

IIT-JEE – 2007 Paper – 1 Solutions

Physics

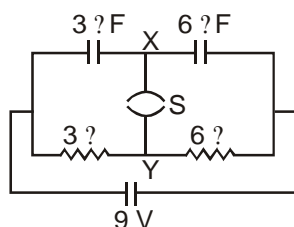
Part – I

Section – I

Straight Objective Type

This section contains 9 multiple choice questions numbered 1 to 9. Each question has 4 choices (A), (B), (C) and (D), out of which only one is correct.

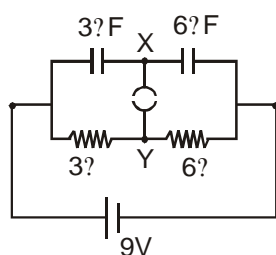
1. A circuit is connected as shown in the figure with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X is



- (A) 0
(B) 54 C
(C) 27 C
(D) 81 C

Sol. (C)

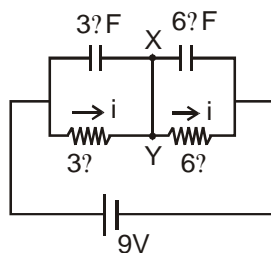
When 'S' is open:



After the capacitors are fully charged, current flows through resistors only.

Total charge on each capacitor is $q = 18 \text{ C}$ $\frac{q}{3} = \frac{q}{6} = 9 \text{ C}$

When 'S' is closed:



$$i = \frac{9}{3 + 6} = 1 \text{ A}$$

Potential drop across the 3Ω resistor $= i \times 3 = 3 \text{ V}$

$\therefore q = 3 \times 3 = 9 \text{ C}$

Potential drop across 6Ω resistor $= 6 \times 1 \text{ V} = 6 \text{ V}$

$\therefore q = 6 \times 6 = 36 \text{ C}$

\therefore Net charge flowing from Y to X

$= 36 + 18 = 54 \text{ C}$

2. In the options given below, let E denote the rest mass energy of a nucleus and n a neutron. The correct option is

(A) $E(^{236}_{92}\text{U}) + E(^{137}_{53}\text{I}) + E(^{97}_{39}\text{Y}) = 2E(n)$

(B) $E(^{236}_{92}\text{U}) + E(^{137}_{53}\text{I}) + E(^{97}_{39}\text{Y}) = 2E(n)$

(C) $E(^{236}_{92}\text{U}) + E(^{140}_{56}\text{Ba}) + E(^{94}_{36}\text{Kr}) = 2E(n)$

(D) $E(^{236}_{92}\text{U}) + E(^{140}_{56}\text{Ba}) + E(^{94}_{36}\text{Kr}) = 2E(n)$

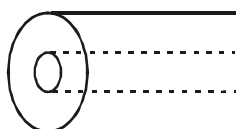
Sol. (A)

$E_{\text{parent nuclei}} = E_{\text{constituents}}$

3. A long, hollow conducting cylinder is kept coaxially inside another long, hollow conducting cylinder of larger radius. Both the cylinders are initially electrically neutral.
- (A) A potential difference appears between the two cylinders when a charge density is given to the inner cylinder
- (B) A potential difference appears between the two cylinders when a charge density is given to the outer cylinder
- (C) No potential difference appears between the two cylinders when a uniform line charge is kept along the axis of the cylinders
- (D) No potential difference appears between the two cylinders when same charge density is given to both the cylinders

Sol. (A)

When an inner cylinder is charged, both negative and positive charges are induced on the outer cylinder.



Using a gaussian enclosing only the inner surface, a radially symmetric electric field exists. Hence a non-zero potential difference exists.

When outer cylinder is charged, no charges are induced on inner cylinder and hence no electric field exists in between. Potential at all places inside the outer cylinder is constant and hence no potential difference.

Since in (C) and (D) a net charge (line charge) exists inside a gaussian surface as chosen earlier, electric field and hence potential difference exists.

4. The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer) is
- (A) 802 nm
 (B) 823 nm
 (C) 1882 nm
 (D) 1648 nm

Sol. (B)

$$\frac{hc}{\lambda} = \frac{13.6 \text{ (eV)}}{n^2}$$

$$\lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{13.6 \times 1.6 \times 10^{-19}} n^2 = \frac{100 \times 6.6 \times 3}{13.6 \times 1.6} n^2 \text{ (nm)}$$

$$\lambda = 91 n^2$$

$$\lambda (n=1) = 91 \text{ nm}, \lambda (n=2) = 364 \text{ nm}$$

$$\lambda (n=3) = 819 \text{ nm}$$

Note that the minimum λ of IR lies close to end of visible range. Hence (C) and (D) cannot represent the smallest λ of IR.

Hence (B)

5. In an experiment to determine the focal length (f) of a concave mirror by the u-v method, a student places the object pin A on the principal axis at a distance x from the pole P. The student looks at the pin and its inverted image from a distance keeping his/her eye in line with PA. When the student shifts his/her eye towards left, the image appears to the right of the object pin. Then.
- (A) $x < f$
 (B) $f < x < 2f$
 (C) $x = 2f$
 (D) $x > 2f$

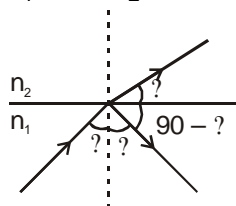
Sol. (B)

From the problem statement it is clear that the image is formed “behind” the object. This means $|v| > |u|$. Thus, $f < x < 2f$.

6. A ray of light traveling in water is incident on its surface open to air. The angle of incidence is θ , which is less than the critical angle. Then there will be
- (A) only a reflected ray and no refracted ray
 (B) only a refraction ray and no reflected ray
 (C) a reflected ray and a refracted ray and the angle between them would be less than $180^\circ - 2\theta$
 (D) a reflected ray and a refracted ray and the angle between them would be greater than $180^\circ - 2\theta$

Sol. (C)

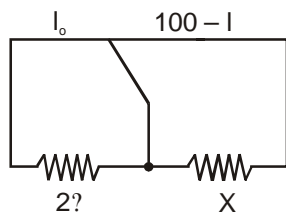
$$n_1 \sin \theta = n_2 \sin(90^\circ - \theta); \quad n_1 > n_2$$



$\therefore 90^\circ = \theta + (90^\circ - \theta)$ Angle between reflected and refracted ray = 90°

7. A resistance of $2\ \Omega$ is connected across one gap of a metre-bridge (the length of the wire is 100 cm) and an unknown resistance, greater than $2\ \Omega$, is connected across the other gap. When these resistances are interchanged, the balance point shifts by 20 cm. Neglecting any corrections, the unknown resistance is
- (A) $3\ \Omega$
(B) $4\ \Omega$
(C) $5\ \Omega$
(D) $6\ \Omega$

Sol. (A)



$$\frac{l}{100-l} = \frac{2}{X}$$

On interchanging $2\ \Omega$ and X ,

$$\frac{l'}{100-l'} = \frac{X}{2}$$

Using $l' = l = 20\text{ cm}$, we have

$$\frac{100x}{2 \times x} = \frac{200}{x \times 2}$$

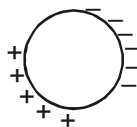
$$\therefore x = 3\ \Omega$$

8. Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then,
- (A) negative and distributed uniformly over the surface of the sphere
(B) negative and appears only at the point on the sphere closest to the point charge
(C) negative and distributed non-uniformly over the entire surface of the sphere
(D) zero

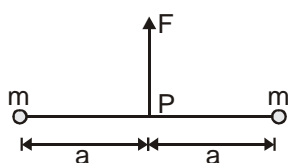
Sol. (D)

Although there will be a charge distribution on the outer surface of sphere, the net charge is still zero.

