

Buoyant Forces

- The resultant fluid force acting on a body that is completely submerged or floating in a fluid is called the buoyant force
- $F_B = \gamma V$
- γ – Sp. Weight of fluid, V – Volume of body
- **Archimedes' principle**

When a body is immersed in a fluid either wholly or partially, it is buoyed or lifted up by a force which is equal to the weight of the fluid displaced by the body.

Fluid Mechanics

Prof. T.I. Eldho

Department Of Civil Engineering

Indian Institute of Technology, Bombay

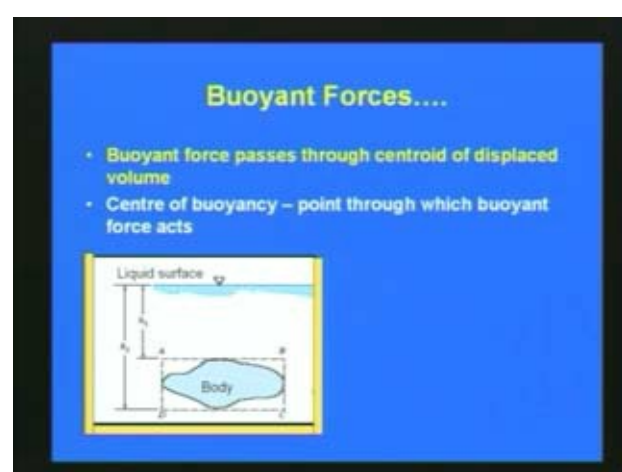
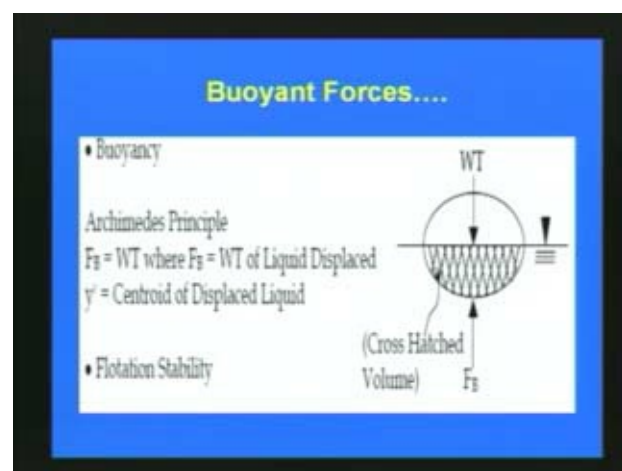
Lecture – 5

Fluid Statics

Welcome back to the video course on fluid mechanics. In the last lecture in fluid statics we were discussing about the buoyant forces and then Archimedes principles and later issues. So in the Archimedes principle you have seen that, when a body is immersed in a fluid.

(Refer Slide Time: 1:36)

Either wholly or partially, it is buoyant or lifted up by a force which is equal to the weight of the fluid displaced by the body. So this we have seen then we have seen that we can equate the buoyant force to the weight of liquid displaced and then we can apply this theory to many engineering problems.



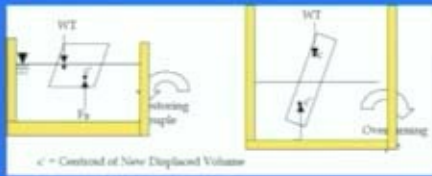
(Refer Slide Time: 1:50)

(Refer Slide Time: 1:57)

The point through which the buoyant force acts as going through the center of buoyancy and the buoyant force passes through the centroid of the displaced volume. So that also we have seen in the last lecture and then we can get to the buoyant force to the weight of the liquid displayed. So that means the F_B is equal to γ into the volume so this volume displaced to be liquid. So this theory we can apply to many engineering problems and also say equipments like hydrometer which is used for the specific determination is based up on this principle.

Stability & Equilibrium

- Stability of a submerged or a floating body meant the tendency for the body to return to the original upright position after it has been displaced slightly.



Now based up on the buoyant force and the Archimedes principle we will discuss the stability and equilibrium in fluid statics. So as we have discussed earlier the position floating body or a submerge body or a body can be either in stable equilibrium or unstable equilibrium or say neutral equilibrium.

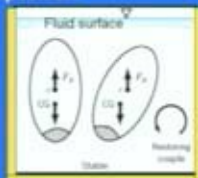
(Refer Slide Time: 3:03)

So a stability of a submerged or a floating body meant the tendency for the body to return to the original upright position after it has been displaced slightly. So here in this slide you can see that here in container there is water and then say here you can see there is a body and then we are just putting a small force on this. Then say we are observing what will happen whether it will restart back or it will over tone, so that is what will discuss in the stability of a submerged or a floating body. So as I mentioned so here if you have water in container so we can put a body here.



Stability & Equilibrium....

- Conditions of equilibrium
- A body is said to be in a state of **stable equilibrium** if a small angular displacement of the body of sets up a couple that tends to oppose the angular displacement of the body, thereby tending to bring the body back to its original position.
- Stable – if CG below CB**

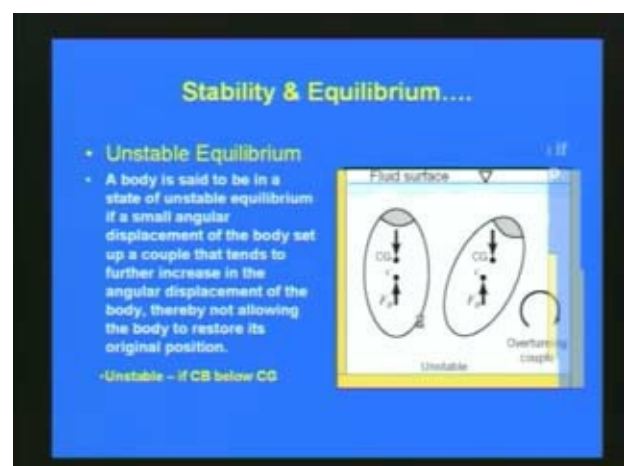


(Refer Slide Time: 3:43)

It is floating you can see that when we give an overtone what is whether a moment or force is applied. So what happens to the body whether it is coming back or it is returning?

So that is what we are discussing here. So as far as the stability and equilibrium is concerned conditions of equilibrium can be stable or unstable or neutral so a body is said to be in a state of stable equilibrium.

(Refer Slide Time: 4:10)



If a small angular displacement of the body sets up a couple that tends to oppose the angular displacement of the body thereby tending to bring the body back to its original position. So here in this slide you can see here there is a fluid and then you can see there is a body and then a force is applied and then a moment is given. So it is the couple tends to oppose the angular displacement of the body and tending to bring in back.

So here in this small experiment here you can see there is water in a container and then body is floating. So here I am just giving a small returning movement like this but you can see that still the body is returning to its original positions by just making it stable.

So this kind of equilibrium when a body is if the small angular displacement of the body sets of a couple that tends to oppose the angular displacement so this kind of equilibrium is called stable equilibrium.

