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A fuzzy state estimator based on uncertain measurements

A.K. AL-Othman

Electrical Engineering Department, College of Technological Studies, P.O. Box 33198, Alrawda 73452, Kuwait

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ABSTRACT

A fuzzy linear state estimation model is employed, which is based on Tanaka's fuzzy linear regression model, for modeling uncertainty in power system state estimation. The estimation process is based on uncertainty measurements as well as uncertainty parametric. The uncertain measurements and the parameters are expressed as fuzzy numbers with a triangular membership function that has middle and spread value reflected on the estimated states. The proposed fuzzy model is formulated as a linear optimization problem, where the objective is to minimize the sum of the spread of the states, subject to double inequality constraints on each measurement. Linear programming technique is employed to obtain the middle and the symmetric spread for every state variable. The estimated middle corresponds to the value of the estimated state, while the symmetric spreads represent the tightest uncertainty interval around that estimated states. For illustrative purposes, the proposed formulation has been applied to various test systems such as, 4-bus, 6-bus, IEEE 30-bus, IEEE 39-bus, IEEE 57-bus and IEEE 118-bus. Furthermore, an assessment of the time convergence of the proposed method has been carried out to demonstrate the applicability of the proposed estimator as an on-line tool for estimating the uncertainty bounds in power system state estimation.

1. Introduction

Having an accurate picture of the state of a system is an important part of the system operations. While a simple SCADA (Supervisory Control and Data Acquisition) system has the ability to provide the system operators with raw information about the system operation conditions, only a state estimator has the ability of filtering the information to supply a more accurate picture of the status of the system.

The conventional purpose of state estimation is to reduce the effect of measurement errors by utilizing the redundancy available in the measurement system. In particular, the objective is to reduce the variance of the estimates and improve their overall accuracy. The other major objectives of state estimation methods include: detection of gross errors, detection of invalid topological information and detection of model parameter errors.

If the inaccuracy (or error) in the measurements, for a given estimator, is modelled by some random probability distribution function, then the set of feasible estimates can also be modelled by a probability distribution function. These estimators are, therefore, probabilistic in nature. In fact probability theory is generally utilized to handle inaccuracy. Due to the fact that statistics of the measurement errors are difficult to be probabilistically characterized in practice, imprecision in error modelling cannot be equated with randomness, [1], and instead can be associated with fuzziness [2]. Thus, fuzzy theory can satisfactorily be deployed in such circumstances to overcome this limitation and address various uncertainties in the modelling of such statistics. That is particularly due to its ability in handling uncertainties and vagueness associated with the observation errors. Generally, in the context of state estimation, fuzzy estimators are possibilistic in nature. If the observation errors are assumed to be fuzzy due to uncertainty that

E-mail address: ak.alothman@paaet.edu.kw

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is inherently present in the system, then the estimates are assumed to be a range of possible values. Consequently, in such situations, it is desirable to provide not just a single 'optimal' estimate of each state variable but also an uncertainty range within which we can be assured that the 'true' state variable must lie. This is attainable by utilizing some a fuzzy function to represent the estimates as fuzzy estimates with their associated uncertainty ranges as opposed to crisp estimates (single point only) produced by the conventional estimators [3].

The main theme of this paper is to model the uncertainties associated with the measured quantities in a way that defines an interval (range) with respect to their nominal values. The range is governed by the tolerance, of the measuring instrument (a quantification of accuracy usually provided by the manufacturer) and other factors that are known to have direct effects on network mathematical model being used in the estimation procedure. By implementing the proposed fuzzy linear techniques the confidence interval (or bounds) of the state variables can be computed. Hence, this study presents an estimator based on fuzzy linear regression formulation for estimating the uncertainty interval around the system state variables. This estimator is based on Tanaka's fuzzy linear regression formulation. The uncertainty is expressed in both measurements and network parameters in a unified fuzzy model. The main objective is to minimize the fuzziness in the estimated states. This can be achieved by minimize the sum of spreads of all fuzzy states, subject to double inequality constraints on each measurement to guarantee that the original membership is included in the estimated membership. Linear programming has been employed to obtain the middle and the symmetric spread of every state variable. The estimated middle corresponds to the value of the estimated state, whereas the symmetric spreads in the membership functions of the state variables represents the uncertainty interval around that estimated state. Thus, the primary goal is to minimize the sums of the uncertainties around the states.

2. Uncertainty and state estimation

Schweppe [4] introduced the concepts of uncertainty in the general context of engineering analysis, estimation and optimization. In [4] the concept of unknown-but-bounded errors for modelling uncertainty in estimation problems was introduced. Measurements are assumed to be inexact and have errors that are unknown but fall within a bounded range.

These concepts have been extended and developed recently and have been applied by a number of researchers. Bargiela and Hainsworth [5] introduced bounds on the measurements, with the intention to increase the robustness of estimation. Brdys and Chen [6], developed a technique based on bounded states, and they introduced the term Set Bounded State Estimation (SBSE). Nagar et al. [7] applied concepts from robust control theory and allowed for uncertainty in both the parameters and the measurements. The uncertainty is isolated with the use of a Linear Fractional Transformation (LFT), which enables the preservation of the structure of the uncertainty and allows for a separate manipulation of the nominal and uncertain part. A Linear Matrix Inequalities (LMI) [8] approach is then used to solve the problem to obtain the upper and lower confidence bounds [9].

In power system state estimation, inequality constraints have been applied in optimization to deal with uncertainties. In [10], inequality constraints are employed in a LAV estimator for handling uncertainty in pseudo-measurements, since they are not measured but are known to vary within bounded intervals. An inequality constrained LAV estimator based on penalty functions, was formulated in [11] to estimate states of external systems. A parameter-bounding model derived from bounded noise measurements was used in [12] with a reformulated constrained WLS, to handle unmeasured loads in the system.