• q

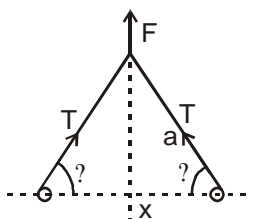
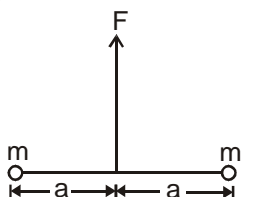


9. Two particles of mass m each are tied at the ends of a light string of length $2a$. The whole system is kept on a frictionless horizontal surface with the string held tight so that each mass is at a distance ' a ' from the center P (as shown in the figure). Now, the mid-point of the string is pulled vertically upwards with a small but constant force F . As a result, the particles move towards each other on the surface. The magnitude of acceleration, when the separation between them becomes $2x$, is



- (A) $\frac{F}{2m} \frac{a}{\sqrt{a^2 - x^2}}$
 (B) $\frac{F}{2m} \frac{x}{\sqrt{a^2 - x^2}}$
 (C) $\frac{F}{2m} \frac{x}{a}$
 (D) $\frac{F}{2m} \frac{\sqrt{a^2 - x^2}}{x}$

Sol. (B)



$$\begin{aligned} 2T \sin \theta &= F \\ T \cos \theta &= ma_x \\ 2 \tan \theta &= \frac{F}{ma_x} \end{aligned}$$

$$a_x \approx \frac{F}{2m \tan \theta} \approx \frac{Fx}{2m\sqrt{a^2 - x^2}}$$

$a_y \approx \frac{F}{2m}$, is very small and hence can be neglected.

$$\therefore a \approx a_x$$

Section – II

Assertion – Reason Type

This section contains 4 questions numbered 10 to 13. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

10. Statement - 1

The formula connecting u , v and f for a spherical mirror is valid only for mirrors whose sizes are very small compared to their radii of curvature.

Because

Statement - 2

Laws of reflection are strictly valid for plane surfaces, but not for large spherical surfaces

(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1

(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is Not a correct explanation for Statement – 1.

(C) Statement – 1 is True, Statement – 2 is False

(D) Statement – 1 is False, Statement – 2 is True

Sol. (C)

S_1 is correct

S_2 is incorrect

Laws of reflection is valid for all surfaces.

11. Statement - 1

A block of mass m starts moving on a rough horizontal surface with a velocity v . It stops due to friction between the block and surface after moving through a certain distance. The surface is now tilted to an angle 30° with the horizontal and the same block is made to go up on the surface with the same initial velocity v . The decrease in the mechanical energy in the second situation is smaller than that in the first situation.

Because

Statement - 2

The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination.

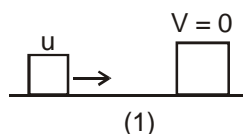
(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1

(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is Not a correct explanation for Statement – 1.

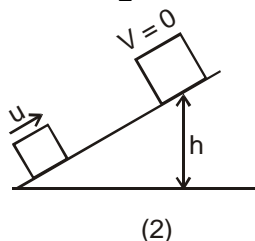
(C) Statement – 1 is True, Statement – 2 is False

(D) Statement – 1 is False, Statement – 2 is True

Sol. (C)



$$? E(1) ? ? \frac{1}{2}mv^2$$



$$? E(2) ? mgh ? \frac{1}{2}mv^2$$

$$? E(2) ? ? E(1)$$

$$? S_1 ? \text{ correct}$$

$$S_2 ? \text{ incorrect}$$

? doesn't change, rather force of friction reduces.

12. **Statement - 1**

If the accelerating potential in an X-ray tube is increased, the wavelengths of the characteristic X-rays do not change.

Because

Statement - 2

When an electron beams strikes the target in an X-ray tube, part of the kinetic energy is converted into X-ray energy.

(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1

(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is Not a correct explanation for Statement – 1.

(C) Statement – 1 is True, Statement – 2 is False

(D) Statement – 1 is False, Statement – 2 is True

Sol. (B)

Accelerating potential affects λ_{\min} and not characteristic X-ray ?

$$? S_1 ? \text{ correct}$$

$$S_2 ? \text{ correct}$$

13. Statement - 1

In an elastic collision between two bodies, the relative speed of the bodies after collision is equal to the relative speed before the collision.

Because

Statement – 2

In an elastic collision, the linear momentum of the system is conserved.

(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1

(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is Not a correct explanation for Statement – 1.

(C) Statement – 1 is True, Statement – 2 is False

(D) Statement – 1 is False, Statement – 2 is True

Sol. (B) Contextually.

NB:

S_2 need not be true in all cases. Counter e.g. ? collision of a ball with a smooth but rigid wall.

Hence S_2 ? incorrect (as a generalized statement)

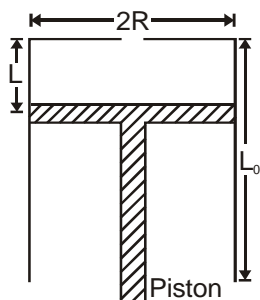
Note: Elastic refers to conservation of kinetic energy only. If S_2 is given as correct, S_1 is generally true.

Section – III Linked Comprehension Type

This section contains 2 paragraphs P14-16 and P 17-19. Based upon each paragraph, 3 multiple choice Questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

P14-16 : Paragraph for Questions Nos. 14 to 16.