So we have already seen in the center of gravity and center of buoyancy so generally a body is said to be in stable equilibrium if center of gravity is below the center of buoyancy. So here in this slide you can see here the center of gravity see and then center of buoyancy just above that so if cg center of gravity below the center of buoyancy then the body is said to be equilibrium and then there will be a restoring couple so that it will be stable.

(Refer Slide Time: 5:41)



Then, unstable equilibrium a body said to be in a state of unstable equilibrium if a small angular displacement of the body set a couple that tends to further increase in the angular displacement of the body thereby not allowing the body to restore its original position. So this is so called the unstable equilibrium. So here we just see a small experiment here. So here there is a bodies floating I am just putting a small weight up on this floating body.

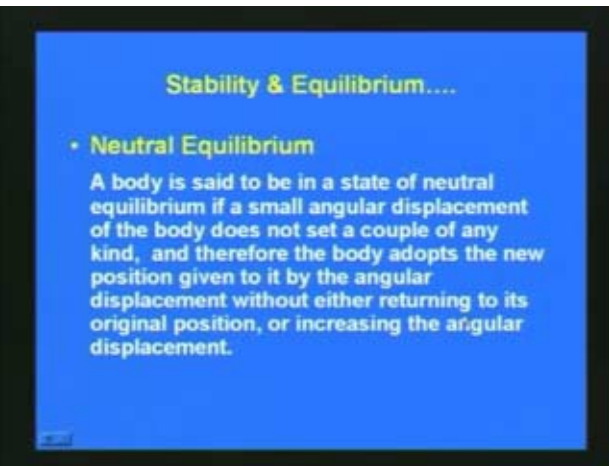
(Refer Slide Time: 6:09)

And then what happens is if I am giving a returning moment to this body here so if say as we have seen it is a stable equilibrium means if it is turning back to its original position but here when if it is returning the returning couple is formed and then it is going down.

So like this you can see now it is the body is said to be unstable equilibrium. Since it has gone from the previous position to a new position and then it is say in returning couple took place and body is not allowed to distort its original positions.

So this is so called the unstable equilibrium. So if we observe in experiments the center of gravity and center of buoyancy. So when a body is floating or submerging a quantity of liquid will be equivalent quantity of liquid will be displaced so that center of buoyancy through where that how much liquid is displaced so that if that center of buoyancy is below the center of gravity then you can see that it will be generally unstable equilibrium.

So a stable equilibrium is when a center of buoyancy above the center of gravity and a body is said to be unstable equilibrium if the center of buoyancy is below the center of



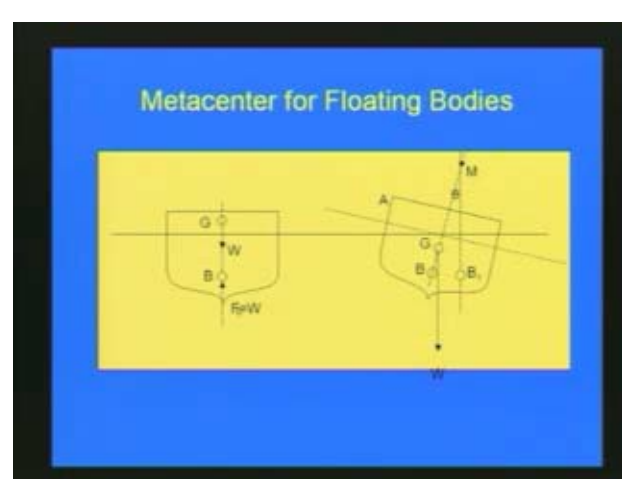
gravity as shown in this figure do that returning couple is formed and then the floating body becomes unstable.

A third category of stability and equilibrium is neutral equilibrium say a body is said to be in a state of neutral equilibrium if a small angular displacement of the body does not set a couple of any kind and therefore the body adopts the new position given to it by the angular displacement without either returning to its original positions or increasing the angular displacement.

(Refer Slide Time: 7:37)

So in this case a body is there so what happens is it is when a small displacement is given if a body does not set a couple of any kind and therefore the body adopts the new position as given like this. A body whether a floating body or submerged body in fluid static and say that if the body is stable equilibrium or the body is unstable equilibrium or the body is in neutral equilibrium.

Especially, when we are using in the vessels and bought learning the stability of the body is very important. So we will be using some of this principle as far as to determine whether the body is stable or unstable in many of the practical engineering field.



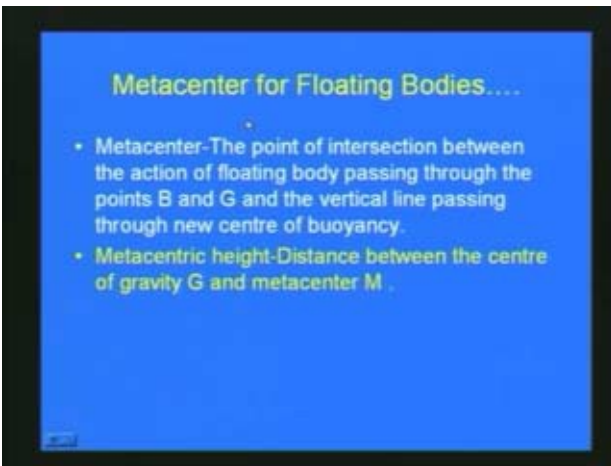
Now related to this say stability and also the buoyant force and then the center gravity and center of buoyancy the one important term is called the metacenter for floating bodies so what is metacenter.

(Refer Slide Time: 8:59)

Here in this slide you can see that there is a small body or a small vessel is say floating in a floating condition and then this is the center of gravity or the body is at g here and then since it is partially submerged the center of buoyancy is at point b here in this figure and the body weight is indicated here also w and then the buoyant force is there so that is fb acting in the opposite direction so since it is in stable equilibrium it is in a condition fb is equal to the weight of the body. So this is the situation.

And now we are just giving a small displacement like this in this second figure we are giving a small displacement to the floating body partially floating body like this a small displacement is given. So then you can see that this line is partially shifted in this direction so that the new position of the body is inclined position like this and then we can see that say due to this effect say the weight of the liquid this based this slide changes and then it is say the position changes and then the center of buoyancy shifting from b to b one.

So this new position is b one and then you can see that when if you put a vertical from b one and then if you draw a line from say the center of buoyancy to the old position center



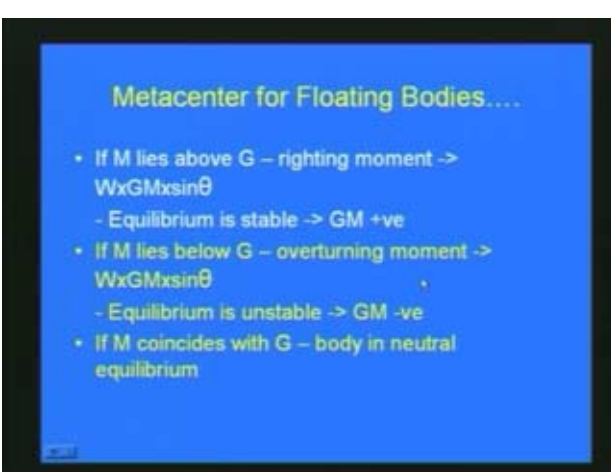
of gravity then it those two lines will coincide at position m. So this is called the metacenter and this you can see that here small angle is formed due to which will be equal to this angle here.