Al-Othman and Irving have introduced in [13–15] different methods for estimating the uncertainty interval around the system state variables. One method is based on using a two-step method is proposed for estimating the uncertainty interval around the system state variables. The first step uses weighted least-squares (WLS) as a point estimator to compute the expected values of the state variables. A linear programming formulation is then utilized to find the tightest possible upper and lower bounds on these estimates [13]. The linear formulation was, however, limited to modelling uncertainty only in the measurements which was due to meters inaccuracies, when in fact other elements (inaccuracies of the network mathematical model) can indeed contribute to the uncertainty. As an extension, authors in [14] have introduced another uncertainty analysis method in which the uncertainties are expressed in both measurements and network parameters. The uncertainties in [14] were assumed to be known and bounded. The problem is formulated as a constrained non-linear optimization problem. To find the tightest possible upper and lower bounds of any state variable, the problem is solved by Sequential Quadratic Programming (SQP) techniques. In [15] authors have conducted a comparison study of both methodologies presented in [13,14] in terms of accuracy in estimating the uncertainty interval with various redundancy levels. The study established that both methods provided almost identical bounds estimates. Also, the study showed that based on CPU execution time analysis WLS-LP was found to be faster than the non-linear method.

The main drawback in those formulations was the major computational burden of the process which arises from the need to perform two (LP) or two (SQP), depending on the formulation used, solutions for every uncertainty interval sought. For example, minimizing a particular state variable of interest, subject to all the measurement inequality constraints, provides the lower bound on that state variable. Likewise, maximizing that state variable, again subject to all the measurement inequalities, provides the upper bound for that state. Consequently, for real world large electrical networks that scenario introduces a significant amount of computation and CPU time, which may ultimately question the practicality of those formulations.

The proposed fuzzy linear state estimator (FLSE) has an attractive feature that combats the above drawback. The proposed (FLSE) computes the interval for all states simultaneously and directly as it converges to the optimal solution. Unlike the methods presented in [13–15] where uncertainty intervals is determined by the successive solution of a series of appropriately formulated linear or nonlinear optimization problems.

Fuzzy theory has also been widely used in power system computation. For example, Shahidehpour et al. in [16,17] have utilized Fuzzy theory to handle the uncertainty in decision making and power purchasing in deregulated environment. In [18,19] authors have applied fuzzy set for multi-area generation scheduling and for optimal reactive power control, respectively. As for state estimation the concept of fuzzy-logic has been employed by Shabani et al. in [20] to improve the over all performance of the WLS estimator. A hybrid WLS and fuzzy-logic estimator was developed in [20] to model residual based on possibility theory. Shahidehpour et al., on the other, have employed fuzzy sets in conjunction with LAV (Least Absolute value) estimator and LMS (Least Medium Squares) estimator to robustly eliminate the bad data in [21]. Furthermore, authors in [22] have developed a fuzzy LAV estimator based on maximizing the sum of individual memberships. This fuzzy LAV estimator out performed the standard WLAV in the presence of leverage point.

3. An overview of Tanaka's fuzzy linear regression

Fuzzy linear regression was introduced by Tanaka et al. [23] in 1982. The general from of Tanaka's formulation is given by:

$$Y_{\sim} = f(x) = A_0 + A_1x_1 + A_2x_2 + \ldots + A_nx_n = Ax \tag{1}$$

where $Y_{\sim}$ is the output (dependant fuzzy variable), $\{x_1, x_2, \ldots, x_n\}$ is a non fuzzy set of crisp independent parameters and $\{A_0, A_1, \ldots, A_n\}$ is a fuzzy set of symmetric members, unknowns, needs to be estimated. Each fuzzy element in that set may be represented by a symmetrical triangular membership function, shown in Fig. 1, defined by a middle and a spread values, p_i and c_i, respectively. The middle is known as the model value and the spread denotes the fuzziness of that model value.

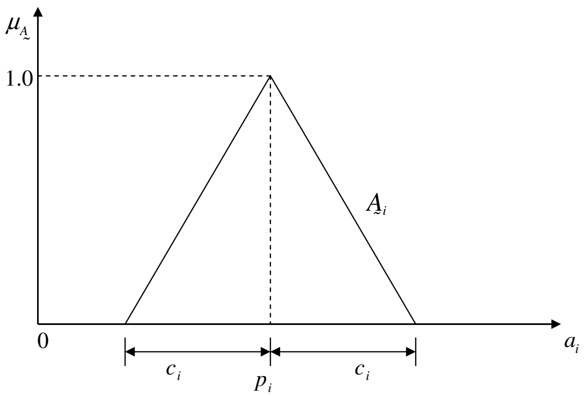

Fig. 1. Membership function of fuzzy coefficient $\underset{\sim}{A}$.

Therefore, since $A_i = (p_i, c_i)$, then Eq. (1) may be rewritten as:

$$Y_{\sim} = f(x) = (p_0, c_0) + (p_1, c_1)x_1 + (p_2, c_2)x_2 + \ldots + (p_n, c_n)x_n \tag{2}$$

The output membership function is given as:

$$\mu_{Y_{\sim}}(y_i) = \begin{cases} 1 - \frac{\left|y - \sum_{i=1}^{n} p_i x_i\right|}{\sum_{i=1}^{n} c_i |x_i|}, & x_i \neq 0 \\ 1, & x_i = 0,\ y_i = 0 \\ 0, & x_i = 0,\ y_i \neq 0 \end{cases} \tag{3}$$

The output membership function is depicted in Fig. 2, and the intermediate mathematical derivations leading to Eq. (3) can be found in [23,24].

