Part III. Sources of market power

Chapter 5. Product differentiation

Slides

Industrial Organization: Markets and Strategies

Paul Belleflamme and Martin Peitz, 2d Edition

- Where does the market power come from?
  - **Consequence of firms' conduct**
    - Marketing mix: **P**rice - **P**roduct - **P**romotion
    - **P**roduct $\rightarrow$ Closer look at different types of product differentiation $\rightarrow$ Chapter 5
    - **P**romotion $\rightarrow$ Advertising strategies $\rightarrow$ Chapter 6
    - **P**rice $\rightarrow$ Discrimination (Part IV)
  - **Consumer inertia (skipped)** $\rightarrow$ Chapter 7
    - Search costs, switching costs, behavioural issues (status-qua bias) etc…

Chapter 5. Learning objectives

- Understand that product differentiation involves two conflicting forces: it relaxes price competition, but it may reduce the demand that the firm faces (niche market).
- Distinguish between horizontal (location) and vertical (quality) product differentiation.
- Reconsider the question of entry into product market.
- Discuss some basic approaches to estimate differentiated product markets.

- Horizontal product differentiation
  - Each product would be preferred by some consumers, depending on their tastes.
- Vertical product differentiation
  - Everybody would prefer one over the other product.
- More formally: if, at equal prices,
  - consumers do not agree on which product is the preferred one $\rightarrow$ products are *horizontally* differentiated;
  - all consumers prefer one over the other product $\rightarrow$ products are *vertically* differentiated.

Location choice with fixed price

- Suppose constant price (e.g., regulated price): $\bar{p} > c$
- Product positioning: Two firms choose where to locate their product in "linear city": $l_1, l_2 \in [0,1]$.
- Consumers are as in the earlier Hotelling model:
  - uniformly distributed on $[0,1]$; location represents the ideal point in product space; linear transportation cost.
  - Need to buy one unit from one of the firms.
  - $u_x(i, p_i)$ is the utility of buying from firm i at the price p_i for the consumer located at the point x:

$$u_x(i, p_i) = r - \tau|x - l_i| - p_i$$

Location choice with fixed price

- Since $p_1 = p_2$, each consumer selects the nearest firm.
- If $l_i < l_j$, there is a unique indifferent consumer:

$$\hat{x} = \frac{l_1 + l_2}{2}$$

$\rightarrow \quad Q_i = \hat{x} = \frac{l_1 + l_2}{2}$ (demand for firm i)

$\rightarrow \quad Q_j = 1 - \hat{x} = 1 - \frac{l_1 + l_2}{2}$ (demand for firm j)

- If $l_1 = l_2$, firms share the market equally: $Q_1 = Q_2 = \frac{1}{2}$
- Firm i's problem: Given l_j,

$$\max_{l_i \in [0,1]} \pi_i(l_1, l_2) = (\bar{p} - c) Q_i(l_1, l_2)$$

Location choice with fixed price

- It follows that:

$$\pi_i(l_1, l_2) = \begin{cases} (\bar{p} - c)(l_1 + l_2)/2 & \text{if } l_i < l_j, \\ (\bar{p} - c)/2 & \text{if } l_i = l_j, \\ (\bar{p} - c)[1 - (l_1 + l_2)/2] & \text{if } l_i > l_j. \end{cases}$$

Note:

- For $l_i < l_j$, π_i is increasing with l_i. Getting closer to firm j brings more customers from the right side without losing anybody from the left side.
- Similarly, when $l_i > l_j$, firm i has an incentive to get closer to the other firm by moving leftward.

$\rightarrow$ There is no Nash equilibrium with $l_1 \neq l_2$.

Location choice with fixed price

- $l_1 = l_2 = 1/2$ is an equilibrium because

$$\pi_i\left(l_i, \frac{1}{2}\right) \leq \frac{\bar{p}-c}{2} = \pi_i\left(\frac{1}{2}, \frac{1}{2}\right) \qquad \forall l_i \in [0,1].$$

- $l_1 = l_2 \neq 1/2$ is not an equilibrium.
  - For example, if $l_1 = l_2 < 1/2$, both firms get half of the market. Any firm i can move slightly rightward, to $l_i + \varepsilon$, so as to increase its market share to $\cong 1 - l_i > 1/2$.

- **CONCLUSION:** In the unique equilibrium, both firms select the midpoint; there is no product differentiation.

Location choice with fixed price

- **Lesson**: If duopolists were not able to entertain distinct prices, they would offer the same product. This is because differentiation reduces the demand for a given product by effectively targeting a smaller niche in the market.

