

Q2 (a)

word	$P(\text{word} \mid \text{spam}) / P(\text{word} \mid \neg\text{spam})$	$P(\neg\text{word} \mid \text{spam}) / P(\neg\text{word} \mid \neg\text{spam})$
w1	$0.8/0.2 = 4$	$0.2/0.8 = 1/4$
w2	$0.5/0.5 = 1$	$0.5/0.5 = 1$
w3	$0.1/0.4 = 1/4$	$0.9/0.6 = 3/2$

(b) An email containing w1 but not w3 is maximally likely to be spam (LR 6). An email containing w3 but not w1 is maximally likely to be non-spam (LR 1/16). The presence or absence of w2 is immaterial.

If spam is 10 times more likely to occur than non-spam, then the first email is still predicted as spam ($10 \cdot 6 > 1$) and the second is still predicted as non-spam ($10 \cdot 1/16 < 1$).

(c) The worst feature to split on is w2: w2 present: (5 spam, 5 non-spam); w2 absent: (5 spam, 5 non-spam). Both subsets have entropy 1, so clearly zero information gain. w1 present: (8 spam, 2 non-spam); w1 absent: (2 spam, 8 non-spam). Both subsets have the same entropy, say E.

w3 present: (1 spam, 4 non-spam), again entropy E; w3 absent: (9 spam, 6 non-spam), with an entropy higher than E.

So the best feature is presence/absence of w1.

(d) In general we could use this information to construct a new feature testing presence/absence of both w1 and w2. However, the numbers indicate that w1 and w2 are independent given the class (spam: $8 \cdot 5/10 = 4$; non-spam: $2 \cdot 5/10 = 1$), so this new feature would not change classification accuracy.