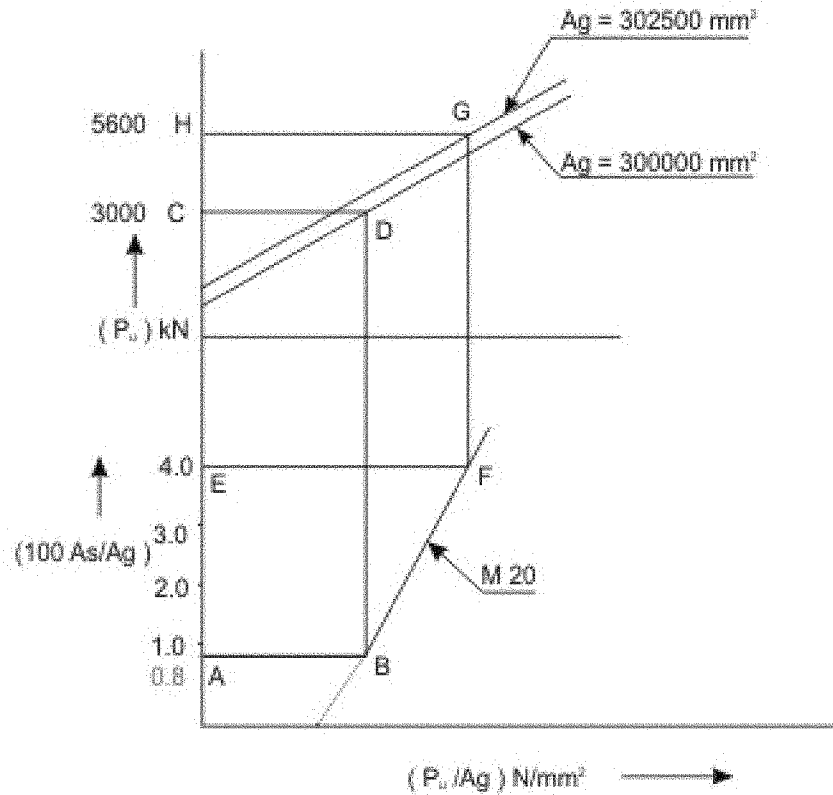
(ii)  $0.05(300) = l/500 + 300/10$  giving  $l = 2500$  mm

Therefore, the column shall have the unsupported length of 2.5 m.

**Q.6:** (a) Suggest five alternative dimensions of square short column with the minimum longitudinal reinforcement to carry a total factored axial load of 3000 kN using concrete of grades 20, 25, 30, 35 and 40 and Fe 415. Determine the respective maximum unsupported length of the column if it is effectively held in position at both ends but not restrained against rotation. Compare the given factored load of the column with that obtained by direct computation for all five alternative columns.

(b) For each of the five alternative sets of dimensions obtained in (a), determine the maximum factored axial load if the column is having maximum longitudinal reinforcement (i) employing SP-16 and (ii) by direct computation.

**A.6:**



**Fig. 10.22.4: Q.6, Chart 25 of SP-16**  
(not to scale)

**Solution of Part (a):**

**Step 1: Determination of  $A_g$  and column dimensions  $b (= D)$**

Chart 25 of SP-16 gives all the dimensions of five cases. The two input data are  $P_u = 3000 \text{ kN}$  and  $100 A_s / A_g = 0.8$ . In the lower section of Chart 25, one horizontal line AB is drawn starting from A where  $p = 0.8$  (Fig.10.22.4) to meet the lines for M 20, 25, 30, 35 and 40 respectively. In Fig.10.22.4, B is the meeting point for M 20 concrete. Separate vertical lines are drawn from these points of intersection to meet another horizontal line CD from the point C where  $P_u = 3000 \text{ kN}$  in the upper section of the figure. The point D is the intersecting point. D happens to be on line when  $A_g = 3000 \text{ cm}^2$ . Otherwise, it may be in between two liens with different values of  $A_g$ . For M 20,  $A_g = 3000 \text{ cm}^2$ . However, in case the point is in between two lines with different values of  $A_g$ , the particular  $A_g$  has to be computed by linear interpolation. Thus, all five values of  $A_g$  are obtained.

The dimension  $b = D = \sqrt{300000} = 550 \text{ mm}$ . Other four values are obtained similarly. Table 10.1 presents the values of  $A_g$  and  $D$  along with other results as explained below.