Socially Efficient Locations:

- Minimize total distance:

$$\min \int_0^{l_1} (l_1 - x)dx + \int_{l_1}^{\frac{l_1+l_2}{2}} (x - l_1)dx + \int_{l_2}^{1} (x - l_2)dx + \int_{\frac{l_1+l_2}{2}}^{l_2} (l_2 - x)dx$$

s.t. $(l_1, l_2) \in [0,1]^2$ and $l_1 \leq l_2$.

- Solution: $l_1 = 1/4$ and $l_2 = 3/4$

$\rightarrow$ Insufficient differentiation in equilibrium.

Hotelling model (full version)

- Firms choose location **and** price.
- 2 stage model
  1. Location choice (long term decision)
  2. Price choice (short term decision)
- We already studied (in Chapter 3) the price stage with extreme locations (i.e., 0 and 1).
- We will "repeat" (not really) the analysis for any pair of locations under two different scenarios:
  - Linear transportation costs
  - Quadratic transportation costs

Linear Hotelling model

- As before, consumers are distributed on $[0,1]$ with the utility function:

$$u_x(i, p_i) = r - \tau|x - l_i| - p_i$$

  - If both products are identical, firms share the market equally. Otherwise, there is **at most** one indifferent consumer.

- Firms:
  - Choose first l_i in $[0,1]$ and then p_i
  - Move simultaneously in both stages.

- Look for subgame perfect equilibria (backward induction).
  - First step: Fix l_1 and l_2. Analyse the ensuing price stage.

Linear Hotelling model (cont'd)

- Price stage:
  - If $l_1 = l_2$, we are back to Bertrand: The cheaper firm gets the whole market.

    $\rightarrow \; p_1 = p_2 = c$
  - If $l_1 < l_2$, depending on prices, either there is an indifferent consumer **between the two locations**, or all consumers select the same firm:

$$r - \tau(\hat{x} - l_1) - p_1 = r - \tau(l_2 - \hat{x}) - p_2$$

$$\rightarrow \quad \hat{x} = \frac{l_1 + l_2}{2} - \frac{p_1 - p_2}{2\tau} = \frac{l_1 + l_2}{2} + \frac{p_2 - p_1}{2\tau}$$

So,
$$\hat{x} \geq l_1 \;\leftrightarrow\; p_1 - p_2 \leq \tau(l_2 - l_1)$$
$$\hat{x} \leq l_2 \;\leftrightarrow\; p_1 - p_2 \geq -\tau(l_2 - l_1)$$

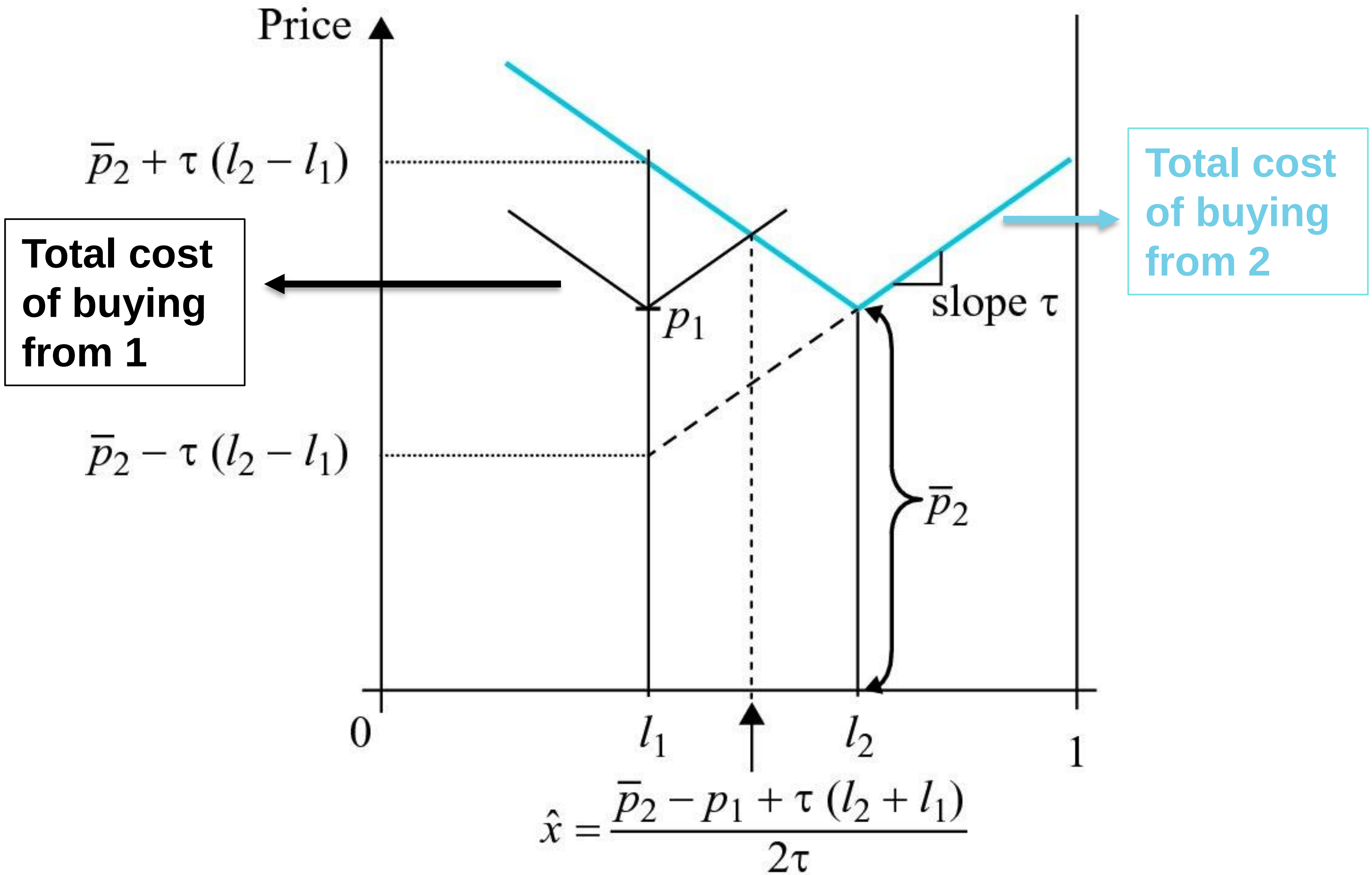
Linear Hotelling model (cont'd)