(Refer Slide Time: 10:54)

Metacenter we can define as metacenter is the point of intersection between action of floating body passing through the point b the center of buoyancy and g the center of gravity and the vertical line passing through the new center of buoyancy.

So for an equilibrium a body in equilibrium in floating or submerge condition we are giving a small team or small displacement and then you can see that the center of buoyancy shifting so the metacenter is the point of intersection between the action of floating body passing through the points b or g the old positions and then vertical liner passing through the new center of buoyancy.

So this is generally the metacenter and the metacentric height is generally used to see whether a body is in stable equilibrium and also how much vessels like ship or above how much weight it can take without any unstable problem. This is important of this metacenter so the metacenter height is the distance between the center of gravity g and metacenter m.



So in the previous slide we have seen so here this distance between this line and this line is intersection is called metacenter and the metacenter height is the distance between g to m so here this distance gm is called the metacenter height. So metacenter height is the distance between the center of

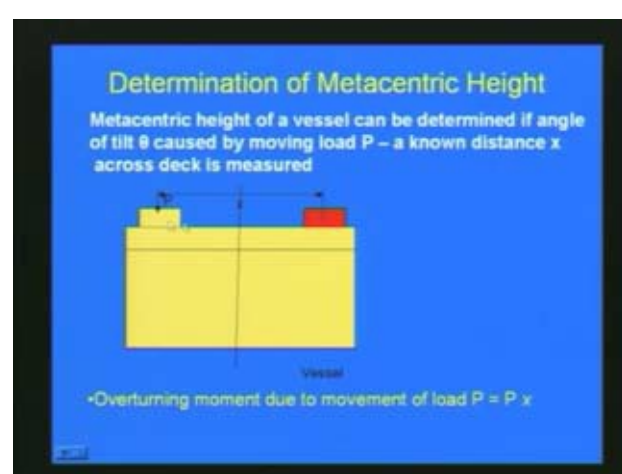
gravity and the metacenter m .

(Refer Slide Time: 12:24)

So the metacenter floating bodies it is very important we have to determine the metacenter and we have to see what is the position of the metacenter that we can determine the weather bodies in stable conditions or unstable conditions Here if m the metacenter lies above g so that righting moment may be w into w the weight of the floating body gm the metacenter height into $\sin \theta$.

So this to make the body is said to be equilibrium is stable the body is stable equilibrium if this gm is positive and if m lies below g that means say in the previous slide. So here if this m lies below g when we are giving a large displacement and due to other problems it can lie below g then there will be a overturning moment and then you can see that again this will be this overturning moment will be w the weight of the body multiplied by the gm into $\sin \theta$ and then the equilibrium is unstable and here gm is negative.

So with respect to the metacenter also we can say that whether there is stable equilibrium or whether there is unstable equilibrium. A stable equilibrium is when gm is positive or when m lies above g it is the body said to be stable equilibrium and if m lies below g then



the body said to be in unstable equilibrium and in the case if m coincides with g the body is said to be neutral equilibrium so that metacenter and the center of gravity of body coincides then we say that the body is said to be in neutral equilibrium.

So if we determine the metacenter position and the metacenter height we can determine whether the body is stable equilibrium or unstable equilibrium or in a neutral equilibrium.

So in the next slides let us see how we can determine this metacentric height So here there is a floating body like this floating vessel.

(Refer Slide Time: 14:28)

You can see there is a weight put on one side of the vessel and then we are moving this vessel from one position say this is old position of the vessel we are moving this vessel this body on the vessel the small body on the vessel from one position to another so that there will be a small displacement will be taking place and then we what will determine the metacentric height.

So metacentric height as we have discussed the height of we can determine if angle of tilt a θ that means with respect to the position of this body, there is a small tilt θ caused by moving load then we can determine it is a distance $non\ x$ across the deck is measured so that is the metacenteric height.

Determination of Metacentric Height...

- If GM is metacentric height & $W=mg$ (including P)
- Righting moment $W GM \theta$ (θ – angle of tilt with horizontal)
- For equilibrium in tilted position,
- Righting moment = overturning moment
- $W GM \theta = Px$
- Metacentric height, $GM = Px / (W \theta)$
- True metacentric height is value of GM as $\theta \rightarrow 0$.

So here in this case the overturning moment of load this load p is moving from one side to another and if x is the distance from the old position to the new position then overturning moment which the moment of the load is equal to p into x .

(Refer Slide Time: 15:34)

Then the next figure shows more clearly about this. So to determination to determine the metacenteric height so this is the floating body and small weight is placed. so when it is in this position it is in stable equilibrium and there is not tilt and when we are shifting this small lot to one side you can see that there will be a tilt for the displacement and there will be therefore the floating body like this and then you can see that a small angle tilt you can see this is the new position of this for horizontal line.

So there will be a small angle of tilt and then small w is the weight of this body here and w is the weight of the floating body which initial pass through the center of gravity. So g and then if b is the buoyancy center of buoyancy initially then if there is w is placed like in this figure then you can see that buoyant force is equal to weight of the body, that is a stable equilibrium. Now we are moving this slightly to one side so that we want to determine the metacenteric type.

If gm is the metacenteric height and the w is the total weight is equal to mg including p that means the p is the load which is moving and this mg is the weight of the floating body. So the writing moment we can write w into gm into θ is very small and θ is the angle of tilt with the horizontal so in equilibrium in tilted position we can write the writing moment is equal to overturning moment that means we are assign is the body is equilibrium we can write writing moment is equal to the overturning moment.

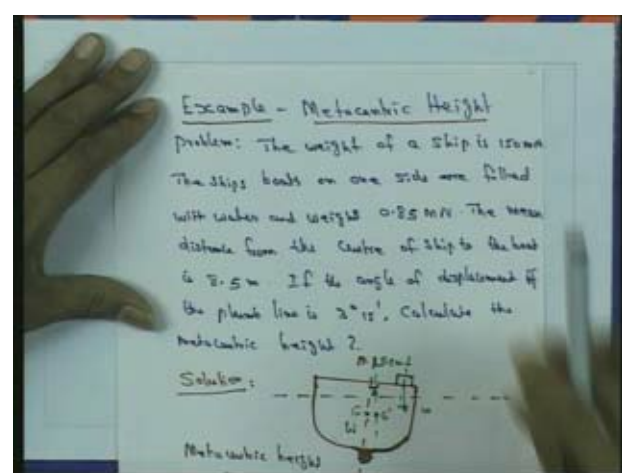
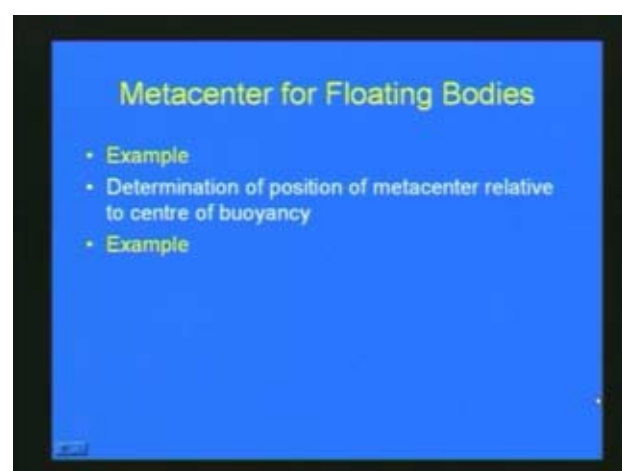
So we can write w into gm the metacenteric height into θ is equal to p into x so where p is the load placed over the floating body and x is the distance between old position and new position. So that metacenteric height we will get gm is equal to p into x by w into θ so where this w is the total weight of the body including the small weight p .