From regression point of view, Eqs. (1)–(3) may be applied to m samples where the output can be either non-fuzzy, (certain or exact), in which no assumption of ambiguity is associated with the output or fuzzy (uncertain), where uncertainty in the output is involved due to human judgment or meters imprecision [25]. In this study both non fuzzy and fuzzy out will be considered.

3.1. Non-fuzzy output model [23]

In this model, Tanaka converted regression model into a linear programming problem [23]. In this case the objective is to solve for the best parameters, i.e. A^*, such that the fuzzy output set is associated with a membership value greater than h as in;

$$\mu_{Y_{\sim j}}(y_j) \geqslant h, \quad j = 1, \ldots, m \tag{4}$$

where $h \in [0,1]$ is the degree of the fuzziness and is normally defined by the user, $Y_{\sim} = A^*_{\sim} x_i$.

Therefore, with Eq. (4) as a condition, the main objective is to find the fuzzy coefficients that minimize the spread of all fuzzy output for all data sets. Note that the fuzziness in the output is due to fuzziness assumed in the system structure A^*. Thus, given non-fuzzy data (y_i, x_i), the fuzzy parameters $A^* = (p,c)$ may be solve for by the linear programming formulation as:

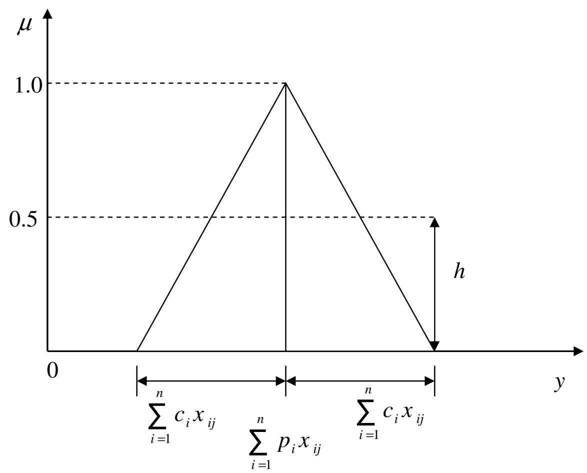

Fig. 2. Membership function of output.

$$F_{non\text{-}fuzzy} = \min\left(\sum_{j=1}^{m}\sum_{i=1}^{n} c_i x_{ij}\right) \tag{5}$$

Subject to:

$$y_j \geqslant \sum_{i=1}^{n} p_i x_{ij} - (1-h)\sum_{i=1}^{n} c_i x_{ij} \tag{6}$$

$$y_j \leqslant \sum_{i=1}^{n} p_i x_{ij} + (1-h)\sum_{i=1}^{n} c_i x_{ij} \tag{7}$$

In the above formulation y_j is the jth observation (constant), x_{ij} is a non fuzzy crisp independent parameter, p_i is the ith fuzzy middle and c_i is its corresponding symmetric spread (both are variables and need to be estimated).

Also, note that in (6) and (7), $\sum_{i=1}^{n} p_i x_{ij}$, defines the middle value and $\sum_{i=1}^{n} c_i x_{ij}$ defines the symmetric spread to the left, constraint (6), and to the right, constraint (7), as illustrated in Fig. 2. As can be seen from the Fig. 2, as $h$increases the fuzziness of the output increases. This is due to the need of a wider spread, c_i, to validate the input measured value in condition of satisfying higher h [2].

3.2. Fuzzy output model [24]

Due to human error and various other sources of imprecision in the measurements, the output may certainly be fuzzy. The uncertainty in the measurements is represented by a fuzzy member as $Y_{\sim j} = (y_j, e_j)$, where y_j is the middle value and e_j represents the uncertainty in measurement j as shown in Fig. 3.

Fig. 4 illustrates the overall membership output function that models uncertainty in the regression parameters along with the output.

The objective of fuzzy linear regression is to determine the fuzzy parameters $A^*_{\sim}$ that minimze the sum of spread as in:

$$F_{fuzzy\text{-}output} = \min\left(\sum_{j=1}^{m}\sum_{i=1}^{n} c_i x_{ij}\right) \tag{8}$$

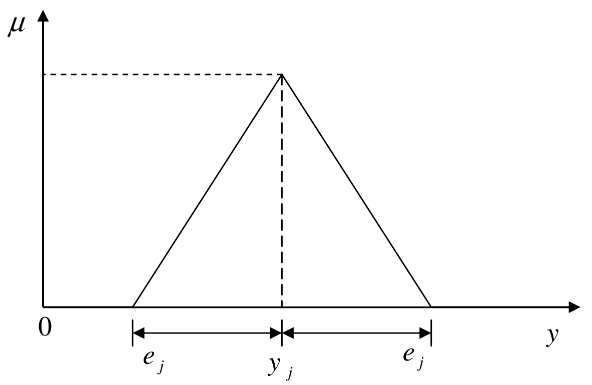

Fig. 3. Membership function of fuzzy output.

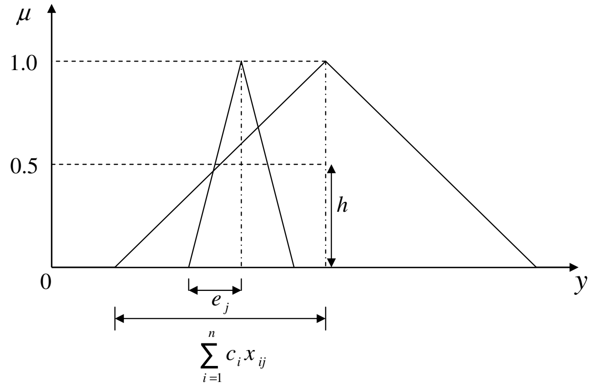

Fig. 4. Membership function of fuzzy output.