- If price difference is "*not too large*" relative to locations, meaning that $|p_1 - p_2| \leq \tau(l_2 - l_1)$, then there is an indifferent consumer given by:

$$\hat{x} = \frac{l_1 + l_2}{2} - \frac{p_1 - p_2}{2\tau}$$

- Otherwise, i.e., if $|p_1 - p_2| > \tau(l_2 - l_1)$, the cheaper firm gets the whole market. (Note the role of linear costs here.)

Linear Hotelling model (cont'd)



Linear Hotelling model (cont'd)

- Profit of firm 1 (assuming $l_1 < l_2$, given p_2):

$$\pi_1(p_1 p_2; l_1, l_2) = \begin{cases} 0 & \text{if } p_1 > p_2 + \tau(l_2 - l_1), \\ (p_1 - c)\left(\frac{l_1 + l_2}{2} + \frac{p_2 - p_1}{2\tau}\right) & \text{if } |p_1 - p_2| \leq \tau(l_2 - l_1), \\ (p_1 - c) & \text{if } p_1 < p_2 - \tau(l_2 - l_1). \end{cases}$$

- **Note:**

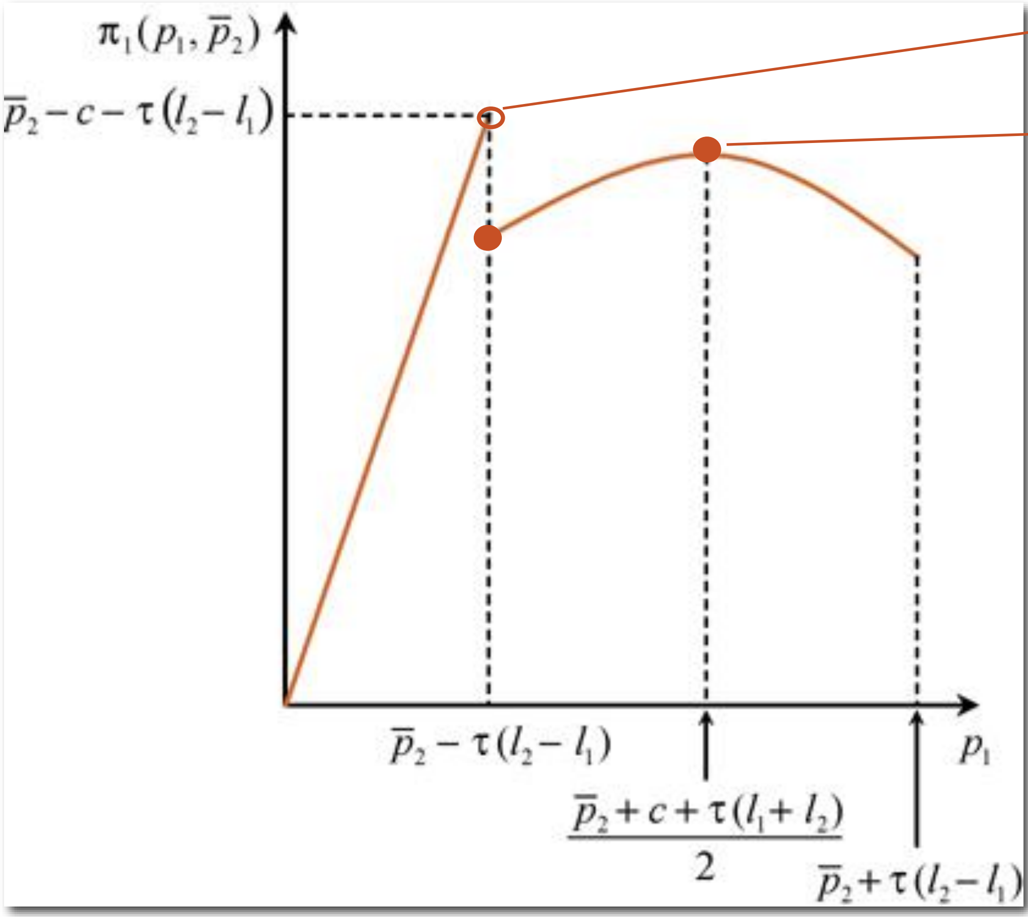
$$p_1 = p_2 - \tau(l_2 - l_1) \rightarrow \pi_1 = (p_1 - c)l_2$$

$$< (p_1 - c)$$ (unless $l_2 = 1$)

$\rightarrow$ Discontinuity at $p_1 = p_2 - \tau(l_2 - l_1)$

Intuition: Firm 2 has zero demand if it can't attract the consumer located at l_2. Otherwise, its demand is at least $1 - l_2$, because any $x \geq l_2$ buys from 2. So, Q_2 falls from $1 - l_2$ to 0, suddenly, at a certain level of p_1.

Linear Hotelling model (cont'd)



A sharp fall in π_1

Local max. Won't be a global max when the arc is narrow, and closer to the point of discontinuity, on the left.

Price equilibrium may fail to exist

Happens when locations selected in stage 1 are too close, so that the arc is narrow.

Linear Hotelling model (cont'd)

- Price equilibrium fails to exist for some pairs of location $\rightarrow$ no subgame perfect equilibrium