So the true metacenteric height is value of gm as θ tends to zero. So here this θ tends to zero here this w is the p which is integrate earlier and capital w is the weight of the floating body and this

small w is the or is that is equal to p . So this will be the metacentric true metacentric height is the value of gm as θ tends to zero. So here you can see θ here in this slide so when that tends to zero to metacentric height. So for this small θ this derivation is valid so that we can write gm is equal to either p in the previous figure or in this figure w is px by $w \tan \theta$ or when θ is more we can write $w \theta$.

So as we have seen here in this slide. So this derivation is with respect to the previous figure here and then this same thing we can write gm is equal to $w \text{ small } w \times$ where this small w is the weight of the body small body on the vessel and then w is the total weight so gm is equal to $w \times$ by $w \theta$ or when it is θ is more we can write $w \theta$ as indicated in this slide.

Now like this we can determine the metacentric height so now just we will discuss a small example.



(Refer Slide Time: 19:17)

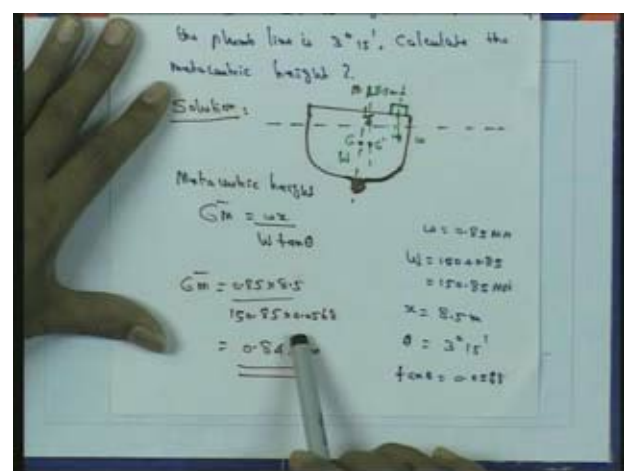
To show how we can determine - we will discuss the small metrical example to determine the metacentric height. So here you can see here in this paper here.

(Refer Slide Time: 19:28)

There is a floating body like this and there is a small weight is placed like here. So we want to determine the metacentric height. So the problem is the weight of the ship is 150

Newton the ship boats or one side are filled with water and weight 0.85 atoms Newton.

The mean distance from the center of ship to the boat is 8.5 meter that means this



distance from the center of the ship to this boat which is placed here is 8.5 meter. If the angle of displacement of the plb line theta is three degree fifteen minutes calculate the metacentric height. So this boat here is placed here is big ship here and a small boat is placed that 8.5 meter find in center line of the ship. So since this ship is placed at a distance 8.5 meter small tilt therefore the ship so that tilt is given as 3 degree 15 minutes.

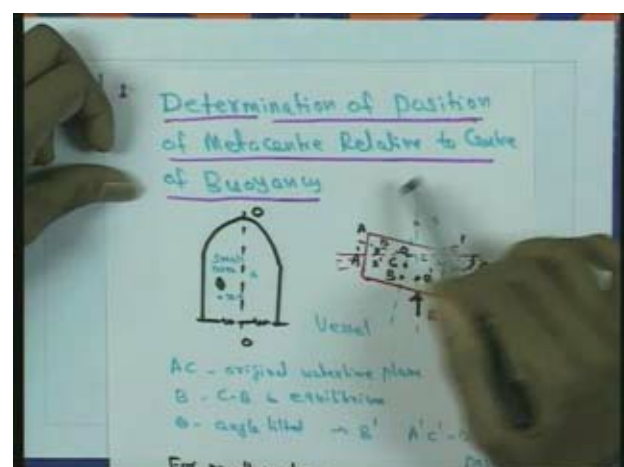
So we want to determine the metacentric height. So this problem here the ship is given and here the small boat is placed 8.5 meter and then initially you can see if say the central line it is in stable equilibrium the boat is not considered then the center of gravity is here and then the weight of the ship is w which is given as 150.

So here this with respect to this position of the boat is g is since the small weight is w is added so g is shifting slightly to g dash. So the metacentric height as per the previous derivation you have seen earlier we can determine metacentric height is gm is equal to w the small w that means the weight of the boat w into this distance x divided by $w \tan \theta$ where here this small w is weight of the boat.

(Refer Slide Time: 22:18)

So that is equal to 0.85 and so here as we have seen that w is capital w is total weight of the ship plus the boat so the capital w is equal to 150 the weight of the ship 0.85 is the weight of the boat. So total capital w is equal to one 50.85 and then x is equal to 8.5

meter and theta is told given as 3 degree 15 minutes. So metacenteric height is gm is



equal to w_x divided by capital w $\tan \theta$. So here the g_m the metacentric height is equal to w is 0.85 multiplied by 8.5 divided by 150.85 that is the w is the total weight the 150.85

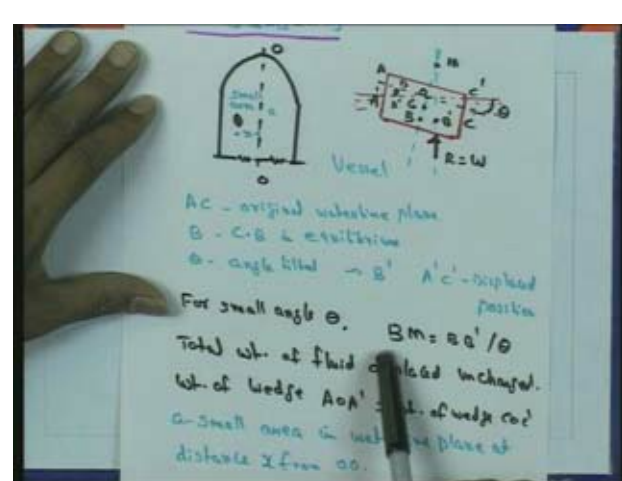
into say $\tan \theta$ $\tan 3^\circ 15'$ 0.05. So, the 150.85 multiplied by 0.05 from which we will get the metacentric height as point eight four three meter.

So like this simple example we can determine the metacentric height by placing a small weight and then we are trying to determine the metacentric height. Similarly, as you have seen in the previous slides so in a very similar way we can determine the metacentric height for the floating bodies.

The next with respect to this metacentric height we want to determine the position of metacenter relating to the center of buoyancy. So here now this paper here we want to determine the position of metacenter.