## Step 2: Unsupported length of each column

The unsupported length  $l$  is determined from two considerations:

(i) Clause 25.1.2 of IS 456 mentions that the maximum effective length  $l_{ex}$  is 12 times  $b$  or  $D$  (as  $b = D$  here for a square column). The unsupported length is related to the effective length depending on the type of support. In this problem Table 28 of IS 456 stipulates  $l = l_{ex}$ . Therefore, maximum value of  $l = 12 D$ .

(ii) The minimum eccentricity of cl. 39.3 should be more than the same as given in cl. 25.4. Assuming them to be equal, we get  $l/500 + D/30 = D/20$ , which gives  $l = 8.33D$ . For the column using M 20 and Fe 415, the unsupported length  $= 8.33(550) = 4581$  mm. All unsupported lengths are presented in Table 10.1 using the equation

$$l = 8.33 D \quad (1)$$

## Step 3: Area of longitudinal steel

Step 1 shows that the area provided for the first case is  $550 \text{ mm} \times 550 \text{ mm} = 302500 \text{ mm}^2$ , slightly higher than the required area of  $300000 \text{ mm}^2$  for the practical aspects of construction. However, the minimum percentage of the longitudinal steel is to be calculated as 0.8 per cent of area required and not area provided (vide cl. 26.5.3.1 b of IS 456). Hence, for this case  $A_{sc} = 0.8(300000)/100 = 2400 \text{ mm}^2$ . Provide 4-25 mm diameter + 4-12 mm diameter bars (area  $= 1963 + 452 = 2415 \text{ mm}^2$ ). Table 10.1 presents this and other areas of longitudinal steel obtained in a similar manner.

## Step 4: Factored load by direct computation

Equation 10.4 is employed to calculate the factored load by determining  $A_c$  from  $A_g$  and  $A_{st}$ . With a view to comparing the factored loads, we will use the values of  $A_g$  as obtained from the chart and not the  $A_g$  actually provided. From Eq.10.4, we have

$$P_u \text{ from direct computation} = 0.4(f_{ck})(0.992 A_g) + 0.67(f_y)(0.008)A_g$$

$$\text{or} \quad P_u = A_g(0.3968 f_{ck} + 0.00536 f_y) \quad (2)$$

For the first case when  $A_g = 300000 \text{ mm}^2$ ,  $f_{ck} = 20 \text{ N/mm}^2$ , and  $f_y = 415 \text{ N/mm}^2$ , Eq.(2) gives  $P_u = 3048.12 \text{ kN}$ . This value and other values of factored loads obtained from the direct computation are presented in Table 10.1.

Table 10.1 Results of Q.6a (Minimum Longitudinal Steel), given factored  $P_u = 3000$  kN

Concrete grade	Gross area of concrete ( $A_g$ )		$b = D$ (cm)	Area of steel ( $A_{sc}$ )			$P_u$ by direct computation	$l$ (m)
	Required ( $\text{cm}^2$ )	Provided ( $\text{cm}^2$ )		Required ( $\text{cm}^2$ )	Provided ( $\text{cm}^2$ )	Bars		
M 20	3000	3025	55	24	24.15	4-25 + 4-12	3048.12	4.581
M 25	2500	2500	50	20	20.60	4-20 + 4-16	3036.10	4.165
M 30	2200	2209	47	17.60	17.85	2-25 + 4-16	3108.25	3.915
M 35	1800	1806	42.5	14.40	14.57	2-28 + 2-12	2900.23	3.540
M 40	1600	1600	40	12.80	13.06	2-20 + 6-12	2895.42	3.332

### Solution of Part (b):

#### Step 1: Determination of $P_u$

Due to the known dimensions of the column section, the  $A_g$  is now known. With known  $A_g$  and reinforcement percentage  $100A_s/A_g$  as 4 per cent, the factored  $P_u$  shall be determined. For the first case, when  $b = D = 550$  mm,  $A_g = 302500 \text{ mm}^2$ . In Chart 25, we draw a horizontal line EF from E, where  $100A_s/A_g = 4$  in the lower section of the chart (see Fig.10.22.4) to meet the M 20 line at F. Proceeding vertically upward, the line FG intersects the line  $A_g = 302500$  at G. A horizontal line towards left from G, say GH, meets the load axis at H where  $P_u = 5600$  kN. Similarly,  $P_u$  for other sets are determined and these are presented in Table 10.2, except for the last case when M 40 is used, as this chart has ended at  $p = 3.8$  per cent.