- Recall that if firms don't expect a price difference in stage 2, they would select the same location in stage 1 to maximize their demand. But when the locations are truly close, there is no price equilibrium. So, firms may indeed want to move towards a zone where price equilibrium does not exist.
  - Instability in competition

- **Lesson**: Although product differentiation relaxes price competition, firms may have an incentive to offer better substitutes to generate more demand, which may lead to instability in competition.

Quadratic Hotelling model

- Transport costs increase quadratically:

$$u_x(i, p_i) = r - \tau(x - l_i)^2 - p_i$$

- Suppose $l_1 < l_2$. Since $x \to \tau(x - l_2)^2$ is strictly convex, even if $x = l_2$ selects firm 1, consumers further on the right may select firm 2 because the additional distance to firm 1 will be costlier for them.
  - Formally, for $l_1 < l_2$ and $x \geq l_2$, the added cost of traveling to the distant firm l_1 increases with x:

$$\frac{d}{dx}[(x - l_1)^2 - (x - l_2)^2] = 2(l_2 - l_1) > 0.$$

  - $\rightarrow$ No discontinuity in demand, in the price stage.

Quadratic Hotelling model (cont'd)

- **Price Stage:**
  - Indifferent consumer (assuming $l_1 < l_2$):

$$r - \tau(x - l_1)^2 - p_1 = r - \tau(x - l_2)^2 - p_2 \quad \leftrightarrow$$

$$\tau[(x - l_2)^2 - (x - l_1)^2] = p_1 - p_2 \quad \leftrightarrow$$

$$\tau[(2x - (l_1 + l_2)) * (l_1 - l_2)] = p_1 - p_2 \quad \leftrightarrow$$

$$\rightarrow \qquad \hat{x} = \frac{l_1 + l_2}{2} + \frac{p_2 - p_1}{2\tau(l_2 - l_1)}$$

  - So, $\hat{x} \uparrow$ with $p_2 - p_1$, given $l_1 < l_2$
  - If $\hat{x} > 1$, we must set $Q_2 \equiv 0$, and similarly for $\hat{x} < 0$.
    - Nevermind: Equilibrium prices will be such that $\hat{x} \in [0,1]$.