Subject to:

$$y_j \geqslant \sum_{i=1}^{n} p_i x_{ij} - (1-h)\sum_{i=1}^{n} c_i x_{ij} + (1-h)e_j \tag{9}$$

$$y_j \leqslant \sum_{i=1}^{n} p_i x_{ij} + (1-h)\sum_{i=1}^{n} c_i x_{ij} - (1-h)e_j \tag{10}$$

Note that an additional term, $(1-h)e_j$, emerged in the formulation due to the introduction of fuzziness in the measurements. As mentioned, the Eq. (9) represents the y_j when it lies in the interval to the left of the middle value with the uncertainty with respect to it added to that interval. In the same manner, Eq. (10) represents the y_j when it lies in the interval to the right of the middle value with the uncertainty with respect to it added to that interval. The proof and detailed derivation for both formulation can be found in [23,24].

4. Proposed power system linear fuzzy state estimation

For a set of measurement equations the well-known state estimation model is:

$$\underline{z} = H(\underline{x}) + \underline{\varepsilon} \tag{11}$$

where:

$\underline{z}$ is the (mx1) measurement vector.

H is a vector of non-linear functions that relate the states to the measurements.

$\underline{x}$ is an (n x1) state vector to be estimated.

$\underline{\varepsilon}$ is an (mx1) measurement error vector.

The measurements are usually obtained from transducers in the electrical network. For the system to be observable, it is necessary that $m \geqslant n$ and that the m measurements are in locations such that the resulting Jacobian (sensitivity matrix with respect to the state variables) has rank n.

For a given set of measurements, where $m \succ n$, $\underline{x}$ can not be exactly determined, instead, $\underline{x}$ can be estimated and it is denoted as $\hat{\underline{x}}$. Eq. (11) is linearized around some operating point x^o using Taylor series expansion, retaining the first two terms and ignoring the higher order terms. This leads to the following relationship:

$$\Delta \underline{z} = J(\underline{x}^o) \cdot \Delta \underline{x} + \underline{\varepsilon} \tag{12}$$

where:

$\Delta \underline{z}$ is = $\underline{z} - J(x^o)$.

J is the Jacobian of $H(\underline{x}^o)$, i.e. $J = \partial H(\underline{x}^o)/\partial \underline{x}$

$\Delta \underline{x}$ is $= \hat{\underline{x}} - \underline{x}^o$.

The Newton–Raphson method is employed as an iterative method, since it is known that power system models are amenable to solution using the N-R approach. The dependence on the iteration index is implicitly assumed for $\Delta \underline{x}$, J and $\Delta \underline{z}$, where the current state vector is updated at each iteration until a stopping criterion is reached.

The linearized power system in (12) for the Jth measurement can be rewritten as:

$$\Delta z_j = \Delta x_1 J_{j1} + \Delta x_2 J_{j2} + \ldots + \Delta x_n J_{jn} \tag{13}$$

If we define the change in the system state variables, Δx, to be a fuzzy member having a middle and a spread values, p_i and c_i, respectively. Then, Eq. (13) can be expressed as:

$$\Delta Z_j = (p_1, c_1)J_{j1} + (p_2, c_2)J_{j2} + \ldots + (p_n, c_n)J_{jn} \tag{14}$$

Note that the modal value p_i (i.e. the middle) for a given unknown, represents the value of the change in the system state variables, Δx_i, at the current iteration of the linearized model. The spread c_i on the other hand, which is symmetric, corresponds to the incremental confidence interval of that state variable. Therefore, $\Delta \underline{x}$ can be defined:

$$\Delta \underline{x} \equiv [(p_1, c_1), (p_2, c_2), \ldots (p_n, c_n)] \tag{15}$$

Tanaka's fuzzy linear regression models are modified in order to be used as state estimator instead. In those linear fuzzy formulations, the optimal state estimate vector $\hat{\underline{x}}$ may be determined by minimizing the sum of the spread of all state variables. In this case the change in state variables, subject to a number of constraint representing measurements can be expressed as:

$$F_{non\text{-}fuzzy} = \min\left(\sum_{j=1}^{m}\sum_{i=1}^{n} c_i J_{ij}\right) \tag{16}$$

Subject to:

$$y_j \geqslant \sum_{i=1}^{n} p_i J_{ij} - (1-h)\sum_{i=1}^{n} c_i J_{ij} \tag{17}$$

$$y_j \leqslant \sum_{i=1}^{n} p_i J_{ij} + (1-h)\sum_{i=1}^{n} c_i J_{ij} \tag{18}$$

Similarly, the fuzzy output model may be given as:

$$F_{fuzzy\text{-}output} = \min\left(\sum_{j=1}^{m}\sum_{i=1}^{n} c_i J_{ij}\right) \tag{19}$$

Subject to:

$$y_j \geqslant \sum_{i=1}^{n} p_i J_{ij} - (1-h)\sum_{i=1}^{n} c_i J_{ij} + (1-h)e_j \tag{20}$$

$$y_j \leqslant \sum_{i=1}^{n} p_i J_{ij} + (1-h)\sum_{i=1}^{n} c_i J_{ij} - (1-h)e_j \tag{21}$$

where h is the degree of the fuzziness and is specified by the decision maker. In the context of power system state estimation e_i may represent the transducer tolerance which is usually provided by the manufacturer of the meter it self. Both models are linear programming models and they can be solved by any linear programming package.

Repeated linearization and solution of (11) then solves the non-linear problem via the Newton–Raphson approach. The solution of the power system state estimation in equation by the proposed fuzzy linear formulation can be explained as:

Suppose that at iterations k, the state variable is updated by

$$\hat{\underline{x}}_{k+1} = \hat{\underline{x}}_k + \Delta \underline{x}_k \tag{22}$$

where the incremental change in state variable Δx_k, is computed by either fuzzy linear models above, Eqs. (16)–(18) or, Eqs. (19)–(21), and it can be expressed

$$\Delta \underline{x}_k = [p_1, p_2, \ldots, p_n]_k^T \tag{23}$$

where p_i correspond to the middle value of the incremental change of the system state variables, i.e. (voltage magnitudes and phase angles) at the at iterations k.