(Refer Slide Time: 23:12)

Relating to the center of buoyancy, so that is the problem. So here you can see there is a big vessel and then the old position the position is horizontal and when there is a small tilt you can see that the new position here is same. This aa indicate this horizontal line indicate the old position and d to the small tilt θ the new position is say here the inclined line. So ac is the original water line ac is the original water line and $a-dash-c-dash$ is the new water line and then b is the center of buoyancy in equilibrium condition so



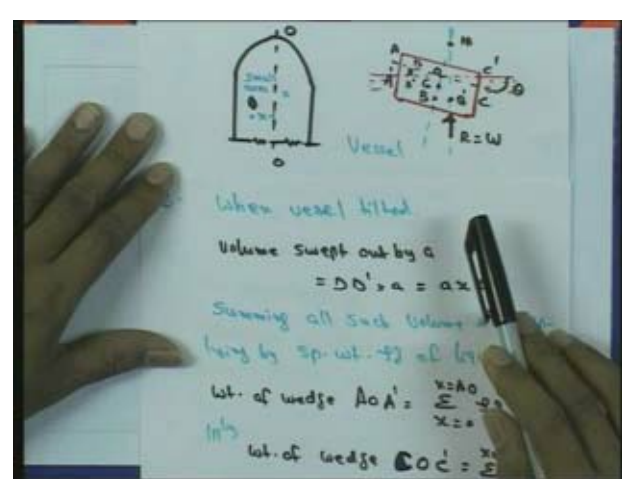
here you can see this b this is the center of buoyancy equilibrium condition and g is the center of gravity.

So a small tilt θ is given and that is the angle of tilt and then with respect to this the vessel is slightly tilted towards the right and then $a-dash-c-dash$ is the displaced position for this vessel. Now we want to determine the position of metacenter relating to the center of buoyancy so that is the problem.

(Refer Slide Time: 24:32)

For small angle θ as shown in this figure, we can write dm which is the distance b can be written as this $b-dash$ that $b-dash$ divided by θ when the angle is very tilt angle of tilt is very small; we can write bm is equal to $b-dash$ divided by θ . So total weight of the fluid displaced unchanged. Since, we are giving a small tilt only with respect to this vessel here.

So the tilt is very small so due to that we can see that the total weight of the fluid displaced is unchanged and then the weight of the wedge we can with respect to this the weight of the wedge with respect to the fluid displace this AOA' is equal to weight of the wedge BOC or BOC' . So AOA' is equal to BOC' and now to determine the position of the metacenter relate to the center of buoyancy.



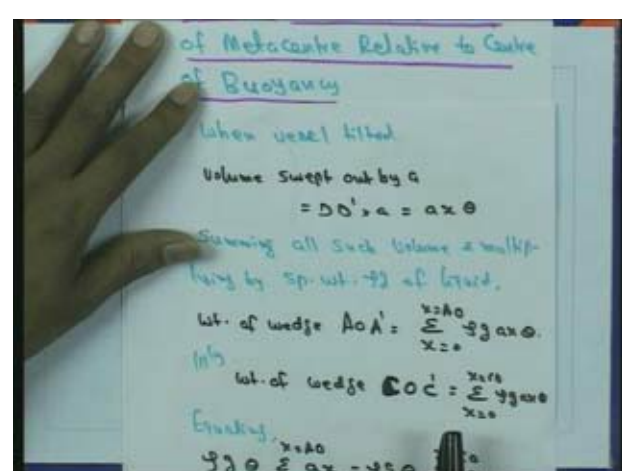
We will consider a small area a like in this figure place a small area is considered in this figure here a small area is considered and it is at distance x from the center line o .

So this is the center line and we are considering a small area at a distance x from oo . Now with respect to this when the vessel is tilted you can see that the volume set by a that means the small area which we have seen here the small area when that vessel is tilted the volume set out by a is equal to d dash.

(Refer Slide Time: 26:29)

So here this is the position of the small area which we are considering and this d dash distances this between the old position new positions. d to d dash so d dash multiplied by a so that can be written as that is equal to the area small area into a distance x into the angle θ so the volume set out by is equal to ax into θ .

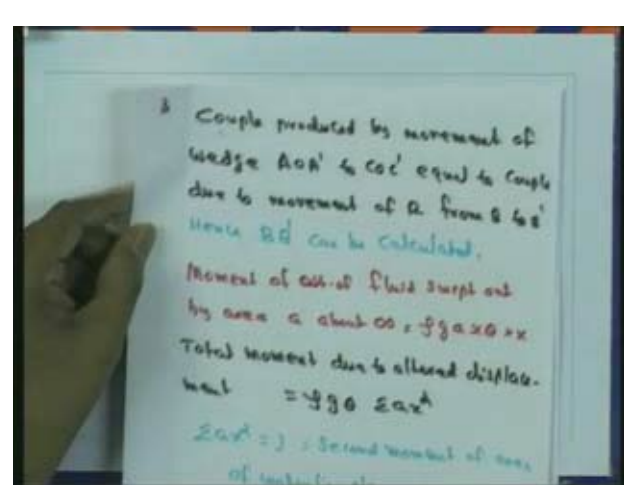
So we are considering a small area now you sum up all such volumes and multiplying specific weight ρg of the liquid you can see that here in the figure the weight of the AOA' is equal to say AOA' is equal to $\int_{x=0}^{x=a_0} \rho g ax \theta$.



(Refer Slide Time: 27:23)

Similarly the weight of the wedge coc dash we can write as σc is equal to 0, x is equal to 0 ρg ax θ . Now since as we have seen here the weight of the wedge a so a dash is equal to $\cos$ dash so that we can equate this weight of the wedge aoa dash is equal to weight of the wedge coc dash. So we can equate so that you will get here ρg θ into σx equal to 0 ax equal to ax is equal to ρg θ σx equal to 0 to co ax .

So that we can find you will write σax is equal to 0 where the small area which h we have consider and x is the distance from the center line of the vessel. Now this we can see that these σax it is the first moment of area of water line plane about the center line o . So this σax is the from the center line o it is the first moment of area so that we can say that axes must pass through the centroid of the waterline so that σax is the first moment of area of water line plane about oo .



(Refer Slide Time: 28:47)

Now, couple produced by moment of wedge aoa dash to coc dash. Since we are providing a small tilt here and the vessel is tilted so when it is the wedge couple produced by moment of wedge aoa dash to coc dash equal to the couple due to moment of this the resultant force r the reaction force r from b to b dash so the couple produced by moment couple produced by moment of wedge aoa dash to coc is equal to the couple due to the moment of this reaction r where r is equal to w .

So due to the moment of r from b to b dash the new position of the center of buoyancy so hence the b dash we can calculate. Now we will take the moment of the weight of fluid said out of bay area at about oo that here the moment of weight of fluid said out by area a at about oo is equal to ρg ax θ into x since you have take in a moment ρg ax into θ into x . Now we can write the total moment due to the altered displacement as ρg θ σax^2 total moment due to the altered displacement.

by area a about $oo = \rho g a x^2 \times x$
 Total moment due to altered displacement
 $= \rho g \theta \sum ax^2$
 $\sum ax^2 = I$ = Second moment of area
 of waterline plane about oo .
 Total moment due to altered displacement
 $= \rho g \theta I$
 moment due to movement of R
 $= R \times oo' = \rho g V \times oo'$
 V = volume of liquid displaced.