#### Step 2: Area of longitudinal steel

The maximum area of steel, 4 per cent of gross area of column =  $0.04(550)(550) = 12100 \text{ mm}^2$ . Provide 12-36 mm diameter bars to have the actual area of steel =  $12214 \text{ mm}^2 > 12100 \text{ mm}^2$ , as presented in Table 10.2.

#### Step 3: Factored $P_u$ from direct computation

From Eq.10.4, as in Step 4 of the solution of Part (a) of this question, we have

$$(3) \quad P_u = 0.4 f_{ck} (A_g - A_{sc}) + 0.67 f_y A_{sc}$$

Substituting the values of  $A_g$  and  $A_{sc}$  actually provided, we get the maximum  $P_u$  of the same column when the longitudinal steel is the maximum. For the first case when  $A_g = 302500 \text{ mm}^2$ ,  $A_{sc} = 12214 \text{ mm}^2$ ,  $f_{ck} = 20 \text{ N/mm}^2$  and  $f_y = 415 \text{ N/mm}^2$ , we get  $P_u = 5718.4 \text{ kN}$ . This value along with other four values are presented in Table 10.2.

### Remarks:

Tables 10.1 and 10.2 reveal that two sets of results obtained from charts of SP-16 and by direct computation methods are in good agreement. However, values obtained from the chart are marginally different from those obtained by direct computation both on the higher and lower sides. These differences are mainly due to personal error (parallax error) while reading the values with eye estimation from the chart.

Table 10.2 Results of Q.6(b) (Maximum Longitudinal Steel) given the respective  $A_g$

Concrete grade	$b = D$ (cm)	Gross concrete area ( $A_g$ ) ( $\text{cm}^2$ )	Area of steel ( $A_{sc}$ )			$P_u =$ Factored load	
			Required ( $\text{cm}^2$ )	Provided ( $\text{cm}^2$ )	Bars (No.)	SP-chart (kN)	Direct Computation (kN)
M 20	55	3025	121	122.14	12-36	5600	5718.4
M 25	50	2500	100	101.06	8-36 + 4-25	5200	5208.9
M 30	47	2209	88.36	88.97	8-32 + 4-28	5000	5017.8
M 35	42.5	1806.25	72.25	73.69	12-28	4500	4474.5
M 40	40	1600	64	64.46	8-28 + 4-32	Not available	4249.2

**Q.7:** Design a short, helically reinforced circular column with minimum amount of longitudinal steel to carry a total factored axial load of 3000 kN with the same support condition as that of Q.6, using M 25 and Fe 415. Determine its unsupported length. Compare the results of the dimension and area of longitudinal steel with those of Q.6(a) when M 25 and Fe 415 are used.

**A.7:**

**Step 1: Diameter of helically reinforced circular column**

As per cl. 39.4 of IS 456, applicable for short spiral column, we get from Eq.10.8

$$P_u = 1.05(0.4 f_{ck} A_c + 0.67 f_y A_{sc}) \quad \dots (10.8)$$

Given data are:  $P_u = 3000 \text{ kN}$ ,  $A_c = \pi/4 (D^2)(0.992)$ ,  $A_{sc} = 0.008(\pi/4) D^2$ ,  $f_{ck} = 25 \text{ N/mm}^2$  and  $f_y = 415 \text{ N/mm}^2$ . So, we have

$$3000(10^3) = 1.05(12.1444)(\pi/4)D^2$$

giving  $D = 547.2 \text{ mm}$  and  $A_g = 235264.2252 \text{ mm}^2$ . Provide diameter of 550 mm.

### Step 2: Area of longitudinal steel

Providing 550 mm diameter, the required  $A_g$  has been exceeded. Clause 26.5.3.1b stipulates that the minimum amount of longitudinal bar shall be determined on the basis of area required and not area provided for any column. Accordingly, the area of longitudinal steel =  $0.008 A_g = 0.008(235264.2252) = 1882.12 \text{ mm}^2$ . Provide 6-20 mm diameter bars (area =  $1885 \text{ mm}^2$ ) as longitudinal steel, satisfying the minimum number of six bar (cl. 26.5.3.1c of IS 456).