Quadratic Hotelling model (cont'd)

- **Price Stage** (cont'd):
  - Assuming $l_1 < l_2$, the firms' problem are:

$$\max_{p_1 \geq 0}(p_1 - c)\hat{x}(p_1, p_2) \text{ and } \max_{p_2 \geq 0}(p_2 - c)\big(1 - \hat{x}(p_1, p_2)\big)$$

$$\rightarrow \quad \begin{aligned} p_1^* &= c + \frac{2\tau}{3}(l_2 - l_1)\left(1 + \frac{l_1 + l_2}{2}\right) \\ p_2^* &= c + \frac{2\tau}{3}(l_2 - l_1)\left(2 - \frac{l_1 + l_2}{2}\right) \end{aligned} \qquad (1)$$

  - **Note:** Holding constant the midpoint $\frac{l_1 + l_2}{2}$, both prices are increasing with $l_2 - l_1$.
    - Illustrates how product differentiation helps relax the price competition.
  - **Note:** Eqn (1) is also valid with $l_1 = l_2$ (Bertrand)

Quadratic Hotelling model (cont'd)

- **Location Stage:**
  - Remember, however, closer locations increase the demand with fixed prices. At the location stage, firms need to take into account both effects simultaneously.
  - Subsitute eqn (1) into profits, and the definition of $\hat{x} \rightarrow$

$$\begin{aligned}\hat{\pi}_1 &= \tfrac{1}{18}\tau(l_2 - l_1)(2 + l_1 + l_2)^2 \\ \hat{\pi}_2 &= \tfrac{1}{18}\tau(l_2 - l_1)(4 - l_1 - l_2)^2\end{aligned} \quad \rightarrow \quad \begin{aligned}&\partial\hat{\pi}_1 / \partial l_1 < 0 \text{ for all } l_1 \in [0, l_2) \\ &\partial\hat{\pi}_2 / \partial l_2 > 0 \text{ for all } l_2 \in (l_1, 1]\end{aligned}$$

  - Subgame perfect equilibrium: firms locate at the extreme points $\rightarrow$ "maximum differentiation"
    - The dominant force here is to relax price competition.

Quadratic Hotelling model (cont'd)

- But this is just a particular example. Different results obtain:
  - if we remove the boundaries, 0 and 1. (Optimal locations will be $-1/4$ and $5/4$.)
  - if we select a non-uniform distribution for the consumers.
  - if we select a different function for cost of traveling.

Quadratic Hotelling model (cont'd)

- **General Conclusions:**
  - 2 forces at play
    - **Competition effect** $\rightarrow$ differentiate to enjoy market power $\rightarrow$ drives competitors apart
    - **Market size effect** $\rightarrow$ meet consumers preferences $\rightarrow$ brings competitors together
    - Balance (equilibrium)depends on distribution of consumers, shape of transportation costs function and feasible product range

- **Lesson**: With endogenous product differentiation, the degree of differentiation is determined by balancing
  - the competition effect (drives firm to $\uparrow$ differentiation)
  - the market size effect (drives firm to $\downarrow$ differentiation).

Vertical product differentiation

- All consumers agree that one product is preferable to another, i.e., has a higher *quality.*

- **Consumers:**
  - Preference parameter for quality: $\theta \in [\underline{\theta}, \overline{\theta}] \subseteq \mathbb{R}_+$
    - larger $\theta \rightarrow$ consumer more sensitive to quality changes
  - Each consumer chooses 1 unit of 1 of the products
  - Distributed uniformly on $[\underline{\theta}, \overline{\theta}]$, total mass is $M = \overline{\theta} - \underline{\theta}$
  - $s_i \in [\underline{s}, \overline{s}] \subseteq \mathbb{R}_+$ stands for the quality of product i

Vertical product differentiation (cont'd)

- Utility of consumer θ from one unit of product i:

$$u_\theta(s_i, p_i) = r + \theta s_i - p_i$$

Key feature: If $s_2 > s_1$, then

$u_\theta(s_2, p) - u_\theta(s_1, p) = \theta(s_2 - s_1)$ is ↑ in θ

→ Higher θ = stronger sensitivity to quality differences

- **Firms:** Duopolists
  - Stage 1: Choose quality: s_1, s_2
  - Stage 2: Choose price: p_1, p_2
  - Simultaneous move in both stages
- Constant marginal cost, $c = 0$