(Refer Slide Time: 30:11)

So here you can see that there is a displacement here. So with respect to this we can write total moment due to the altered displacement is equal to $\rho g \theta \sum ax^2$. So here you can see the $\sum ax^2$ is I which is the second moment of area waterline plane about the center line.

So this is second moment of area of water line is I so that $\sum ax^2$ is equal to I .

Finally, we can write the total moment due to the altered displacement is equal to ρ the density multiplied by g acceleration into gravity into θ into i . So the total moment of due to the altered displacement is equal to $\rho g \theta i$. So moment due to the movement of r finally we can write r into bb' is equal to $\rho g i$ into v into bb' , where v is the displaced volume so $\rho g i$ into v into bb' where v is the volume of the liquid displaced. Now with respect to this finally, we can find out the bm that means the metacenter relating to the center of buoyancy that is what we are trying to find out.

1. Determination of position of Metacenter Relative to Centre of Buoyancy
 4. Equating,
 $\rho g V \times bb' = \rho g \theta I$
 $bb' = \frac{\theta I}{V}$
 $\therefore BM = bb' / \theta = I / V$
 Distance BM called as Metacentric radius.

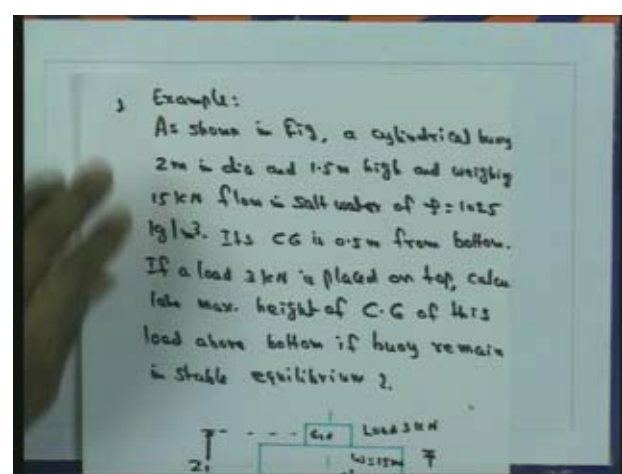
(Refer Slide Time: 31:25)

Finally you can equate $\rho g i$ into bb' is equal to $\rho g \theta i$. Finally we get bb' is equal to this $\rho g i$ is canceled from this side and then bb' is equal to θ the angle of displacement into i the second moment of inertia divided by the volume of the liquid displacement.

So bb' is here you can see is the new distance from the old position of center of buoyancy due to the new position bb' is equal to θ the angle of displacement and i is the second moment of the area and v is the volume of liquid displacement the volume the liquid displaced so bb' is equal to

theta I by v and then finally we get bm that is what we are looking for bm is the distance from the center of buoyancy to the metacenter bm is equal to bb dash divided by theta or this is equal to I by v where I is the second moment of area of waterline plane about oo . Finally we get bm is equal to bb dash divided by theta that is equal to I by v so this distance bm is called as the metacentric radius.

So like this we can determine the position of metacenter relating to the center of buoyancy. So what we are doing here is say we are considering in a small tilt and with respect to how much liquid is displaced and then the new position and then taking the moment and finally we get this bm that means the distance from the center of buoyancy

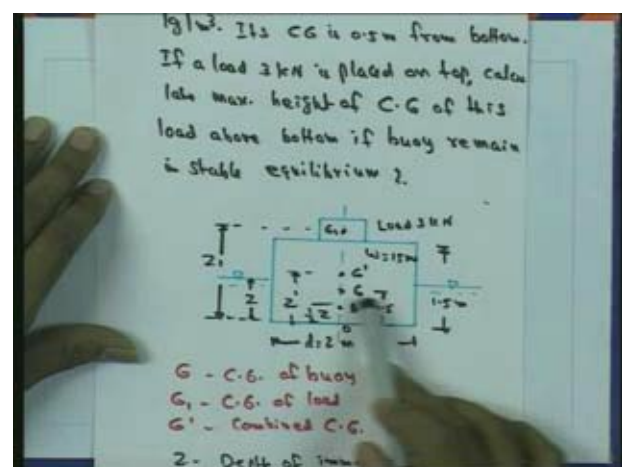


to the metacentric distance as the second moment of area of waterline about oo divided by the volume of the liquid displaced.

So that is what we are getting as called the metacentric radius so like this we can determine the position of metacenter relating to the center of buoyancy. Now with respect to the center of buoyancy and then with respect to the metacenter which we are discussed one more metrical example. So here is the problem.

(Refer Slide Time: 33:38)

Here you can see there is a floating vessel and then a small load is placed over that so we want to determine the maximum weight of the center of gravity of this load that means the placed load above the vessel. So that the vessel remaining stable equilibrium. So the problem statement is as shown in figure a cylindrical buoy two meter in diameter and one point five meter height and weighing fifteen kilo in Newton float in salt water of density one thousand kilo gram per meter cube.



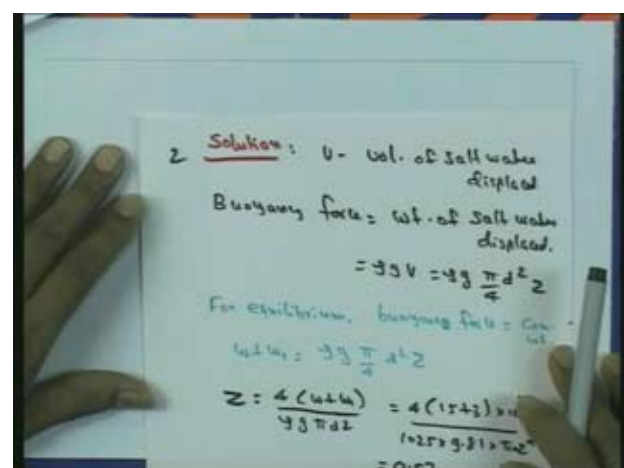
(Refer Slide Time: 33:44)

Its center of gravity is point five meter from the bottom if a load 3 Kilo Newton is placed on the top. Calculate the maxim height of the center of gravity of this load above the bottom if buoy remain in stable equilibrium?

So here a vessel or a buoy is there as shown in this figure. So it is a cylindrical in shape and its diameter is 2 meter and then its height is 1.5 meter height and then is 15 Kilo Newton and it is floating on a salt water of density 1025 kilogram per meter cube and initially it is stable and then its center of gravity is 0.5 meter its center of gravity is which is the center of gravity g so it is 0.5 meter from the bottom.

Now we have placing a load three Kilo Newton just on the center to the buoy likes this. A new load is placed three Kilo Newton on the top. We want to calculate the maxim height of the center of gravity that means the position of when we place a load new load the center of gravity is shifting from g to g dash. So you want to determine we want to calculate the maxim height of the center of gravity of this load when from the bottom in buoy remains in stable equilibrium. So this is the problem you want to determine the new position of the center of gravity.