### Step 3: Helical reinforcement

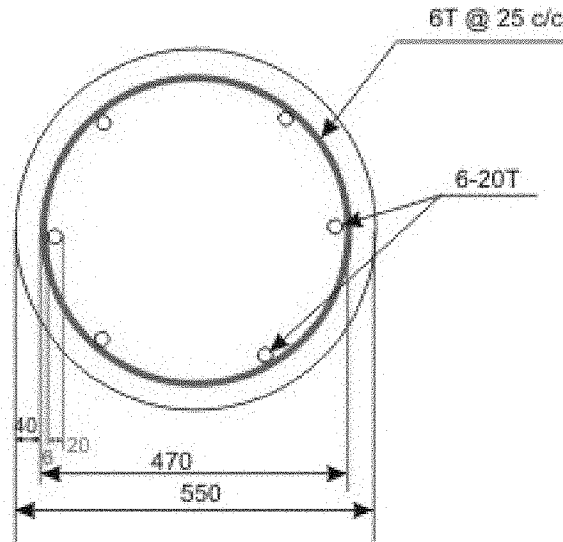


Fig. 10.22.5: Q.7

Minimum diameter of helical reinforcement is greater of (i)  $20/4$  or (ii) 6 mm. So, provide 6 mm diameter bars for the helical reinforcement (cl. 26.5.3.2d-2 of IS 456). The pitch of the helix  $p$  is determined from Eq.10.11 as follows:

$$p \leq 11.1(D_c - \phi_{sp}) a_{sp} f_y / (D^2 - D_c^2) f_{ck} \quad \dots (10.11)$$

Using  $D_c = 550 - 40 - 40 = 470$  mm,  $\phi_{sp} = 6$  mm,  $a_{sp} = 28$  mm<sup>2</sup>,  $D = 550$  mm,  $f_{ck} = 25$  N/mm<sup>2</sup> and  $f_y = 415$  N/mm<sup>2</sup>, we get

$$\rho \leq 11.1(470 - 6)(28)(415)/(550^2 - 470^2)(25) \leq 29.34 \text{ mm}$$

Provide 6 mm diameter bar @ 25 mm c/c as helix. The reinforcement bars are shown in Fig. 10.22.5. Though use of Eq.10.11 automatically checks the stipulation of cl. 39.4.1 of IS 456, the same is checked as a ready reference in Step 4 below.

#### Step 4: Checking of cl. 39.4.1 of IS 456

$$\text{Volume of helix in one loop} = \pi (D_c - \phi_{sp}) a_{sp} \quad \dots (10.9)$$

$$\text{Volume of core} = (\pi/4) D_c^2 (\rho) \quad \dots (10.10)$$

$$\begin{aligned} \text{The ratio of Eq.10.9 and Eq.10.10} &= 4(D_c - \phi_{sp}) a_{sp}/D_c^2 \rho \\ &= 4(470 - 6)(28)/(470)(470)(25) = 0.009410230874 \end{aligned}$$

This ratio should not be less than  $0.36(A_g/A_c - 1)(f_{ck}/f_y)$

$$= 0.36\{(D^2/D_c^2) - 1\} (f_{ck}/f_y) = 0.008011039177$$

Hence, the stipulation of cl. 39.4.1 is satisfied.

#### Step 5: Unsupported length

The unsupported length shall be the minimum of the two obtained from (i) short column requirement given in cl. 25.1.2 of IS 456 and (ii) minimum eccentricity requirement given in cls. 25.4 and 39.3 of IS 456. The two values are calculated separately:

$$(i) \quad l = l_e = 12D = 12(550) = 6600 \text{ mm}$$

$$(ii) \quad l/500 + D/30 = 0.05 D \text{ gives } l = 4583.3 \text{ mm}$$

So, the unsupported length of this column = 4.58 m.

#### Step 6: Comparison of results

Table 10.3 presents the results of required and actual gross areas of concrete and area of steel bars, dimensions of column and number and diameter of longitudinal reinforcement of the helically reinforced circular and the square

columns of Q.6(a) when M 20 and Fe 415 are used for the purpose of comparison.

Table 10.3 Comparison of results of circular and square columns with minimum longitudinal steel ( $P_u = 3000$  kN, M 25, Fe 415)

Column shape and Q.No.	Gross concrete area			Area of steel		
	Required (cm <sup>2</sup> )	Provided (cm <sup>2</sup> )	Dimension $D$ (cm)	Required (cm <sup>2</sup> )	Provided (cm <sup>2</sup> )	Bar dia. and No. (mm, No.)
Circular (Q.7)	2352.64	2376.78	55	18.82	18.85	6-20
Square (Q.6(a))	2500	2500	50	20	20.6	4-20 + 4-16

## 10.22.7 References

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16. Design Aids for Reinforced Concrete to IS: 456 – 1978, BIS, New Delhi.