Vertical product differentiation (cont'd)

- **Price stage:** Suppose $s_1 < s_2$
  - The indifferent consumer $\hat{\theta}$ is given by (if it exists):

$$r + \hat{\theta}s_1 - p_1 = r + \hat{\theta}s_2 - p_2$$

$\rightarrow \hat{\theta} = \frac{p_2 - p_1}{s_2 - s_1}$;

$\theta < \hat{\theta}$ prefers the lower quality s_1
$\theta > \hat{\theta}$ prefers the higher quality s_2

Compare with the endogenous sunk-costs/quality augmented Cournot:

- In that model price-quality ratio is the same for every firm; consumers are indifferent between all products.
- Here, a given consumer prefers one or the other good, depending on their sensitivity to quality. (Consumers are more heterogeneous.)

Vertical product differentiation (cont'd)

- **Price stage** (cont'd)
  - An indifferent consumer truly exists iff:

$$\underline{\theta} \leq \hat{\theta} \leq \overline{\theta} \quad \leftrightarrow \quad \underline{\theta}(s_2 - s_1) \leq p_2 - p_1 \leq \overline{\theta}(s_2 - s_1)$$

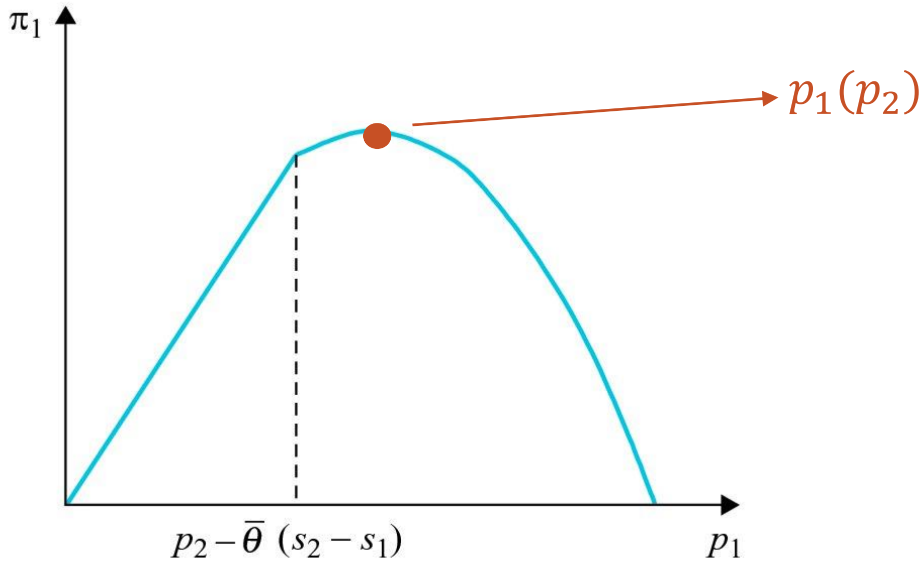
Hence (assuming $s_2 > s_1$):

$$\pi_1 = \begin{cases} 0 & \text{if } p_1 > p_2 - \underline{\theta}(s_2 - s_1), \\ p_1\left(\dfrac{p_2 - p_1}{s_2 - s_1} - \underline{\theta}\right) & \text{if } \underline{\theta}(s_2 - s_1) \leq p_2 - p_1 \leq \overline{\theta}(s_2 - s_1), \\ p_1(\overline{\theta} - \underline{\theta}) & \text{if } p_1 < p_2 - \overline{\theta}(s_2 - s_1). \end{cases}$$

- No discontinuity because $p_1 = p_2 - \overline{\theta}(s_2 - s_1) \rightarrow \pi_1 = p_1(\overline{\theta} - \underline{\theta})$
  - Moreover, π_1 is increasing in p_1 up to this point.

Vertical product differentiation (cont'd)

- **Price stage** (cont'd)



- The quadratic part $p_1\left(\frac{p_2-p_1}{s_2-s_1} - \underline{\theta}\right)$ is maximized at

$$p_1(p_2) = \frac{1}{2}[p_2 - \underline{\theta}(s_2 - s_1)]$$

  - This is the best response if it is positive. Otherwise, $\pi_1 = 0$ because firm 1 has no demand for any $p_1 \geq 0$, and a best response is $p_1 = 0$.