So here all the problem dimensions are given and the various loads here the buoy weight is 15kilonewton and a new placed load is 3kilonewton. So here let g be the center of gravity of the buoy as shown here. This g is the center of gravity of the buoy and then g



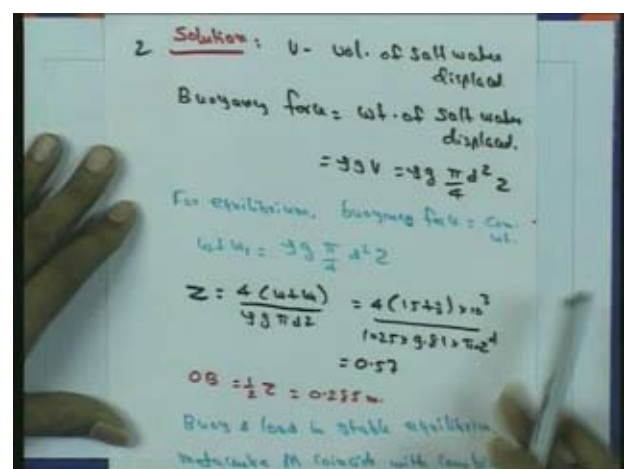
one is the center of gravity of the load. So that load is placed just on the line with symmetric to the buoy. So that g and g one is the same straight line and g one is the center of gravity of the load and then g dash is the combined here. Now load is placed so the center of gravity is shifting so g dash is the combined center of gravity with respect to the load and then buoy that means a 15kilonewton is the weight of the weight of the buoy and 3 kilo Newton is the load. So with respect to the new position of the combine center of gravity let it be g dash and the z is the depth of immersion of buoy that means say this is the position of the sea water.

So the buoy is immerse that to a depth of z as shown in this figure z it is not given we have to find this z so z is the depth of immersion of the buoy. So with respect to this the problem statement now the solution let v is the volume of the salt water displaced since we are placing buoy and then extra large there will be a liquid will be displaced So let v be the volume of the salt water displaced.

(Refer Slide Time: 37:29)

Finally, we can find out the buoyancy force is equal to weight of the salt water displaced.

We have seen earlier the buoyancy is the weight of the liquid displaced. So here in this case the buoyancy force is equal to the weight of the salt water displaced. Since the buoy is placed on sea so this buoyancy force is equal to the weight of the salt water displaced



2 Solution: V - Vol. of salt water displaced
Buoyancy force = wt. of salt water displaced.
 $= \rho V = \rho g \frac{\pi d^2 z}{4}$
For equilibrium, buoyancy force = Comb. wt.
 $w + w_1 = \rho g \frac{\pi d^2 z}{4}$
 $z = \frac{4(w + w_1)}{\rho g \pi d^2} = \frac{4(15 + 3) \times 10^3}{1025 \times 9.81 \times \pi \times 2^2}$
 $= 0.57$
 $OB = \frac{1}{2} z = 0.285 \text{ m}$
Buoy & load in stable equilibrium
Metacenter M coincides with center of buoyancy

so that is equal to rho the density of the salt water and multiplied g the acceleration due to gravity into the volume of the salt water displaced.

So that is equal to rho into g into v. Finally you can write what we said is in this figure z is the depth of the immersion of the buoy. So we can write the buoyancy rho into g into pie d square d is the diameter of the buoy so pie d square z is the reposition of the buoy multiplied by the depth of immersion of the buoy. Finally buoyancy force is rho g pie square by four into z now for equilibrium.

(Refer Slide Time: 38:36)

Here buoyancy force is equal to the combine weight, Now we have the buoy weight of the 15 kilo Newton and then we have putting an extra load of 3 kilo Newton. So the combined weight is equal to fifteen plus three so that 18 kilo Newton that is equal to the buoyancy force.

So you can equate w plus w one is equal to w one is the extra added weight w is the weight of buoy. So that is equal to rho into g into pie by four d square into z. So we want to determine this z that means the depth of immersion of buoy.

So z is equal to four times w plus w one divided by rho into g into pie d square. So once we substitute all the values capital w is the weight of buoy is equal to fifteen and small w one is the added load is three. So 4 into 15 plus 3 then we say it is Kilo Newton we want to determine in terms of Newton. So we have multiplied by 1000 divided by rho g is the salt water with respect to the density 1025 into g 9.81 and d square is say pied square to meter is the diameter of the buoy so pie in to two square so that will give z as 0.57 meter so we get the depth of immersion of the buoy as point five seven meter.

Now we want to determine the center of buoyancy. So in this figure now we can see that this z is older determine as this 0.57 meters so z is 0.57 meters. So ob is equal to half of this z so that we can write this ob is equal to half of z so that is equal to point two eight five meter.

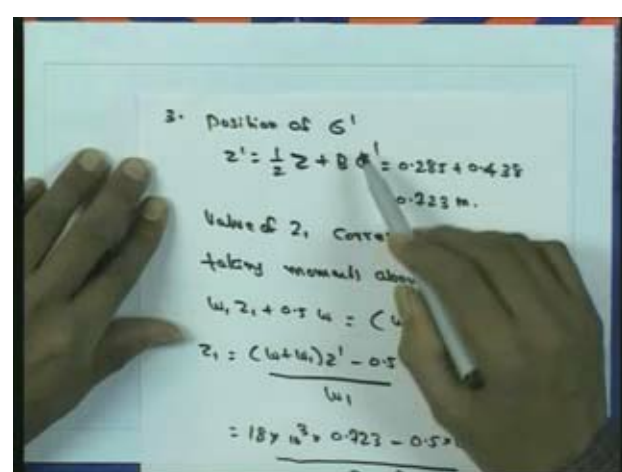
So the buoy and load is stable equilibrium. So as far as the problem statement the buoy and load is in stable equilibrium and the metacenter m coincide with the combined center of gravity g dash. So the metacenteric height in the case g dash is equal to zero.

Since it is stable equilibrium say this metacenter m coincide with the combined cg g dash so m and g dash coincide, so that we can write mg dash g dash m is equal to 0 and now bg dash this the new position of this center of gravity g dash from to that position from g dash g equal to bm that means since the g dash is equal to the met center m .

So bg dash is equal to bm or bg dash is equal to bm is equal to as per the formula which we have derived is equal to second moment of the area I divided by v so the moment of immersion I divided by v the volume of that displacement f the liquid.

So here I is equal to πd to the power four d to the power four divided by sixty four πd to the power four divided by 64 and the volume of displaced liquid is πd square z by 4. Finally, bg dash is equal to bm is equal to this d square divided by 16 into z . So d is power two so 2 square divided by 16 into 0.57, which is the z . We have determined we have finally we get bg dash is equal to bm is equal to 0.438 meter.

Now we want to determine the position g dash the new due to the added weight the new position g dash we want to determine. So the position of g dash is say from the bottom of the buoy that is equal to z dash. This is the new position of z dash is equal to half z that means up to b this ob plus BG dash. So ob is equal to half z .



