7. Describing motion, part III: Acceleration

If the velocity of a motion changes from one moment to another, the rate of change is also a vector quantity, called acceleration a:

$$a = \frac{v - u}{t}$$

where u = initial velocity (sometimes the symbol v_0 is used), v = final velocity and t = the time it took for the velocity to change from u till v. If $u > v$ then the acceleration is negative (also called retardation or deceleration). The unit of acceleration is **1 m/s²** (this is mathematically the same as 1 m/s per second: 1 (m/s)/s = 1 m/(ss) = 1 m/s^2).

Example: If a car accelerates from 0 m/s to 28 m/s (approx. 100 km/h) in 12 seconds, then its (mean) acceleration is

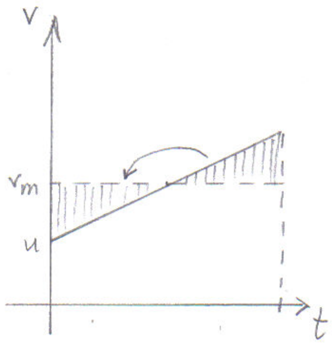
$$a = \frac{v-u}{t} = \frac{28-0}{12} = 2.3\ m/s^2$$

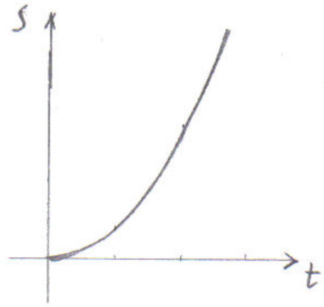
If the acceleration is the same over a period of time, we talk about *uniformly accelerated motion*. (UAM).

The equation above can be rewritten as

$$v = u + at$$

This can equivalently be written as $v = at + u$, which in a (t,v)-graph corresponds to a line like, e.g., $y = 2x + 3$ where the gradient 2 shows how steep the line is (corresponds to the acceleration a here) and the number 3 shows the point where the line passes through the y-axis (the y-intercept, which corresponds to the initial velocity u here):





Exactly as for uniform motion, the area under the graph corresponds to the displacement. For uniform motion, the area was a rectangle, but here it is a trapezoid. Its area is easily calculated by noting that the area of the trapezoid is the same as the area of the rectangle one gets by moving the shadowed triangle. The result will use the mean velocity v_m

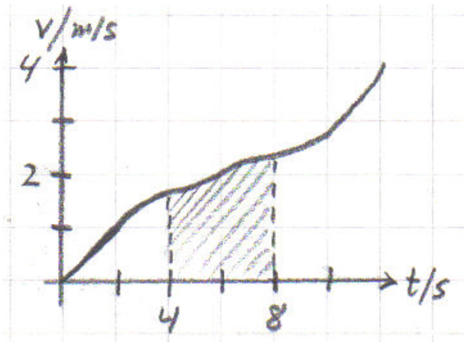
$$v_m = \frac{u + v}{2}$$

The total displacement (the area of the trapezoid) is then

$$s = v_m t = \frac{(u + v) \cdot t}{2}$$

The graph to the right shows s as a function of t in uniformly accelerated motion. Since the velocity changes all the time, we get a curve that becomes steeper all the time. Thus, it cannot be a straight line, but an upward bending curve (for positive acceleration). Mathematically, such a shape is described by squaring the x-variable (a parabola).

Example: The graph to the right illustrates the velocity of a toy car moving along a track. The velocity rises from 0 m/s to 1 m/s during the first 2 seconds, so the average acceleration during that time is ½ = 0.5 m/s^2. The area under the curve between t = 4 s and t = 8 s is approximately 4 "squares" (and every square is 2 s × 1 m/s = 2 m) so the displacement during those 4 seconds is 4×2 = 8 m.



Equation of free fall

An object in free fall near the surface of the Earth will accelerate with acceleration g = 9.8 m/s². The distance that the object falls in time t, starting from rest, can be calculated from the formula

$$s = \frac{1}{2}gt^2$$

if the air resistance is so small that it can be ignored. In realistic situations, air resistance can be ignored if the object is small and heavy, and the velocity is not very big.

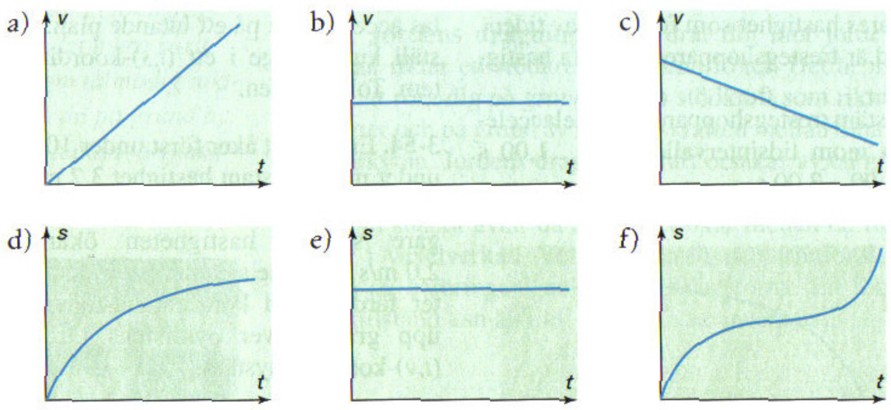
Example: If a stone is dropped from a rooftop, and it takes 2.5 seconds for it to reach the ground, then the height of the building is

$$s = \frac{1}{2}gt^2 = \frac{1}{2} \times 9.81 \frac{m}{s^2} \times (2.5\ s)^2 = 30.624\ m \approx 31\ m$$