Since the optimal spreads represent a quantified measure of how uncertain we are about their respective middles i.e. state variables, and then the interval of confidence due to uncertainty can be constructed by adding or subtracting the spreads to or from their respective middles. For instance, the lower bound of the incremental changes at iterations k can be calculated as:

$$\Delta \underline{x}_k^- = \Delta \underline{x}_k - [c_1, c_2, \ldots, c_n]_k^T \tag{24}$$

And likewise, the upper bound of the incremental changes at iterations k can be calculated as:

$$\Delta \underline{x}_k^+ = \Delta \underline{x}_k + [c_1, c_2, \ldots, c_n]_k^T \tag{25}$$

Ultimately, the lower bound of the interval at iterations k of all states can be computed:

$$\hat{\underline{x}}_{k+1}^- = \hat{\underline{x}}_k + \Delta \underline{x}_k^- \tag{26}$$

And the similarly, the upper bound of that interval is computed as:

$$\hat{\underline{x}}_{k+1}^+ = \hat{\underline{x}}_k + \Delta \underline{x}_k^+ \tag{27}$$

Upon choosing an appropriate initial guess x^o, an arbitrary initial guess of considered state variables, N-R should iterate until the stopping criterion is reached. Thus the nonlinear problem is solved and eventually not only the states are computed by the fuzzy linear estimator but also, an uncertainty range of the state variables (voltage magnitudes and phase angles) are contracted within which we can be assured that the "true" state may lie with high confidence.

It is important to mention that the fundamental concept of power system state estimation is to determine the estimated $\hat{\underline{x}}$ which best fits the redundant set of measurements $\underline{z}$. The proposed fuzzy formulation provides the set of estimates $\hat{\underline{x}}$ (middle values) along with an upper bound of $\hat{\underline{x}}^+$ and lower bound $\hat{\underline{x}}^-$ for the estimated middle values. Determination of estimated middle values is extremely crucial. Upper and lower bounds (which are computed with the help of the spread) are added features that generally offer two relevant indications:

- An extended confidence in the particular system states estimates.
- A possible violation of some operation limits or system closeness to dangerous states, i.e. in case when the middle estimate looks acceptable, but the spread is already approaching the system limits.

4.1. Implementation of case studies

This section presents some typical results obtained by applying the proposed algorithms to the 4-bus system from [26], 6-bus test system from [27], IEEE 30-bus, IEEE 39-bus, IEEE 57-bus and IEEE 118-bus test network data from [28]. A set of MATLAB™ files has been developed to facilitate the computation of all state variables to illustrate the concepts. The LP problems have been solved by the function *linprog()* incorporated in the MATLAB™ optimization toolbox [29].

Selected measurements, (i.e. active and reactive power injections, active and reactive power flow and current magnitudes with redundancy levels ≈1.8 to 2.2), have been acquired from load-flow solution for all test cases. To simulate parametric uncertainty, elements of the admittance matrix have been perturbed by adding uniformly distributed random values to the nominal values of those elements over an interval (for example [−1%, 1%]) and therefore, approximately representing typical inaccuracies related to the acquisition and computation of network transmission lines resistances *R*, reactances *X* and the total line charging values *B* (susceptance). As a matter of fact variation or, (ambiguity), in the network parameters is mainly a function of line loading and other factors like ambient temperature and wind speed [30–32]. While measurements used for in all the test cases were acquired from base case load-flow, a 1–5% uncertainty in the parameters seems to be appropriate for the small and medium size test cases. A relatively larger uncertainty range, i.e. 7–10%, is acceptable for the IEEE 57 & IEEE118 test cases due to the increase in transmission lines and, therefore leading to an increase in the overall degree of the parameters uncertainty. Hence, all implementation based on those ranges of parameters uncertainty have been carried out and presented.

The fuzziness in the output (uncertainty in the measurements) due to meters inaccuracies is modelled in the adopted formulation by the e_i coefficient. These coefficients correspond to the overall accuracy of the meter, (such as ±3%), and can usually be provided by the manufacturer. Nonetheless, different values for the elements of positive and negative tolerances are permissible so that a transducer can be specified to have asymmetric accuracy if required (e.g. an accuracy of −3% to +5% of the nominal value) [33]. In fact, in all test cases the meters accuracies were obtained by generating normally distributed values multiplied by symmetric meter tolerances and are therefore approximately modelling unknown uncertainty in a given reading of measurement, but it is bounded between + or − the value of the meter tolerance.

This assumption corresponds to real-life situation where acquired measurement values are not exact but are contained within the range specified by the accuracy of meters. It is important to mention that the transducer tolerances e_i are assumed to be known and fixed. In realty the instrument inaccuracies will increase as the instruments age under the action of various processes and as the instruments may not be recalibrated. It should be noted that measurement recalibration is rarely carried out in a systematic manner by utilities [34,35], mainly due to the fact that large numbers of measurements exist in a power network and the time and expertise required to check each individual transducer would be expensive.

4.2. Application of FLSE on 4-bus test system

Table 1 present typical results obtained by the proposed FLSE, when applied to the 4-bus network from [26] and shown in Fig. 5. The transducer tolerance is assumed to

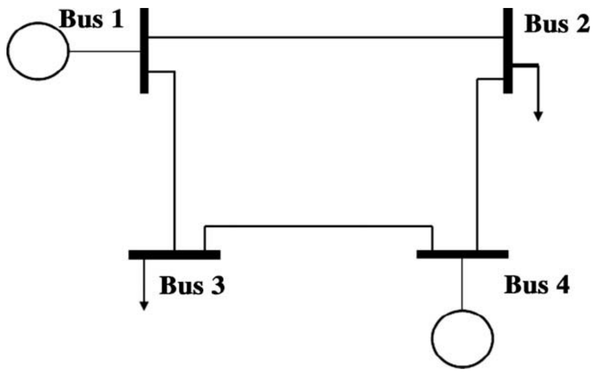

Fig. 5. Single-line diagram of 4-bus system.