A fixed thermally conducting cylinder has a radius R and height L_0 . The cylinder is open at its bottom and has a small hole at its top. A piston of mass M is held at a distance L from the top surface, as shown in the figure. The atmospheric pressure is P_0



14. The piston is now pulled out slowly and held at a distance $2L$ from the top. The pressure in the cylinder between its top and the piston will then be

- (A) P_0
 (B) $\frac{P_0}{2}$
 (C) $\frac{P_0}{2} + \frac{Mg}{\pi R^2}$
 (D) $\frac{P_0}{2} + \frac{Mg}{\pi R^2}$

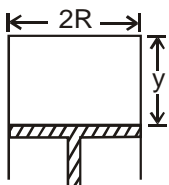
Sol. (A)

Since the top is open to the atmosphere, pressure inside is equal to pressure outside. Since the piston is moved slowly, the pressure equalizes to that of atmosphere.

15. While the piston is at a distance $2L$ from the top, the hole at the top is sealed. The piston is then released, to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is

- (A) $\frac{2P_0\pi R^2}{\pi R^2 P_0 + Mg} \cdot 2L$
 (B) $\frac{P_0\pi R^2 - Mg}{\pi R^2 P_0} \cdot 2L$
 (C) $\frac{P_0\pi R^2 + Mg}{\pi R^2 P_0} \cdot 2L$
 (D) $\frac{P_0\pi R^2}{\pi R^2 P_0 - Mg} \cdot 2L$

Sol. (D)



Since the cylinder is thermally conducting, the expansion is isothermal. Let P be the pressure inside the cylinder at equilibrium.

$$P_0(2L)\pi R^2 = P(y)\pi R^2$$

$$P = \frac{2P_0L}{y}$$

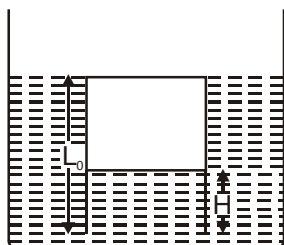
Equilibrium Forces are balanced.

$$P\pi R^2 = Mg + P_0\pi R^2$$

$$\frac{2P_0L}{y} = P_0 + \frac{Mg}{\pi R^2}$$

$$y = \frac{2P_0\pi R^2 L}{\pi R^2 P_0 + Mg}$$

16. The piston is taken completely out of the cylinder. The hole at the top is sealed. A water tank is brought below the cylinder and put in a position so that the water surface in the tank is at the same level as the top of the cylinder as shown in the figure. The density of water is ρ . In equilibrium, the height H of water column in the cylinder satisfies



- (A) $\rho g L_0 - H \rho \neq P_0$, $\rho L_0 - H \rho \neq L_0 P_0 \neq 0$
 (B) $\rho g L_0 - H \rho \neq P_0$, $\rho L_0 - H \rho \neq L_0 P_0 \neq 0$
 (C) $\rho g L_0 - H \rho \neq P_0$, $\rho L_0 - H \rho \neq L_0 P_0 \neq 0$
 (D) $\rho g L_0 - H \rho \neq P_0$, $\rho L_0 - H \rho \neq L_0 P_0 \neq 0$

Sol. (C)

Let the air pressure be P (inside the cylinder). Then

$$P L_0 + H \rho R^2 = P_0 L_0 + \rho R^2$$

$$P = P_0 + \frac{\rho L_0}{L_0} \rho H$$

This pressure is balanced by the hydrostatic pressure. So

$$\frac{\rho L_0}{L_0} \rho H = \rho g L_0 + H \rho \neq P_0$$

$$\rho g L_0 + H \rho \neq P_0 L_0 + H \rho \neq P_0 L_0 \neq 0$$

P17-19 : Paragraph for Questions Nos. 17 to 19

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and $2I$ respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1 . Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2 . Both the discs rotate in the clockwise direction

17. The ratio x_1/x_2 is

- (A) 2
 (B) $\frac{1}{2}$
 (C) $\sqrt{2}$
 (D) $\frac{1}{\sqrt{2}}$

Sol. (C)

$$\frac{1}{2} kx_1^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} I \omega_0^2$$

$$x_1^2 = \frac{4I \omega_0^2}{k}$$

$$\text{Also, } \frac{1}{2} kx_2^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} I \omega_0^2$$

$$x_2^2 = \frac{2I \omega_0^2}{k}$$

$$\frac{x_1}{x_2} = \sqrt{2}$$

18. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t . The average frictional torque on one disc by the other during this period is

(A) $\frac{2I\omega_0}{3t}$

(B) $\frac{9I\omega_0}{2t}$

(C) $\frac{9I\omega_0}{4t}$

(D) $\frac{3I\omega_0}{2t}$

Sol. (A)

If the common angular velocity is ω_0 ,

$$I \omega_0 + 2I \omega_0 = 3I \omega_0$$

$$\omega_0 = \frac{4}{3} \omega_0$$

Now, for disc A,

$$\omega_0 = 2\alpha t$$

$$\alpha = \frac{2\omega_0}{t} = \frac{2}{3t}$$

$$\tau = I \alpha = \frac{2I\omega_0}{3t}$$

19. The loss of kinetic energy during the above process is

(A) $\frac{I\omega_0^2}{2}$

(B) $\frac{I\omega_0^2}{3}$

(C) $\frac{I\omega_0^2}{4}$

(D) $\frac{I\omega_0^2}{6}$

Sol. (B)

$$KE_i = \frac{1}{2}mv^2 = \frac{1}{2}m(2l)^2 = 2ml^2$$

$$KE_f = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{4}{3}l\right)^2 = \frac{8}{3}ml^2 \quad \text{Loss of KE} = \frac{ml^2}{3}$$

Section – IV

Matrix-Match Type

This section contains 3 questions. Each question contains statement given in two columns which have to be matched. Statement (A, B, C, D) in **column I** have to be matched with statements (p, q, r, s) in **column II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A-p, A-s, B-q, B-r, C-p, C-q and D-s then the correct bubbled 4 × 4 matrix should be as follows

	p	q	r	s
A	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
B	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
C	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
D	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

20. Some physical quantities are given in **column I** and some possible SI units in which these quantities may be expressed are given in **column II**. Match the physical quantities in **column I** with the units in **column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I		Column II	
(A)	$\frac{GM_e M_s}{R_e}$ G – universal gravitational constant, M_e – mass of the earth M_s – Mass of the Sun	(p)	(volt) (coulomb) (metre)
(B)	$\frac{3RT}{M}$ R – universal gas constant T – absolute temperature M – molar mass	(q)	(kilogram)(metre) ³ (second) ⁻²
(C)	$\frac{F^2}{q^2 B^2}$ F – force Q – charge B – magnetic field	(r)	(metre) ² (second) ⁻²
(D)	$\frac{GM_e}{R_e}$ G – universal gravitational constant, M_e – mass of the earth R_e – radius of the earth	(S)	(farad)(volt) ² (kg) ⁻¹

Sol.

- (A) ? (p), (q)
 (B) ? (r), (s)
 (C) ? (r), (s)
 (D) ? (r), (s)

- 21.** **Column I** gives certain situations in which a straight metallic wire of resistance R is used and **column II** gives some resulting effects. Match the statement in **column I** with the statements in **column II** and indicate your answer by darkening appropriate bubbles in the 4×4 matrix given in the ORS.