(Refer Slide Time: 42:32)

So z dash is the position of g dash the new center of gravity position z dash is equal to half z plus bg dash so half z is already determine a point two eight five meter plus bg dash. We have already determined here as 0.438 meter we have already determine here this is bg dash 0.438 meter. Finally position of g dash z dash is equal to 0.285 plus 0.438

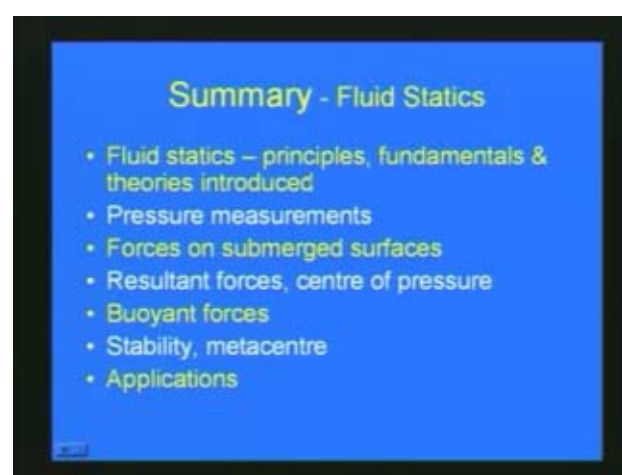
is equal to 0.723 meter.

Now this value of this mater one say here from this figure, you can see that say to the center of gravity of the extra load g one that z_1 from the bottom of the buoy this z_1 . We want to determine so value of z_1 is corresponds to the z dash that means with respect to the new center of gravity we can take moment about the point o .

So here you can take moment about this particular point o so that we can determine this z_1 so w_1 into z_1 where w_1 is the weight of the buoy. So, w_1 into z_1 plus $0.5 w$ is equal to w plus $w_1 z$ dash so z_1 . We finally get as w plus w_1 into z dash minus $0.5 w$ divided by w_1 .

So here in this gates the total weight is fifteen of the buoy and three Kilo Newton of the extra load placed so it is eighteen ten to the power in terms of Newton multiply z dash.

We have already determined the z dash as point seven two three meter into point seven two three meter into 0.723 minus $0.5 w$ is the weight of the buoy.



So 0.5 into 15 into 10 to the power three divided by w_1 is the extra load 3 into 10 to the power three. Finally, we get z_1 that means from the bottom of the buoy to the position of the center gravity of the extra load placed as 1.838 meter.

So here finally we have determine in this figure we have to determine z as 1.838 meter and we have to determine the position of g dash this z dash. We have to determine as 0.723 meters. So this gives the solution of the problem. Now before closing this chapter, we will just have an overview of what we have studied the fluid statics, a brief summary.

What we have studied in fluid static?

As I mentioned at the beginning of this lecture the fluid statics is what we are studying what is happening in the fluid is at rest say with respect to the forces at in the pressure and then buoyancy then all this things what we have studied. We have studied in fluid statics principles the fundamentals and theories have been introduced as far as this fluid statics is concerned and then, we have studied about the pressure measurement.

(Refer Slide Time: 45:44)

So we have seen that the various say metrologies like given by using manometer then, we have seen the automatic gauge equipments like Borden gauges. We have seen the pressure measurement using the various equipments and then we have also discussed the forces on submerged surfaces like inclined surfaces horizontal surfaces vertical surfaces then curved surfaces.

We have seen how we can determine the forces on submerged bodies and then we have seen how we can determine the resultant forces say when the body is either in submerged floating body in horizontal position or vertical position or inclined position. How we can determine the resultant force

and then correspondingly how we can determine the center of pressure and then we have discussed about the buoyant forces and we have also seen the Archimedes principle and then how we can determine the center of buoyancy and then with respect to its various applications we have seen in this fluid statics.

Finally at the end of the chapter we have seen with respect to buoyant force say when a body is submerged or when a body is a floating it can be stable equilibrium unstable equilibrium neutral equilibrium.

So this also we have seen and then we have seen with respect to the position of the center of gravity position of the center of buoyancy how we can define whether the body is stable or the whether the body is unstable or whether the body is in neutral equilibrium and then finally, we have defined term metacenter.

There is small tilt is given an then whether it is in stable equilibrium which is time to come back so with respect to the small tilt given we have also seen what is metacenter and also we have seen how we can determine the metacentric height.

So this are some of the important topics we have discussed in fluid statics and as we have seen in this lecture there are a large number of applications as far as the fluid is static is concerned starting from with respect to the pressure measurement say we have seen, how we can with respect static fluid how we can determine the pressure measurement and including from the how the doctor is determine the human pressure the body pressure?

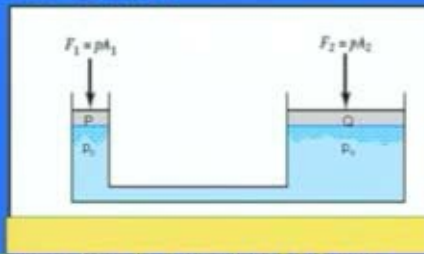
How the application is there and then also we have seen with respect to the Pascal law how the application side like a hydraulic lift how it is lifting it is put a small force it is making it's a trying to say rise larger bodies.

So that also we have seen as applications as in some of the previous slides we have seen.

So like this a large number of applications are there as far as the fluid statics is concerned. So fluid statics is one of the important branch of fluid mechanics the



Forces vs. Area



- Pressure intensity at same level P & Q: $p_1 = p_2 = p_3$
- Transmission of pressure through stationary fluid
- Larger area A_2 – large force F_2 – eg. Lifts, Hydraulics jacks, Presses etc.

advantage here, we can say, since the fluid is say not in moving since it is just static conditions. So the analysis is much easier.

(Refer Slide Time: 49:19)

So the application but large number of obligation here we have a man can car or hydrolic leaves so like that various applications are there as far as,

(Refer Slide Time: 49:33)

The fluid statics is concerned and then also the ships and the movement of ships and boats with respect to this stability very importance. So the stability the stable the equilibrium and then also the metacenter height which is we have discussed it has also got large number of applications as far as say this fluid statics concerned.

Fluid statics is one of the important branches of fluid mechanics and here over analysis is much easier compared to fluid dynamics. In this case in the fluid is moving the fluid is just static so that, we can easily determine the various forces by just taking and just using the equations of equilibrium like $\sum F_x = 0$, $\sum F_y = 0$ or $\sum F_z = 0$ that means the force in xyz direction. Since it is static and we can determine $\sum F_x = 0$, $\sum F_y = 0$, $\sum F_z = 0$ and also, we can take a moment as we have doing in mechanics most of this principles used in mechanics directly applicable as for as the fluid statics concerned.

Various applications as we have seen here in various slides where large number of applications and fluid statics one of the most important branch of fluid mechanics and advantage here is the analysis much easier compared to the dynamic fluid dynamics which will be discussing the coming lectures.

So with this, we are finishing the topic on fluid statics and next we will be discussing kinematics of fluid and then fluid dynamics before going to the turbulant fluent laminate fluent and other theories.