Table 1
Estimated state variables and uncertainty bounds for the four-bus network with $h = 0.5$.

Bus_i	WLS		Fuzzy-LP⁻		Fuzzy-LP middle		Fuzzy-LP⁺	
	$\lvert V_i \rvert$	θ_i	$\lvert V_i \rvert$	θ_i	$\lvert V_i \rvert$	θ_i	$\lvert V_i \rvert$	θ_i
1	0.9666	0	0.9557	0	0.9666	0	0.9776	0
2	0.9476	−0.9349	0.9362	−0.9503	0.9476	−0.9349	0.9591	−0.9196
3	0.9354	−1.9639	0.9244	−1.9951	0.9354	−1.9639	0.9464	−1.9326
4	0.9870	1.7687	0.9761	1.6823	0.9870	1.7687	0.9978	1.8550

be of ±3% of nominal values, while parametric uncertainties are bounded by ±1%. For comparison purposes, the Weighted Least Squares (WLS) estimates have been obtained for the perturbed set of measurements. Note that the estimated center points of the interval, (Fuzzy-LP Middle), obtained by the FLSE are identical to those obtained by WLS estimates. This particular outcome is expected since the FLSE aims to minimize the sum of spreads of all fuzzy parameters and states which is analogous to the least squares criterion [2]. It must be stressed that no generalization may be made based upon this outcome, particularly if WLS assumptions (normality and homogeneity of error terms) are violated [2].

As for the uncertainty interval, the estimated upper and lower bound are shown in Table 1, where the estimated center points appear to be fuzzy, (uncertain) or non crisp (a crisp estimate occurs when its corresponding spread or width is 0), since during convergence a spread was produced and therefore an upper bound and a lower bound have been eventually produced by Eqs. (24–27). In this particular test, if the supplied set of measurements is noise-free, the FLSE spread will definitely be zero indicating no uncertainty in the estimates. It is also apparent that the estimated center points lie exactly in the middle of the confidence interval. This particular outcome is expected since a symmetric spread was adopted by FLSE to model the uncertainties.

The FLSE has been found to perform reliably, with convergence occurring in 4 iterations. This is consistent with the behaviour of the Newton Raphson process in solving other types of power system state estimation problems. Furthermore, the execution time was found to be 0.4490 s, see Table 4. With the same initial guess the state estimation problem was solved for the 4-bus system by (WLS). Table 4 shows that WLS required 5 iterations to converge (with the same tolerance of 10^{-7}) and considerably less CPU time.

4.3. Application of FLSE on 6-bus test system

Table 2 shows the fuzzy state estimates for the 6-bus network from [27] and shown in Fig. 6, where the transducer tolerance is assumed to be of ±3% of nominal values, while parametric uncertainties are bounded by ±1%. The algorithm converged in 3 iterations, with execution time 0.4106 s, see Table 4.

4.4. Application of FLSE on IEEE 30-bus test system

Table 3 shows the fuzzy state estimates for the IEEE30-bus network from [28], where the transducer tolerance is assumed to be of ±3% of nominal values, while parametric uncertainties are bounded by ±1%. The algorithm converged in 3 iterations, with execution time 0.6474 s see Table 4.

4.5. Application of FLSE on IEEE 39-bus test system

The proposed FLSE has been applied to the IEEE39-bus network, from [28], where the transducer tolerance is assumed to be of ±5% of nominal values, while parametric uncertainties are bounded by ±3%. The algorithm converged in 3 iterations with execution time 0.7111 s, see Table 4.

4.6. Application of FLSE to IEEE 57-bus and 118-bus test systems

The proposed Fuzzy LP algorithm has been applied on the IEEE57-bus and IEEE118-bus systems. The CPU time as well as the number of iterations required for convergence of the IEEE57-bus and IEEE118-bus systems is shown in Table 4.

Note that in this study it is found that the degree of fuzziness h seems to have no significant effect on the computation of spreads c_i, which appears to be rather counter intuitive. One reason is due to the fact that having to estimate incremental changes of state variable (in the linearized domain) that are relatively very small. Had there been any change in the value of the degree of fuzziness h prior the estimation at any given iteration, this would yield a very small change in the values of spreads c_i, (to the order of 10-5). This small change is really insignificant and is likely to be trivial in the computation of the final incremental changes of the spreads c_i.

4.7. Discussion and results analysis

Based on the time performance shown in the previous section in Table 4, the proposed fuzzy LP estimator was found to converge in either one less or an equal number of the conventional WLS. On the other hand, the CPU execution time of the fuzzy LP estimator required for convergence is relatively higher that WLS estimator. This slightly more CPU time of the fuzzy LP may be attributed to having to solve a constrained state estimation linear programming problem, where each measurement considered in the estimation process is represented by two constraints in the fuzzy domain. This in turn leads to the construction of $2m$ constraints leading to a slightly more computational

Table 2
Estimated state variables and uncertainty bounds for the six-bus network with $h = 0.5$.