Column I		Column II	
(A)	A charged capacitor is connected to the ends of the wire	(p)	A constant current flows through the wire
(B)	The wire is moved perpendicular to its length with a constant velocity in a uniform magnetic field perpendicular to the plane of motion	(q)	Thermal energy is generated in the wire
(C)	The wire is placed in a constant electric field that has direction along the length of the wire	(r)	A constant potential difference develops between the ends of the wire
(D)	A battery of constant emf is connected to the ends of the wire	(S)	Charges of constant magnitude appear at the ends of the wire

Sol.

- (A) ? (q)
 (B) ? (r), (s),
 (C) ? (s)
 (D) ? (p), (q), (r)

- 22.** Some laws/ processes are given in **Column I**. Match these with physical phenomena given in **Column II** and indicate your answer by darkening appropriate bubbles in the 4×4 matrix given in the ORS.

Column I		Column II	
(A)	Transition between two atomic energy levels	(p)	Characteristic X-rays
(B)	Electron emission from a material	(q)	Photoelectric effect
(C)	Mosley's law	(r)	Hydrogen spectrum
(D)	Change of photon energy into kinetic energy of electrons	(S)	? – decay

Sol.

- (A) ? (p), (r)
 (B) ? (q), (s),
 (C) ? (p)
 (D) ? (q)

Chemistry

Part – II

Useful data:

Gas Constant, $R = 8.314 \text{ K JK}^{-1}\text{mol}^{-1}$

$1^\circ \text{F} = 9/5^\circ \text{C}$

Atomic Numbers:

H = 1, Li = 3, B = 5, C = 6, N = 7, O = 8, F = 9, Na = 11, P = 15, S = 16, Cl = 17, Ar = 18,
 K = 19, V = 23, Cr = 24, Mn = 25, Fe = 26, Co = 27, Ni = 28, Cu = 29, Zn = 30, Ge = 32, Br = 35, Ag = 47,
 I = 53, Xe = 54, Pt = 78, Hg = 80, Pb = 82

Section – I Straight Objective Type

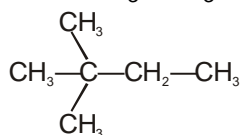
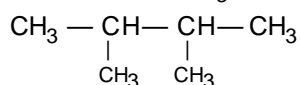
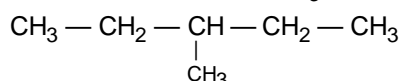
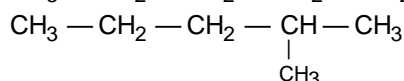
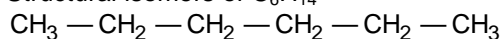
This section contains 9 multiple choice questions numbered 23 to 31. Each question has 4 choices (A), (B), and (D), out of which **only one** is correct

23. The number of structural isomers for C_6H_{14} is

- (A) 3
- (B) 4
- (C) 5
- (D) 6

Sol. (C)

Structural isomers of C_6H_{14}



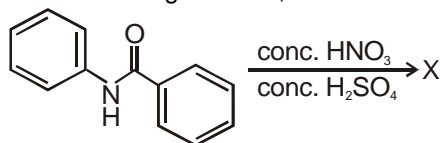
24. When 20 g of naphthoic acid $[\text{C}_{11}\text{H}_8\text{O}_2]$ is dissolved in 50 g of benzene $[K_f = 1.72 \text{ K kg mol}^{-1}]$, a freezing point depression of 2 K is observed. The van't Hoff factor (i) is

- (A) 0.5
- (B) 1
- (C) 2
- (D) 3

Sol. (A)

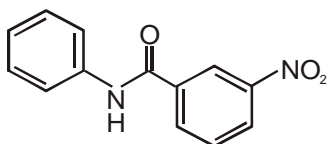
$$i = \frac{T_f}{K_m} = \frac{2 \times 172 \times 50}{1.72 \times 20 \times 2000} = 0.5$$

25. In the following reaction,

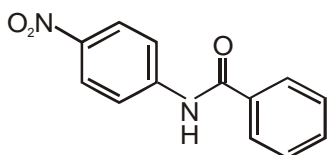


the structure of the major product 'X' is

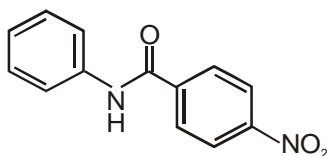
(A)



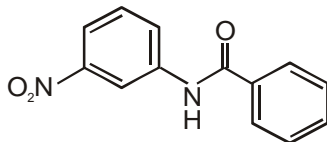
(B)



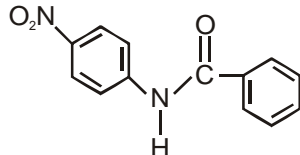
(C)



(D)



Sol. (B)



Because >NH group is activator and p-directing towards electrophilic aromatic substitution.

26. The value of $\log_{10} K$ for a reaction $A \rightarrow B$ is
(Given ${}_rH_{298K}^\circ = -54.07 \text{ kJ mol}^{-1}$, ${}_rS_{298K}^\circ = 10 \text{ JK}^{-1}\text{mol}^{-1}$ and
 $R = 8.314 \text{ JK}^{-1}\text{mol}^{-1}$; $2.303 \times 298 = 5705$)
(A) 5
(B) 10
(C) 95
(D) 100

Sol. (B)

$$\begin{aligned} \Delta G^\circ &= -2.303 RT \log K \\ \Delta G^\circ &= \Delta H^\circ - T \Delta S^\circ \\ &= -54070 - 2980 \\ &= -54070 \text{ J} \\ &= -2.303 \times 8.314 \times 298 \log K \\ \log K &= 10 \end{aligned}$$

27. Among the following, the paramagnetic compound is
(A) Na_2O_2
(B) O_3
(C) N_2O
(D) KO_2

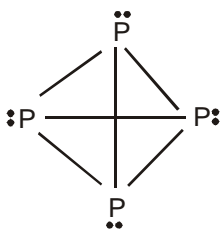
Sol. (D)

KO_2 is paramagnetic in nature because it contains unpaired electrons.

28. The percentage of p-character in the orbitals forming P-P bonds in P_4 is
(A) 25
(B) 33
(C) 50
(D) 75

Sol. (D)

Phosphorous will show sp^3 hybridisation having 75% p-character.

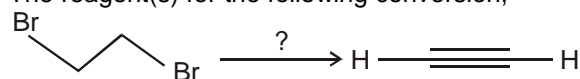


29. The species having bond order different from that in CO is
(A) NO^-
(B) NO^+
(C) CN^-
(D) N_2

Sol. (A)

Bond order of $\text{NO}^?$ is 2 while bond order of CO is 3.