Vertical product differentiation (cont'd)

- **Price stage** (cont'd). Similarly:

$$\pi_2 = \begin{cases} p_2\left(\overline{\theta} - \underline{\theta}\right) & \text{if } p_1 > p_2 - \underline{\theta}(s_2 - s_1), \\ p_2\left(\overline{\theta} - \dfrac{p_2 - p_1}{s_2 - s_1}\right) & \text{if } \underline{\theta}(s_2 - s_1) \leq p_2 - p_1 \leq \overline{\theta}(s_2 - s_1), \\ 0 & \text{if } p_1 < p_2 - \overline{\theta}(s_2 - s_1). \end{cases}$$

$\rightarrow$ $p_2(p_1) = \frac{1}{2}[\overline{\theta}(s_2 - s_1) + p_1]$ (best response)

- **Equilibrium:**

$$p_1^* = \frac{1}{3}\left(\overline{\theta} - 2\underline{\theta}\right)(s_2 - s_1)$$
$$p_2^* = \frac{1}{3}\left(2\overline{\theta} - \underline{\theta}\right)(s_2 - s_1)$$

(assuming $\overline{\theta} > 2\underline{\theta}$) (2)

$\rightarrow$ **Even the price of the low-quality firm increases with the quality difference!**

Vertical product differentiation (cont'd)

- **Price stage** (cont'd)

Note: Suppose $\overline{\theta} \leq 2\underline{\theta}$. Set $p_2 \equiv p_2(0) = \overline{\theta}(s_2 - s_1)/2$.

$$\rightarrow \qquad \hat{\theta} = \frac{p_2 - p_1}{s_2 - s_1} \leq \frac{p_2}{s_2 - s_1} = \frac{1}{2}\overline{\theta} \leq \underline{\theta}$$

- That is, firm 1 cannot get a positive demand even if it sets $p_1 = 0$. This makes $p_1^* = 0$ a best response.
  - Then, $p_2^* \equiv p_2(0)$ is a best response too.

$\rightarrow$ **Conclusion:** If $\overline{\theta} \leq 2\underline{\theta}$ and $s_1 \neq s_2$, the high quality firm gets the whole market, the other firm shuts down in the price stage.

- Henceforth, assume $\overline{\theta} > 2\underline{\theta}$, so that both firms remain active in stage 2.

Vertical product differentiation (cont'd)

- **Quality stage**
  - Substitute p_1^* and p_2^* from eqn (2) into stage 1 profit function:

$$\pi_1(s_1, s_2) = \frac{1}{9}\left(\overline{\theta} - 2\underline{\theta}\right)^2 (s_2 - s_1)$$

$$\pi_2(s_1, s_2) = \frac{1}{9}\left(2\overline{\theta} - \underline{\theta}\right)^2 (s_2 - s_1)$$

  - Both profits ↑ in the quality difference $s_2 - s_1$.
  - → equilibrium quality choices: $s_1^* = \underline{s}$ and $s_2^* = \overline{s}$
    - The converse is also possible by symmetry in this simultaneous move game.
    - **Note:** Sequential quality selection would imply a first mover advantage, because of strategic substitutability. The first mover would select $\overline{s}$ and get a higher profit as in π_2 above.

Vertical product differentiation (cont'd)

- Note that the marginal cost of quality is assumed to be 0 here. If we were to take into account the cost of producing a high quality product, optimal quality choices may not be so extreme. The general conclusion is the following:

- **Lesson**: In markets in which products can be vertically differentiated, the firms offer different qualities in equilibrium so as to relax price competition.

Case. **VLJ industry: "Battle of bathrooms"**

- Very Light Jets
  - 4 to 8 passengers, city-to-city, 60 to 90-minute trips

You are not going to have women on a plane unless it has a lavatory.

Jim Burns,
Founder of Magnum Air

Vertical differentiation

Having a bath-room on board is not an issue for short trips.

Ed Iacobucci,
CEO of DayJet Corp.

VS

Adam Aircraft A700
Bigger, more expensive
Has a lavatory

Eclipse 500
Less expensive
No lavatory

Vertical differentiation: Entry problem revisited

- Recall Chapter 4: Quality augmented Cournot may bound the number of firms in oligopolistic markets. (Requires costly quality choice.)

- The present model predicts a limited number of firms even for **costless quality choice and arbitrarily small entry costs**.
  - The presence of a small entry cost creates a small economies of scale, which turns out to be sufficient to limit the number of active firms. We may even have a **natural monopoly**.

- But the equilibrium number of firms goes to ∞ as the mass of consumers $M = \overline{\theta} - \underline{\theta} \uparrow \infty$.

Vertical differentiation: Entry problem revisited

- Formally, recall that if $\overline{\theta} \leq 2\underline{\theta}$ the low quality firm shuts down.
  - No other firm will have an incentive to enter.