Bus_i	WLS		Fuzzy-LP$^-$		Fuzzy-LP middle		Fuzzy-LP$^+$	
	$\lvert V_i\rvert$	θ_i	$\lvert V_i\rvert$	θ_i	$\lvert V_i\rvert$	θ_i	$\lvert V_i\rvert$	θ_i
1	0.9922	0	0.9735	0	0.9922	0	1.0110	0
2	0.9901	−3.8625	0.9706	−3.9269	0.9901	−3.8625	1.0096	−3.7980
3	1.0120	−4.5438	0.9930	−4.6370	1.0120	−4.5438	1.0309	−4.4507
4	0.9258	−4.5463	0.9050	−4.6692	0.9258	−4.5463	0.9466	−4.4233
5	0.9217	−5.7330	0.9009	−5.8938	0.9217	−5.7330	0.9426	−5.5722
6	0.9383	−6.4765	0.9165	−6.6631	0.9383	−6.4765	0.9600	−6.2898

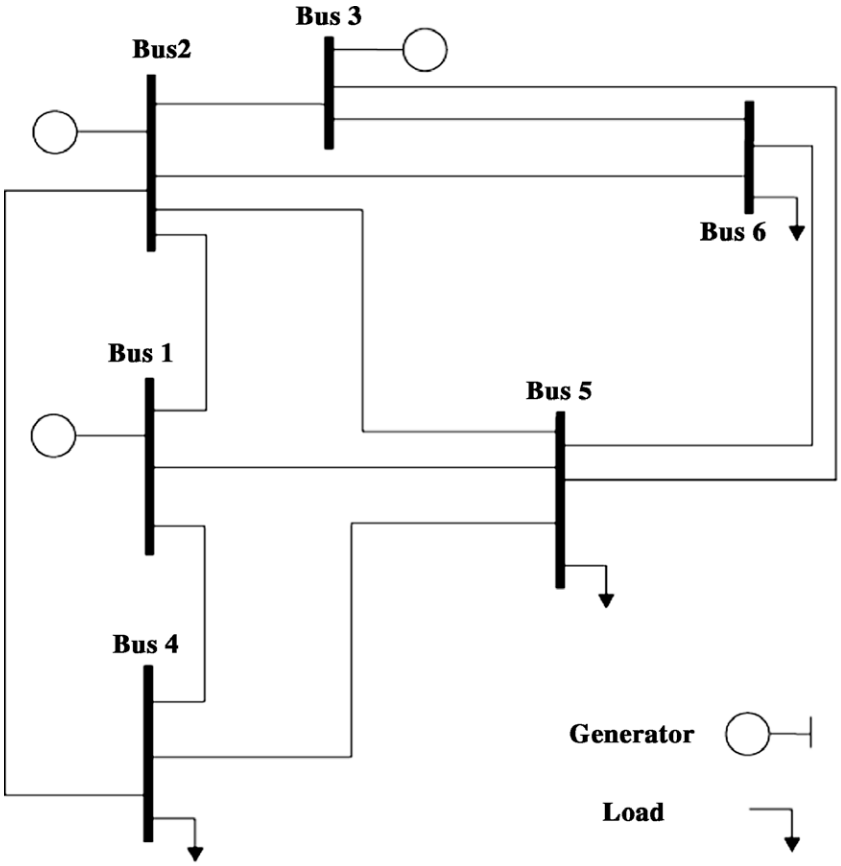

Fig. 6. Single-line diagram of 6-bus system.

effort than the conventional WLS estimator. Nonetheless, looking at the CPU time required for execution of test systems considered in this study, it can be said that the proposed fuzzy estimator posses no significant computational burden. Furthermore, for improved computational efficiency, particularly for large actual systems, the dual formulation may be employed [36]. Tanaka et al., in [24], derived the dual formulation of their primal formulation presented in Eqs. 7,8,9 or 12,13,14, where the number of equations is related to the number variables, n, as opposed to the number of constraints, i.e. $2m$.

4.8. Advantages and practicalities

The availability of the upper and lower bounds on state estimates can have practical advantages for the power system operator. For critical quantities, such as a power flow which is close to its thermal, stability or contractual limit, the operator can gain confidence that the true value is not exceeding the constraint provided that the state estimate and both bounds are all within the limit. The uncertainty range on the estimate also gives a useful indication of the quality of the metering configuration for the relevant part of the power system. For example, where a voltage level often has a wide estimated uncertainty range, this would suggest that the metering in that area is insufficient. This type of additional information could be very useful during the installation or upgrading of an online state estimator. In addition with the introduction of parametric variation in the formulation, a more realistic and accurate uncertainty range is attainable now about the different system quantities.

Some other advantages of FLSE are (as apposed to other conventional estimators):

- It has the ability to provide "interval" estimation rather than "point" estimation.
- Execution time is reasonable, which makes the proposed FLSE efficient and applicable to large electric networks.
- The FLSE is able to provide direct estimates along with their bound without having to relay on any other estimator i.e. WLS or LAV as an intermediate stage.
- The FLSE is more suitable for uncertainty modelling and analysis due to its possibilistic nature.
- Parametric uncertainty may be introduced and modeled along with the measurement uncertainty in a combined framework. As a consequence, computation of more realistic analysis of the bounds is possible.

However, the FLSE may have the following disadantages:

- Linearization is needed. Therefore, the construction of Jacobian is required in every iteration of N-R.

Table 3
Estimated state variables and uncertainty bounds for the 30-bus network with $h = 0.5$.