30. The reagent(s) for the following conversion,



is/are

- (A) alcoholic KOH
- (B) alcoholic KOH followed by NaNH_2
- (C) aqueous KOH followed by NaNH_2
- (D) $\text{Zn}/\text{CH}_3\text{OH}$

Sol. (B)



31. Extraction of zinc from zinc blende is achieved by

- (A) electrolytic reduction
- (B) roasting followed by reduction with carbon
- (C) roasting followed by reduction with another metal
- (D) roasting followed by self-reduction

Sol. (B)

Roasting followed by reduction with carbon.

Section – II

Assertion – Reason Type

This section contains 4 questions numbered 32 to 35. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which only one is correct.

32. Statement - 1

p-Hydroxybenzoic acid has a lower boiling point than o-hydroxybenzoic acid

Because

Statement - 2

o-Hydroxybenzoic acid has intramolecular hydrogen bonding

- (A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1
- (B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is Not a correct explanation for Statement – 1.
- (C) Statement – 1 is True, Statement – 2 is False
- (D) Statement – 1 is False, Statement – 2 is True

Sol. (D)

O-hydroxybenzoic acid has intramolecular hydrogen bonding hence has lower boiling point than p-hydroxybenzoic acid.

33. Statement - 1

Boron always forms covalent bond

Because

Statement - 2

The small size of B^{3+} favours formation of covalent bond.

- (A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1
- (B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is Not a correct explanation for Statement – 1.
- (C) Statement – 1 is True, Statement – 2 is False
- (D) Statement – 1 is False, Statement – 2 is True

Sol. (A)

Boron always forms covalent bond, because its compound will have zero resultant dipole moment due to its trigonal planar shape.

34. Statement - 1

Micelles are formed by surfactant molecules above the critical micellar concentration (CMC)

Because

Statement - 2

The conductivity of a solution having surfactant molecules decreases sharply at the CMC.

- (A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1
- (B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is Not a correct explanation for Statement – 1.
- (C) Statement – 1 is True, Statement – 2 is False
- (D) Statement – 1 is False, Statement – 2 is True

Sol. (B)

35. Statement - 1

In water, orthoboric acid behaves as a weak acid monobasic acid.

Because

Statement – 2

In water, orthoboric acid acts as a proton donor.

- (A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1
- (B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is Not a correct explanation for Statement – 1.
- (C) Statement – 1 is True, Statement – 2 is False
- (D) Statement – 1 is False, Statement – 2 is True

Sol. (C)

Section – III

Linked Comprehension Type

This section contains 2 paragraphs C36-38 and C39-41. Based upon each paragraph, 3 multiple choice Questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which **Only one** is correct.

P36-38 : Paragraph for Questions Nos. 36 to 38.

Chemical reactions involve interaction of atoms and molecules. A large number of atoms/molecules (approximately 6.023×10^{23}) are present in a few grams of any chemical compound varying with their atomic/molecular masses. To handle such large numbers conveniently, the mole concept was introduced. This concept has implications in diverse areas such as analytical chemistry, biochemistry, electrochemistry and radiochemistry. The following example illustrates a typical case, involving chemical/electrochemical reaction, which requires a clear understanding of the mole concept.

A 4.0 molar aqueous solution NaCl is prepared and 500 mL of this solution is electrolysed. This leads to the evolution of chlorine gas at one of the electrodes (atomic mass : Na = 23, Hg = 200; 1 Faraday = 96500 coulombs).

36. The total number of moles of chlorine gas evolved is
 (A) 0.5
 (B) 1.0
 (C) 2.0
 (D) 3.0

Sol. (B)

37. If the cathode is a Hg electrode, the maximum weight (g) of amalgam formed from this solution is
 (A) 200
 (B) 225
 (C) 400
 (D) 446

Sol. (D)

In presence of amalgam cathode sodium ion will discharge in place of hydrogen gas due to over voltage is the form of amalgams.

$$\text{Weight of amalgam} = 2 \times (23 + 200) = 446 \text{ g}$$

38. The total charge (coulombs) required for complete electrolysis is
 (A) 24125
 (B) 48250
 (C) 96500
 (D) 193000

Sol. (D)

$$\text{Total charge} = 2 \times 96500 = 193000 \text{ C}$$

P39-41 : Paragraph for Questions Nos. 39 to 41.

The noble gases have closed-shell electronic configuration and are monoatomic gases under normal conditions. The low boiling points of the lighter noble gases are due to weak dispersion forces between the atoms and the absence of other interatomic interactions.

The direct reaction of xenon with fluorine leads to a series of compounds with oxidation numbers +2, +4 and + 6. XeF_4 reacts violently with water to give XeO_3 . The compounds of xenon exhibit rich stereochemistry and their geometries can be deduced considering the total number of electron pairs in the valence shell

- 39.** Argon is used in arc welding because of its
 (A) low reactivity with metal
 (B) ability to lower the melting point of metal
 (C) flammability
 (D) high calorific value

Sol. (A)

Argon may be used to provide an inert atmosphere in welding.

- 40.** The structure of XeO_3 is
 (A) linear
 (B) planar
 (C) pyramidal
 (D) T-shaped

Sol. (C)

Pyramidal

- 41.** XeF_4 and XeF_6 are expected to be
 (A) oxidizing
 (B) reducing
 (C) unreactive
 (D) strong basic

Sol. (A)

Section – IV

Matrix-Match Type

This section contains 3 questions . Each question contains statement given in two columns which have to be matched. Statement (A, B, C, D) in **column I** have to be matched with statements (p, q, r, s) in **column II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A-p, A-s, B-q, B-r, C-p, C-q and D-s then the correct bubbled 4 × 4 matrix should be as follows

	p	q	r	s
A	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
B	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
C	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
D	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

42. Match the complexes in **column I** with their properties listed in **column II**. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I		Column II	
(A)	$[\text{Co}(\text{NH}_3)_4(\text{H}_2\text{O})_2\text{Cl}_2]$	(p)	geometrical isomers
(B)	$[\text{Pt}(\text{NH}_3)_2\text{Cl}_2]$	(q)	paramagnetic
(C)	$[\text{Co}(\text{H}_2\text{O})_5\text{Cl}_2]$	(r)	diamagnetic
(D)	$[\text{Ni}(\text{H}_2\text{O})_6\text{Cl}_2]$	(s)	metal ion with +2 oxidation state

Sol.

A — p, A — q, A — s.
B — p, B — s, B — r
C — s, C — q
D — s, D — q

43. Match the chemical substances in **column I** with type of polymers/type of bonds in **column II**. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I		Column II	
(A)	Cellulose	(p)	natural polymer
(B)	nylon-6,6	(q)	synthetic polymer
(C)	Protein	(r)	amide linkage
(D)	Sucrose	(s)	glycoside linkage

Sol.