  $\rightarrow$ Natural monopoly.

- More generally, it can be shown that, for arbitrarily small entry costs, the equilibrium number of active firms is the smallest integer n such that

$$\overline{\theta} \leq 2^n \underline{\theta}$$

  - See the book for the details.

- **Note:** In contrast to the earlier model, equilibrium number of firms "slowly" $\rightarrow$ ∞ as the mass of consumers, $\overline{\theta} - \underline{\theta}$, goes to ∞.

Probabilistic choice

- Discrete choice problem: Choose one among few options.
- Empirical analysis of discrete choice problems are based on the so called “random” or “probabilistic” choice models.
  - Random component is meant to capture consumer heterogeneity in tastes or quality sensitivity etc.
    - There can also be unpredictable variations in the behaviour of a given consumer.

Probabilistic choice & horizontal differentiation

- Suppose the utility of a product i is a random variable: $v_i \equiv \bar{v}_i + \varepsilon_i$
- $\bar{v}_i \equiv u(r, p_i)$ is the (mean) utility, including the effect of price
  - The observable part of utility that we can estimate.
- ε_i is the random part: Exogenous.
  - Think of it as a random taste parameter: For example, this can be the distance between a particular product location i and a randomly chosen consumer.
  - *Assumption:* The expected value of ε_i is 0
  - $\rightarrow \mathrm{E}(v_i) = \bar{v}_i$. So, $\bar{v}_i$ is the **mean** or **expected** utility from product i. In the "linear city" this is the utility of the consumer $x = 1/2$ from the product i

Probabilistic choice & horizontal diff. (cont'd)

- Let e_i denote the realization of $\varepsilon_j - \varepsilon_i$.
- Our randomly chosen consumer selects the product i over j iff $v_i > v_j$, iff

$$\bar{v}_i - \bar{v}_j > e_i$$

- Thus, with two products, and assuming continuous distributions, the **choice probability of $\boldsymbol{i}$** is:

$$\Pr(e_i \leq \bar{v}_i - \bar{v}_j) \equiv F_i(\bar{v}_i - \bar{v}_j),$$

  where F_i is the distribution function of $\varepsilon_j - \varepsilon_i$.

- Typically ε_1 and ε_2 are assumed to be i.i.d. with a well behaved distribution (e.g., logistic distribution).
  - $\rightarrow$ Particular functional form for $F_i(\bar{v}_i - \bar{v}_j)$ (optional: see the book)

Probabilistic choice & horizontal diff. (cont'd)

- Let α_i denote the **market share** of product i.
- Our first demand equation is:

$$\alpha_i = F_i(\bar{v}_i - \bar{v}_j) \qquad \text{(D1)}$$

  - LHS is observable, RHS is an exogenously given function of the variables $\bar{v}_1$ and $\bar{v}_2$.

- Second demand equation decomposes $\bar{v}_i$:

$$\bar{v}_i = \beta x_i - \gamma p_i + \xi_i \qquad \text{(D2)}$$

  - x_i is the vector of observed product characteristic (location, level of sugar or alcohol etc.)
  - γ measures the effect of price
  - ξ_i is an error term, that will be left unexplained

- Use (D1) and (D2) to estimate (β, γ) and thereby $(\bar{v}_1, \bar{v}_2)$

Probabilistic choice & product diff.: Final remarks

- First order conditions of the firms will also depend on the demand function/market share, which will give one more equation that depends on γ.

- If there are n products, choice probability of i will be:

$$\Pr(v_i = \max\{v_1, \ldots, v_n\}) = \Pr\left(\bar{v}_i - \bar{v}_j \geq \varepsilon_j - \varepsilon_i \;\forall j \neq i\right)$$

  - This can be computed as a function of $(\bar{v}_1, \ldots, \bar{v}_n)$ given the joint distribution of $(\varepsilon_1, \ldots, \varepsilon_n)$.

- In case of **vertical differentiation**, we need an additional random variable θ_k that represents the quality-sensitivity of consumer k. (The main methodological ideas are similar.)

Review questions

- What makes firms locate close to each other in the product space? And what does it make them differentiate themselves from their competitors?
- Explain the main difference between horizontal and vertical product differentiation.
- Determine if the following statements are true or false. Explain your answer.
  - In horizontal product differentiation, firms always select most extreme positions.
  - In a model of vertical product differentiation with sequential moves, the firm that selects the quality first is advantageous.
  - The number of firms in an industry with constant marginal costs necessarily converges to infinity as the entry cost goes to zero.