### 10.22.8 Test 22 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions carrying equal marks.

**TQ.1:** Derive the expression of determining the pitch of helix in a short axially loaded spiral column which satisfies the requirement of IS 456. (20 marks)

**A.TQ.1:** See third paragraph onwards up to the end of sec. 10.22.4.

**TQ.2:** Design a square, short tied column of  $b = D = 500$  mm to carry a total factored load of 4000 kN using M 20 and Fe 415. Draw the reinforcement diagram. (30 marks)

**A.TQ.2:**

#### Step 1: Area of longitudinal steel

Assuming  $p$  as the percentage of longitudinal steel, we have  $A_c = (500)(500)(1 - 0.01p)$ ,  $A_{sc} = (500)(500)(0.01p)$ ,  $f_{ck} = 20$  N/mm<sup>2</sup> and  $f_y = 415$  N/mm<sup>2</sup>. Using these values in Eq.10.4

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{st} \quad \dots (10.4)$$

$$\text{or} \quad 4000000 = 0.4(20)(250000)(1 - 0.01p) + 0.67(415)(250000)(0.01p)$$

we get  $p = 2.9624$ , which gives  $A_{sc} = (2.9624)(500)(500)/100 = 7406$  mm<sup>2</sup>. Use 4-36 + 4-25 + 4-22 mm diameter bars  $(4071 + 1963 + 1520) = 7554$  mm<sup>2</sup> > 7406 mm<sup>2</sup> as longitudinal reinforcement.

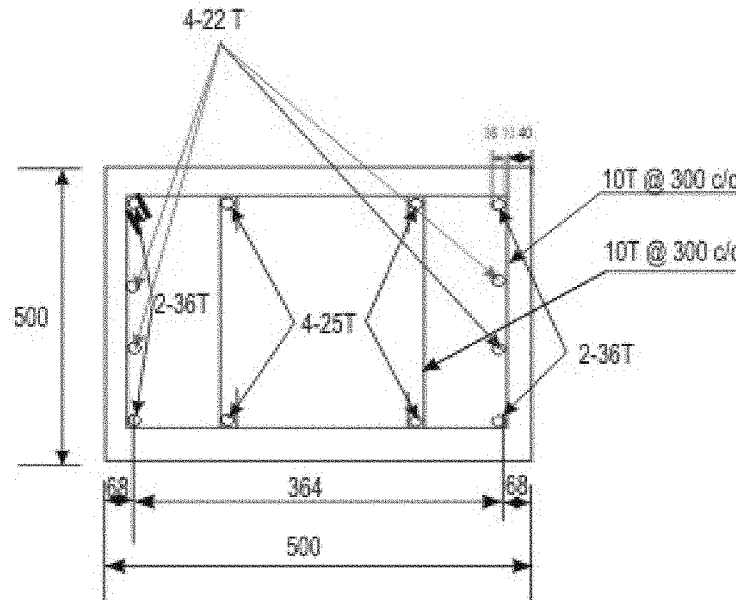


Fig. 10.22.6: TQ.2

### Step 2: Lateral ties

Diameter of tie is the greater of (i)  $36/4$  and (ii) 6 mm. Provide 10 mm diameter lateral ties.

The pitch of the lateral ties is the least of (i) 500 mm, (ii)  $16(22) = 352$  mm, and (iii) 300 mm. Provide 10 mm diameter @ 300 mm c/c. The reinforcement bars are shown in Fig.10.22.6. It is evident that the centre to centre distance between two corner bars = 364 mm is less than 48 times the diameter of lateral ties = 480 mm (Fig.9 of cl. 26.5.3.2b-2 of IS 456).

### 10.22.9 Summary of this Lesson

This lesson illustrates the additional assumptions made regarding the strengths of concrete and steel for the design of short tied and helically reinforced columns subjected to axial loads as per IS 456. The governing equations for determining the areas of cross sections of concrete and longitudinal steel are derived and explained. The equation for determining the pitch of the helix for circular columns is derived. Several numerical examples are solved to illustrate the applications of the derived equations and use of charts of SP-16 for the design of both tied and helically reinforced columns. The results of the same problem by direct computation are compared with those obtained by employing the charts of SP-16 are compared. The differences of results, if any, are discussed. Understanding the illustrative examples and solving the practice and test problems will explain the applications of the equation and use of charts of SP-16 for designing these two types of columns.