A — p, A — s
 B — q, B — r
 C — r, C — p
 D — s

- 44.** Match gases under specified conditions listed in **column I** with their properties/laws in **column II**. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I		Column II	
(A)	hydrogen gas ($P = 200 \text{ atm}$, $T = 273 \text{ K}$)	(p)	compressibility factor > 1
(B)	hydrogen gas ($P = 0$, $T = 273 \text{ K}$)	(q)	attractive forces are dominant
(C)	CO_2 ($P = 1 \text{ atm}$, $T = 273 \text{ K}$)	(r)	$PV = nRT$
(D)	real gas with very large molar volume	(s)	$P(V - nb) = nRT$

Sol.

A — p, A — s, B — r, C — q, C — p, D — p, D — q

Mathematics

Section – III Straight Objective Type

This section contains 9 multiple choice questions numbered 45 to 53. Each question has 4 choices (A), (B), (C) and (D), out of which **only one** is correct

45. A hyperbola, having the transverse axis of length $2\sin\theta$, is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is
- (A) $x^2 \operatorname{cosec}^2\theta + y^2 \sec^2\theta = 1$
 (B) $x^2 \sec^2\theta + y^2 \operatorname{cosec}^2\theta = 1$
 (C) $x^2 \sin^2\theta + y^2 \cos^2\theta = 1$
 (D) $x^2 \cos^2\theta + y^2 \sin^2\theta = 1$

Sol. (A)

$$x^2 \operatorname{cosec}^2\theta + y^2 \sec^2\theta = 1$$

$$a = \sin\theta \text{ for hyperbola}$$

$$ae = \sqrt{4 + 3} = 1$$

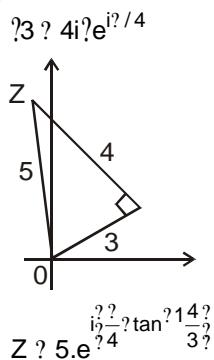
$$b^2 = \sin^2\theta (\operatorname{cosec}^2\theta - 1) = \cos^2\theta$$

$$\frac{x^2}{\sin^2\theta} + \frac{y^2}{\cos^2\theta} = 1$$

$$x^2 \operatorname{cosec}^2\theta + y^2 \sec^2\theta = 1$$

46. A man walks a distance of 3 units from the origin towards the north-east ($N 45^\circ E$) direction. From there, he walks a distance of 4 units towards the north-west ($N 45^\circ W$) direction to reach a point P. Then the position of P in the Argand plane is
- (A) $3e^{i\pi/4} + 4i$
 (B) $3 - 4i e^{i\pi/4}$
 (C) $4 + 3i e^{i\pi/4}$
 (D) $3 + 4i e^{i\pi/4}$

Sol. (D)



$$5e^{i \tan^{-1} \frac{4}{3}} \cdot e^{i\pi/4}$$

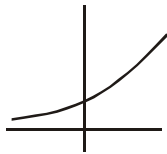
$$5 \frac{3}{5} - i \frac{4}{5} \cdot e^{i\pi/4}$$

$$3 - 4i \cdot e^{i\pi/4}$$

47. The tangent to the curve $y = e^x$ drawn at the point (c, e^c) intersects the line joining the points $(c-1, e^{c-1})$ and $(c+1, e^{c+1})$
- (A) on the left of $x = c$
 (B) on the right of $x = c$
 (C) at no point
 (D) at all points

Sol. (A)

On the left of $x = c$



Shortcut: Let $c = 0$

48. The number of solutions of the pair of equations
- $$2\sin^2 \theta + \cos 2\theta = 0$$
- $$2\cos^2 \theta + 3\sin \theta = 0$$
- in the interval $[0, 2\pi]$ is
- (A) zero
 (B) one
 (C) two
 (D) four

Sol. (C)

Add two equations

$$2 + 1 + 2\sin^2 \theta + 3\sin \theta = 0$$

$$2\sin^2 \theta + 3\sin \theta + 1 = 0$$

$$(2\sin \theta + 1)(\sin \theta + 1) = 0$$

$$\sin \theta = -\frac{1}{2}, 1$$

If $\sin \theta = 1$, $\cos \theta = 0$ not satisfies second equation.

$$\sin \theta = -\frac{1}{2} \quad \cos \theta = \pm \frac{\sqrt{3}}{2}$$

$$\theta = \frac{7\pi}{6}, \frac{5\pi}{6}$$

49. Let $f(x)$ be differentiable on the interval $(0, \infty)$ such that $f(1) = 1$, and $\lim_{t \rightarrow x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ for each $x > 0$. Then $f(x)$ is
- (A) $\frac{1}{3x} + \frac{2x^2}{3}$
 (B) $\frac{1}{3x} + \frac{4x^2}{3}$
 (C) $\frac{1}{x} + \frac{2}{x^2}$
 (D) $\frac{1}{x}$

Sol. (A)

$$\frac{1}{3x} + \frac{2x^2}{3}$$

$$\lim_{t \rightarrow x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1 \quad \begin{matrix} \frac{0}{0} \\ \frac{0}{0} \end{matrix}$$

$$\lim_{t \rightarrow x} \frac{x^2 f(x) - x^2 f(t)}{1} = 1$$

$$-2xf(x) - x^2 f'(x) = 1$$

$$\frac{dy}{dx} + \frac{2}{x}y = -\frac{1}{x^2}$$

$$\text{I.F.} = e^{\int \frac{2}{x} dx} = e^{\ln x^2} = x^2$$

Solution is given by

$$yx^2 = \int -\frac{1}{x^4} dx + c$$

$$= \frac{x^3}{3} + c$$

$$\text{at } x = 1, y = 1 \Rightarrow c = \frac{2}{3}$$

50. The number of distinct real values of λ , for which the vectors $\lambda \hat{i} + \hat{j} + \hat{k}$, $\hat{i} + \lambda \hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + \lambda \hat{k}$ are coplanar, is
- (A) zero
 (B) one
 (C) two
 (D) three

Sol. (C)

$$\begin{vmatrix} x^2 & 1 & 1 \\ 1 & x^2 & 1 \\ 1 & 1 & x^2 \end{vmatrix} = 0$$

$$x^4 - 2x^2 + 1 = 0 \quad x = \pm \sqrt{2}$$

51. Let α, β be the roots of the equation $x^2 + px + r = 0$ and $\frac{\alpha}{2}, \frac{\beta}{2}$ be the roots of the equation $x^2 + qx + r = 0$. Then the value of r is

- (A) $\frac{2}{9}p + q$
 (B) $\frac{2}{9}q + p$
 (C) $\frac{2}{9}q + 2p$
 (D) $\frac{2}{9}2p + q$