Bus_i	WLS		Fuzzy-LP$^-$		Fuzzy-LP middle		Fuzzy-LP$^+$	
	$\lvert V_i \rvert$	θ_i	$\lvert V_i \rvert$	θ_i	$\lvert V_i \rvert$	θ_i	$\lvert V_i \rvert$	θ_i
1	1.0696	0	1.0405	0	1.0696	0	1.0986	0
2	1.0719	−0.0943	1.0422	−0.1871	1.0719	−0.0943	1.1017	−0.0014
3	1.0601	−0.9809	1.0288	−1.1479	1.0601	−0.9809	1.0914	−0.8139
4	1.0567	−1.2286	1.0255	−1.4056	1.0567	−1.2286	1.0879	−1.0517
5	1.0504	−1.0305	1.0218	−1.2822	1.0504	−1.0305	1.0791	−0.7789
6	1.0493	−1.5273	1.0182	−1.7576	1.0493	−1.5273	1.0805	−1.2970
7	1.0407	−1.7776	1.0104	−2.0500	1.0407	−1.7776	1.0710	−1.5052
8	1.0382	−1.9067	1.0066	−2.1644	1.0382	−1.9067	1.0698	−1.6489
9	1.0575	−2.3387	1.0263	−2.5542	1.0575	−2.3387	1.0888	−2.1232
10	1.0585	−2.9667	1.0282	−3.1171	1.0585	−2.9667	1.0889	−2.8162
11	1.0670	−2.2262	1.0329	−2.4723	1.0670	−2.2262	1.1011	−1.9802
12	1.0651	−1.5697	1.0331	−1.5770	1.0651	−1.5697	1.0971	−1.5625
13	1.0799	0.8656	1.0479	0.6788	1.0799	0.8656	1.1119	1.0524
14	1.0564	−1.9683	1.0243	−2.0865	1.0564	−1.9683	1.0884	−1.8501
15	1.0585	−2.4010	1.0268	−2.4020	1.0585	−2.4010	1.0901	−2.4000
16	1.0559	−2.3555	1.0242	−2.4643	1.0559	−2.3555	1.0876	−2.2467
17	1.0516	−2.9974	1.0210	−3.1444	1.0516	−2.9974	1.0821	−2.8504
81	1.0459	−3.7323	1.0144	−3.7621	1.0459	−3.7323	1.0773	−3.7024
19	1.0436	−4.3441	1.0119	−4.4041	1.0436	−4.3441	1.0753	−4.2841
20	1.0480	−4.2075	1.0162	−4.2551	1.0480	−4.2075	1.0798	−4.1598
21	1.0643	−2.9044	1.0350	−3.1048	1.0643	−2.9044	1.0936	−2.7039
22	1.0708	−2.8385	1.0416	−3.0295	1.0708	−2.8385	1.1000	−2.6475
23	1.0793	−1.8738	1.0475	−1.9351	1.0793	−1.8738	1.1112	−1.8124
24	1.0665	−2.3494	1.0351	−2.4557	1.0665	−2.3494	1.0980	−2.2431
25	1.0762	−0.7157	1.0424	−0.9989	1.0762	−0.7157	1.1100	−0.4325
26	1.0638	−1.2811	1.0282	−1.5384	1.0638	−1.2811	1.0993	−1.0238
27	1.0868	0.3060	1.0528	−0.0102	1.0868	0.3060	1.1208	0.6223
28	1.0515	−1.4348	1.0202	−1.6898	1.0515	−1.4348	1.0828	−1.1799
29	1.0827	−0.6183	1.0438	−1.0490	1.0827	−0.6183	1.1216	−0.1876
30	1.0785	−1.5834	1.0373	−2.0106	1.0785	−1.5834	1.1197	−1.1562

Table 4
CPU and execution time (CPU: Pentium 4, 1.7 GHZ).

Test system	Fuzzy LP		WLS	
	# iterations	CPU time (s)	# iterations	CPU time (s)
Four-bus	4	0.4490	5	0.21252
Six-bus	3	0.4106	3	0.17996
IEEE 30-bus	4	0.6474	4	0.28292
IEEE 39-bus	3	0.7111	4	0.27158
IEEE 57-bus	3	0.8595	3	0.16113
IEEE 118-bus	3	1.7449	3	0.36153

- Bounds are always symmetric which is due to the triangular membership functions adopted by the FLSE. Representing the uncertainties by an asymmetric membership function would provide a better estimation of the bounds.
- The FLSE is unable to handle faulty measurements and outliers. The FLSE is very sensitive to outliers [1,37]. Based on this fact, it is expected that the FLSE would produce deceptive estimation of the center points and their respective upper and lower bounds.

In general, the *Breakdown Point*[1] is a known concept used to quantify the robustness of an estimator which was introduced by Donoho and Huber in [38]. In theory the highest breakdown point one can achieve is 0.5 (or 50%) because for any higher contamination level, one is not guaranteed to be able to distinguish the good points from the bad.

Based on experimentation, FLSE was found to fail with a single outlier in the measurement set and therefore leading to having a 0% Breakdown Point (Note that both the Least squares (LS) and the Least absolute value (LAV) also have 0% Breakdown Point [39] and may fail with a single outlier). That breakdown percentage clearly shows how vulnerable the FLSE is to outliers. Nonetheless, this weakness may be overcome by using a high breakdown point static estimator, (such as Least Median Squares (LMS) or Least Trimmed Squares (LTS) [39–41]), where any outliers would be identified and eliminated from the measurement set, prior to the estimation process for the uncertainty bounds.

5. Conclusion

An analysis of uncertainty in power system state estimation is presented in this paper. The uncertainty is modelled and is assumed to be present in the system parameters and in the measurements which take into account known meter accuracies. A Fuzzy linear estimator was employed to estimate the both the states and their respective upper and lower bounds. The provision of bounds by the proposed FLSE offers useful additional information to the power system operator. By examining bounds on the estimates one can infer the quality of the metering configuration and

[1] *Breakdown Point* may be defined as the smallest fraction of contaminations that critically offsets the estimator from the true measurements [1,5].

determine the proximity of estimated quantities to voltage and flow limits with greater confidence.

When applied and tested on various standard systems, the proposed estimator can be considered as a very efficient tool in estimating the unknowns and their confidence interval due to uncertainty and imprecision. Based on the convergence and the time assessment, the advocated estimator proved to be could be used as a valuable on-line tool for power system state estimation.

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