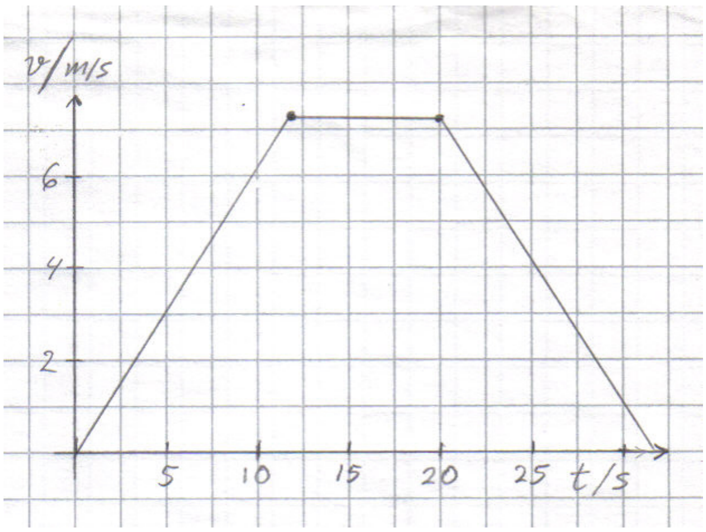
Note that the acceleration of free fall is independent of the mass of the object. In practice, air resistance will affect different objects differently – the formula considers the ideal case when there is no air resistance at all.

Exercises

7.1 Describe in words the motions illustrated by the six diagrams below.



7.2 (a) Describe in words the motion described by the graph below.
(b) The graph describes the motion of a ride at a fun fair. Can you guess what ride it is?



7.3 A swimmer in a 50 m race had the following times during her race:

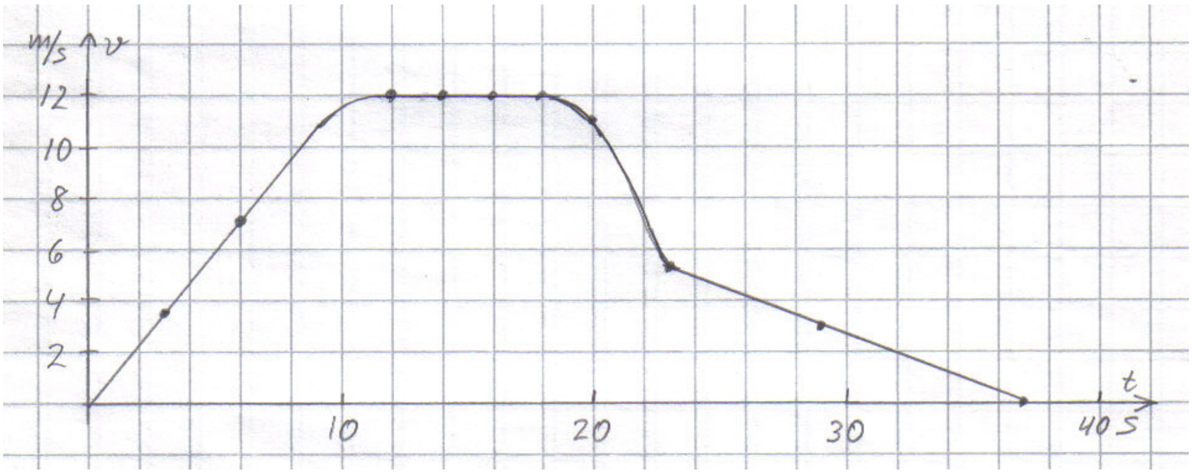
s/m	0	5	10	20	30	40	50
t/s	0.0	1.4	3.9	9.4	16.8	24.9	32.6

(a) Plot the data points in a (t,s) graph.
(b) Create a new table with the swimmer's mean velocities in the intervals 0..10 s, 10..20 s etc.
(c) Sketch a (t,v) graph
(d) Why does the swimmer's velocity vary the way it does?

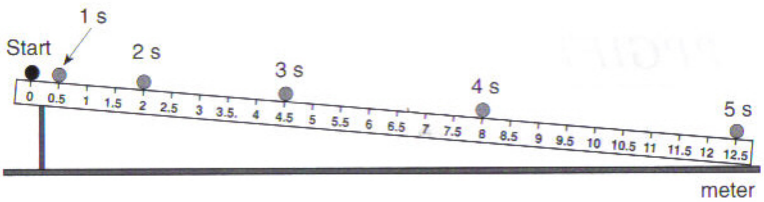
7.4 If a car goes from 0 km/h to 100 km/h in seven seconds, what is its acceleration (in m/s^2)? How many ***g*** is this, when ***g*** = 9.8 m/s^2?

7.5 The graph below describes a car starting at a traffic light.

(a) Interpret the graph in words.
(b) What is the average acceleration of the car in the interval 0..10 s?
(c) What is the average acceleration of the car in the interval 18..24 s?
(d) How far does the car travel during the interval 12..18 s?
(e) How far does the car travel during the interval 0..10 s?



7.6 The picture below illustrates an investigation of how a marble accelerates on an inclined plane. Plot the motion in a time-distance graph and interpret it.



7.7 A stone falls from a rooftop 46 m above the ground (starting from rest).

(a) How long does it take before it hits the ground?
(b) What is its velocity when it touches down?

7.8 A stone with mass 200 g is thrown straight up with initial speed 25 m/s.

(a) How high does it reach (air resistance is ignored)?
(b) How long does it take before it hits the ground?