Sol. (D)

$$\frac{2}{9}2p + q$$

$$= p$$

$$= r$$

$$\frac{2}{9}2p + q$$

$$= 4p + 2q$$

$$= \frac{2q + p}{3}$$

$$= \frac{4p + 2q}{3}; \quad r = \frac{2}{9}2q + p$$

52. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\int_0^x \sec^2 t \, dt}{x^2 - \frac{\pi^2}{16}}$ equals

- (A) $\frac{8}{\pi} f(2)$
 (B) $\frac{2}{\pi} f(2)$
 (C) $\frac{2}{\pi} f\left(\frac{1}{2}\right)$
 (D) $4f(2)$

Sol. (A)

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sec^2 x \int_{\frac{\pi}{4}}^x f(t) dt}{x^2 - \frac{\pi^2}{16}} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{2f(\sec^2 x) \sec^2 x \tan x}{2x}$$

$$= \frac{2f(2) \cdot 2 \cdot 1}{2 \cdot \frac{\pi}{4}} = \frac{8}{\pi} f(2)$$

53. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is

- (A) $\frac{1}{2}$
 (B) $\frac{1}{3}$
 (C) $\frac{2}{5}$
 (D) $\frac{1}{5}$

Sol. (C)

$A_1 A_1' A_2 A_2' A_3 A_3' A_4 A_4' II'$
 Total $5! \cdot 2^4$
 $A_1 A_1' A_2 A_2' A_3 A_3' A_4 A_4' II'$
 Favourable $4! \cdot 2^5$
 $\frac{4! \cdot 2^5}{5! \cdot 2^4} = \frac{2}{5}$

Section – II

Assertion – Reason Type

This section contains 4 questions numbered 54 to 57. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **only one** is correct.

54. Let the vectors \vec{PQ} , \vec{QR} , \vec{RS} , \vec{ST} , \vec{TU} and \vec{UP} represent the sides of a regular hexagon.

Statement -1: $\vec{PQ} + \vec{RS} + \vec{ST} = \vec{0}$

Because

Statement - 2

$\vec{PQ} + \vec{RS} = \vec{0}$ and $\vec{PQ} + \vec{ST} = \vec{0}$

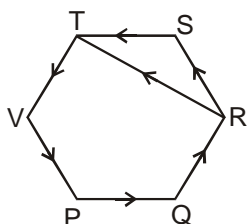
- (A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is Not a correct explanation for Statement – 1.
(C) Statement – 1 is True, Statement – 2 is False
(D) Statement – 1 is False, Statement – 2 is True

Sol. (C)

$S_1 = \text{True}, S_2 = \text{False}$

$\vec{RS} + \vec{ST} = \vec{RT}$

$\vec{PQ} + \vec{RT} = \vec{0}$ as $\vec{PQ} = \vec{RT}$



$\vec{PQ} + \vec{RS} = \vec{0}$

55. Let H_1, H_2, \dots, H_n be mutually exclusive and exhaustive events with $P(H_i) > 0, i = 1, 2, \dots, n$. Let E be any other event with $0 < P(E) < 1$.

Statement - 1

$P(H_i | E) = P(E | H_i) \cdot P(H_i)$ for $i = 1, 2, \dots, n$.

Because

Statement - 2

$\sum_{i=1}^n P(H_i) = 1$

- (A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is Not a correct explanation for Statement – 1.
(C) Statement – 1 is True, Statement – 2 is False
(D) Statement – 1 is False, Statement – 2 is True

Sol. (D)

S_1 ? False, S_2 ? True ,

$$P(H_i / E) = \frac{P(H_i \cap E)}{P(E)}$$

$$P(E / H_i) = \frac{P(E \cap H_i)}{P(H_i)} \cdot P(H_i) = P(E \cap H_i)$$

$$P(H_i / E) = P(E / H_i) \cdot P(H_i)$$

as $P(E) = 1$

56. Let $F(x)$ be an indefinite integral of $\sin^2 x$.

Statement - 1

The function $F(x)$ satisfies $F(x + \pi) = F(x)$ for all real x .

Because

Statement - 2

$\sin^2(x + \pi) = \sin^2 x$ for all real x .

- (A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1
 (B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is Not a correct explanation for Statement – 1.
 (C) Statement – 1 is True, Statement – 2 is False
 (D) Statement – 1 is False, Statement – 2 is True

Sol. (D)

S_1 ? False

$$f(x) = \frac{x}{2} - \frac{\sin 2x}{4} + c$$

57. Tangents are drawn from the point $(17, 7)$ to the circle $x^2 + y^2 = 169$

Statement - 1

The tangents are mutually perpendicular.

Because

Statement - 2

The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$.

- (A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1
 (B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is Not a correct explanation for Statement – 1.
 (C) Statement – 1 is True, Statement – 2 is False
 (D) Statement – 1 is False, Statement – 2 is True

Sol. (A)

S_1 : True, S_2 : True, S_2 explains S_1

Director circle of $x^2 + y^2 = r^2$ is $x^2 + y^2 = 2r^2$

Section – III

Linked Comprehension Type

This section contains 2 paragraphs M58-60 and M61-63. Based upon each paragraph, 3 multiple choice Questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which **Only one** is correct.

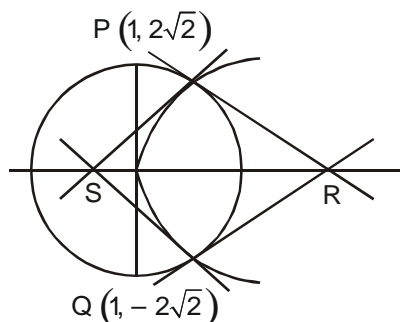
M58-60 : Paragraph for Questions Nos. 58 to 60.

Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in the first and the fourth quadrants, respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents to the parabola at P and Q intersect the x-axis at S.

58. The ratio of the areas of the triangles PQS and PQR is

- (A) $1 : \sqrt{2}$
(B) $1 : 2$
(C) $1 : 4$
(D) $1 : 8$

Sol. (C)



$$PR : x + 2\sqrt{2}y = 9$$

$$QR : x + 2\sqrt{2}y = 9$$

$$R(9, 0)$$

$$PS : 2\sqrt{2}y + 4x = 1$$

$$QS : -2\sqrt{2}y + 4x = 1 \quad S(1, 0)$$

$$\frac{\text{Area PQS}}{\text{Area PQR}} = \frac{\frac{1}{2} \cdot 4\sqrt{2} \cdot 2}{\frac{1}{2} \cdot 4\sqrt{2} \cdot 8} = 1 : 4$$

59. The radius of the circumcircle of the triangle PRS is
 (A) 5
 (B) $3\sqrt{3}$
 (C) $3\sqrt{2}$
 (D) $2\sqrt{3}$

Sol. (B)

$$R = \frac{abc}{4\Delta}$$

60. The radius of the incircle of the triangle PQR is
 (A) 4
 (B) 3
 (C) $\frac{8}{3}$
 (D) 2

Sol. (D)

$$r = \frac{\Delta}{s}$$

M61-63 : Paragraph for Questions Nos. 61 to 63.

Let V_r denote the sum of the first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is $(2r + 1)$.

Let $T_r = V_{r+1} - V_r + 2$ and $Q_r = T_{r+1} - T_r$ for $r = 1, 2, \dots$

61. The sum $V_1 + V_2 + \dots + V_n$ is
 (A) $\frac{1}{12}n(n+1)(3n^2 + n + 1)$
 (B) $\frac{1}{12}n(n+1)(3n^2 + n + 2)$
 (C) $\frac{1}{2}n(2n^2 + n + 1)$
 (D) $\frac{1}{3}n(2n^3 + 2n + 3)$

Sol. (B)

$$V_r = r^2 + \frac{r^2}{2} + \frac{r}{2}; \quad V_r = \frac{1}{12}n(n+1)(3n^2 + n + 2)$$

62. T_r is always
 (A) an odd number
 (B) an even number
 (C) a prime number
 (D) a composite number

Sol. (D)

Always composite number

$$T_r = 3r^2 + 1, r = 1$$

63. Which one of the following is a correct statement?

- (A) Q_1, Q_2, Q_3, \dots are in A.P. with common difference 5
 (B) Q_1, Q_2, Q_3, \dots are in A.P. with common difference 6
 (C) Q_1, Q_2, Q_3, \dots are in A.P. with common difference 11
 (D) $Q_1 = Q_2 = Q_3 = \dots$

Sol. (B)

Q_1, Q_2, Q_3, \dots are in A.P. with common difference 6

$$Q_r = 6r + 5$$

Section – IV Matrix-Match Type

This section contains 3 questions. Each question contains statement given in two columns which have to be matched. Statement (A, B, C, D) in **column I** have to be matched with statements (p, q, r, s) in **column II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A-p, A-s, B-q, B-r, C-p, C-q and D-s then the correct bubbled 4×4 matrix should be as follows

	p	q	r	s
A	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
B	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
C	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
D	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

64. Consider the following linear equations

$$ax + by + cz = 0$$

$$bx + cy + az = 0$$

$$cx + ay + bz = 0$$

Match the conditions/expressions in Column I with statement in Column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

Column I		Column II	
(A)	$a \neq b \neq c \neq 0$ and $a^2 \neq b^2 \neq c^2 \neq ab \neq bc \neq ca$	(p)	the equations represent planes meeting only at a single point.
(B)	$a \neq b \neq c \neq 0$ and $a^2 \neq b^2 \neq c^2 \neq ab \neq bc \neq ca$	(q)	the equations represent the line $x = y = z$
(C)	$a \neq b \neq c \neq 0$ and $a^2 \neq b^2 \neq c^2 \neq ab \neq bc \neq ca$	(r)	the equations represent identical planes.
(D)	$a \neq b \neq c \neq 0$ and $a^2 \neq b^2 \neq c^2 \neq ab \neq bc \neq ca$	(s)	the equations represent the whole of the three dimensional space.

Sol.

A ? r, B ? q, C ? p, D ? s

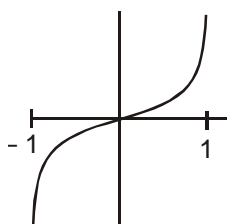
65. In the following $[x]$ denotes the greatest integer less than or equal to x .

Match the functions in Column I with the properties in Column II and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS

Column I		Column II	
(A)	$x x $	(p)	continuous in $(-1, 1)$
(B)	$\sqrt{ x }$	(q)	differentiable in $(-1, 1)$
(C)	$x - [x]$	(r)	strictly increasing in $(-1, 1)$
(D)	$ x - 1 - x + 1 $	(s)	not differentiable at least at one point in $(-1, 1)$

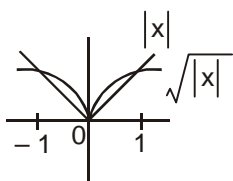
Sol.

A ? p, q, r



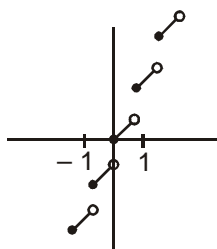
$x|x|$? continuous in $(-1, 1)$ differentiable in $(-1, 1)$ strictly increasing in $(-1, 1)$

(B) ? p, s



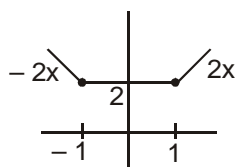
$\sqrt{|x|}$? continuous in $(-1, 1)$ and differentiable in $(-1, 1)$

(C) ? r, s



$x - [x]$? strictly increasing in $(-1, 1)$ not differentiable at least at one point in $(-1, 1)$

(D) ? p, q



$|x|$ is continuous in $(-1, 1)$ and differentiable in $(-1, 1)$

A \rightarrow p, q, r

B \rightarrow p, s

C \rightarrow r, s

D \rightarrow p, q

66. Match the integrals in Column I with the values in Column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the CRS.

Column I		Column II	
(A)	$\int_{-1}^1 \frac{dx}{1+x^2}$	(p)	$\frac{1}{2} \log \frac{2}{3}$
(B)	$\int_0^1 \frac{dx}{\sqrt{1+x^2}}$	(q)	$2 \log \frac{2}{3}$
(C)	$\int_2^3 \frac{dx}{1+x^2}$	(r)	$\frac{2}{3}$
(D)	$\int_1^2 \frac{dx}{x\sqrt{1+x^2}}$	(s)	$\frac{2}{3}$

Sol.

A \rightarrow s

$$\int_{-1}^1 \frac{dx}{1+x^2} = \left[\tan^{-1} x \right]_{-1}^1 = \frac{\pi}{2}$$

B \rightarrow s

$$\int_0^1 \frac{dx}{\sqrt{1+x^2}} = \left[\sinh^{-1} x \right]_0^1 = \sinh^{-1} 1 = \frac{1}{2} \log \frac{1+x}{1-x}$$

C \rightarrow p

$$\int_2^3 \frac{dx}{1+x^2} = \left[\frac{1}{2} \log_e \left| \frac{1+x}{1-x} \right| \right]_2^3 = \frac{1}{2} \log_e \frac{2}{3}$$

D \rightarrow r

$$\int_1^2 \frac{dx}{x\sqrt{x^2-1}} = \left[\sec^{-1} x \right]_1^2 = \sec^{-1} 2 = \frac{\pi}{3}$$