Module
9
Staircases

Lesson
20
Types and Design of Staircases

Instructional Objectives:

At the end of this lesson, the student should be able to:

- classify the different types of staircases based on geometrical configurations,
- name and identify the different elements of a typical flight,
- state the general guidelines while planning a staircase,
- determine the dimensions of trade, riser, depth of slab etc. of a staircase,
- classify the different staircases based on structural systems,
- explain the distribution of loadings and determination of effective spans of stairs,
- analyse different types of staircases including the free-standing staircases in a simplified manner,
- design the different types of staircases as per the stipulations of IS 456.

9.20.1 Introduction

Staircase is an important component of a building providing access to different floors and roof of the building. It consists of a flight of steps (stairs) and one or more intermediate landing slabs between the floor levels. Different types of staircases can be made by arranging stairs and landing slabs. Staircase, thus, is a structure enclosing a stair. The design of the main components of a staircase-stair, landing slabs and supporting beams or wall – are already covered in earlier lessons. The design of staircase, therefore, is the application of the designs of the different elements of the staircase.

9.20.2 Types of Staircases

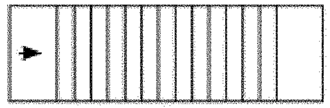

Fig. 9.20.1(a): Single flight staircase

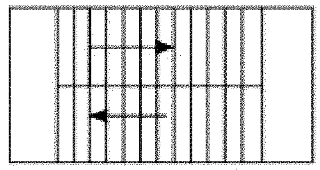

Fig. 9.20.1(b): Two flight staircase

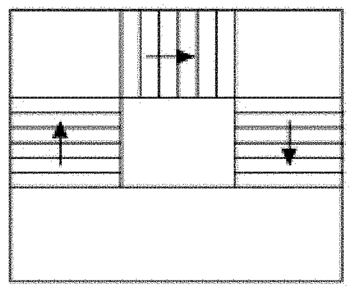

Fig. 9.20.1 (c): Open-well staircase

Fig. 9.20.1: Types of staircases

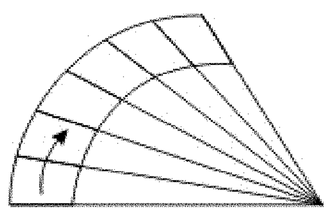

Fig. 9.20.1(e): Helicoidal staircase

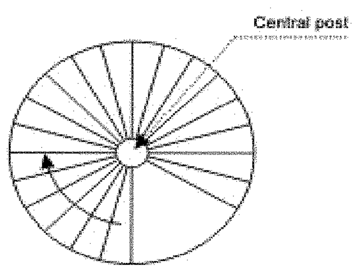

Fig. 9.20.1(d): Spiral staircase

Fig. 9.20.1: Types of staircases

Figures 9.20.1a to e present some of the common types of staircases based on geometrical configurations:

(a) Single flight staircase (Fig. 9.20.1a)

(b) Two flight staircase (Fig. 9.20.1b)
(c) Open-well staircase (Fig. 9.20.1c)
(d) Spiral staircase (Fig. 9.20.1d)
(e) Helicoidal staircase (Fig. 9.20.1e)

Architectural considerations involving aesthetics, structural feasibility and functional requirements are the major aspects to select a particular type of the staircase. Other influencing parameters of the selection are lighting, ventilation, comfort, accessibility, space etc.

9.20.3 A Typical Flight

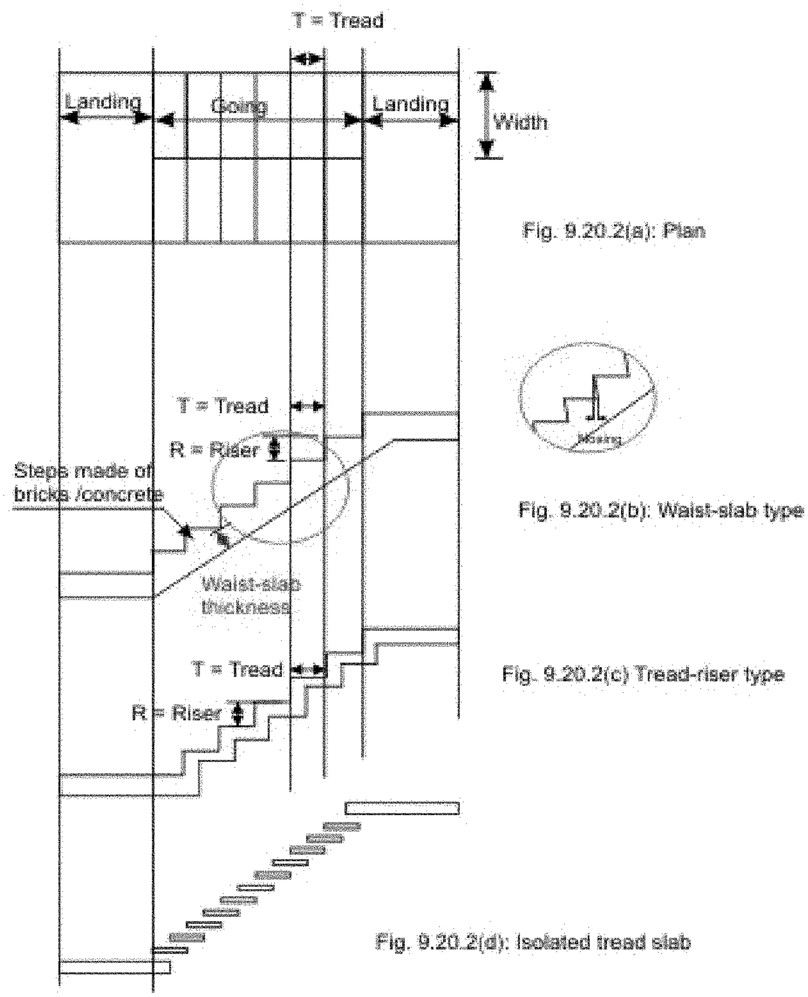

Fig. 9.20.2: A typical flight

Figures 9.20.2a to d present plans and sections of a typical flight of different possibilities. The different terminologies used in the staircase are given below:

(a) Tread: The horizontal top portion of a step where foot rests (Fig.9.20.2b) is known as tread. The dimension ranges from 270 mm for residential buildings and factories to 300 mm for public buildings where large number of persons use the staircase.

(b) Nosing: In some cases the tread is projected outward to increase the space. This projection is designated as nosing (Fig.9.20.2b).

(c) Riser: The vertical distance between two successive steps is termed as riser (Fig.9.20.2b). The dimension of the riser ranges from 150 mm for public buildings to 190 mm for residential buildings and factories.

(d) Waist: The thickness of the waist-slab on which steps are made is known as waist (Fig.9.20.2b). The depth (thickness) of the waist is the minimum thickness perpendicular to the soffit of the staircase (cl. 33.3 of IS 456). The steps of the staircase resting on waist-slab can be made of bricks or concrete.

(e) Going: Going is the horizontal projection between the first and the last riser of an inclined flight (Fig.9.20.2a).

The flight shown in Fig.9.20.2a has two landings and one going. Figures 9.2b to d present the three ways of arranging the flight as mentioned below:

(i) waist-slab type (Fig.9.20.2b),
(ii) tread-riser type (Fig.9.20.2c), or free-standing staircase, and
(iii) isolated tread type (Fig.9.20.2d).

9.20.4 General Guidelines

The following are some of the general guidelines to be considered while planning a staircase:

- The respective dimensions of tread and riser for all the parallel steps should be the same in consecutive floor of a building.

- The minimum vertical headroom above any step should be 2 m.

- Generally, the number of risers in a flight should be restricted to twelve.

- The minimum width of stair (Fig.9.20.2a) should be 850 mm, though it is desirable to have the width between 1.1 to 1.6 m. In public building, cinema halls etc., large widths of the stair should be provided.

9.20.5 Structural Systems

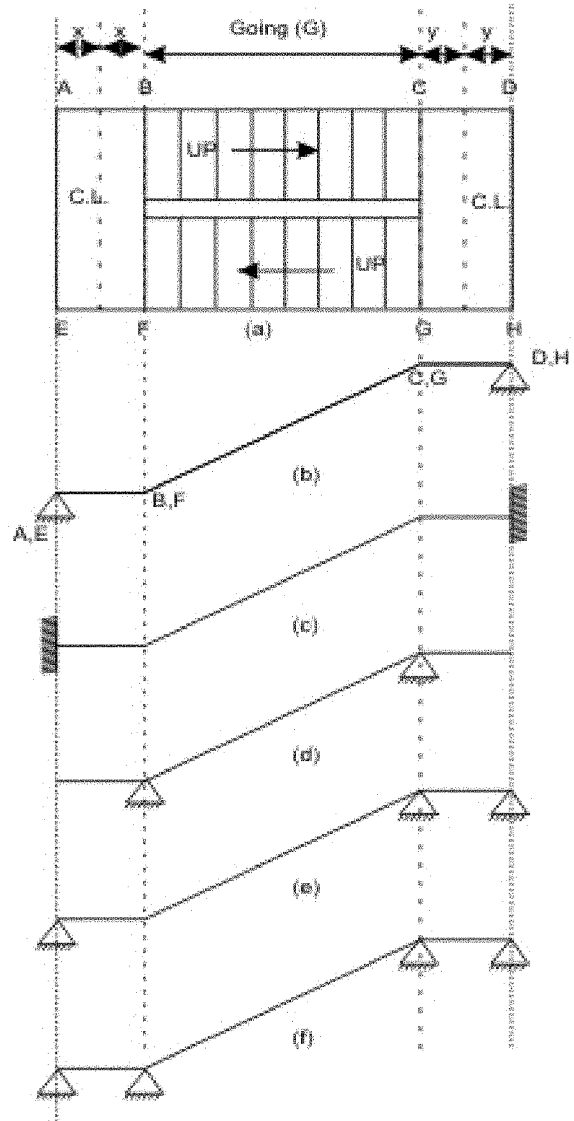

Fig. 9.20.3: Longitudinally spanning staircases

Different structural systems are possible for the staircase, shown in Fig. 9.20.3a, depending on the spanning direction. The slab component of the stair spans either in the direction of going i.e., longitudinally or in the direction of the steps, i.e., transversely. The systems are discussed below:

(A) Stair slab spanning longitudinally

Here, one or more supports are provided parallel to the riser for the slab bending longitudinally. Figures 9.20.3b to f show different support arrangements of a two flight stair of Fig.9.20.3a:

(i) Supported on edges AE and DH (Fig.9.20.3b)
(ii) Clamped along edges AE and DH (Fig.9.20.3c)
(iii) Supported on edges BF and CG (Fig.9.20.3d)
(iv) Supported on edges AE, CG (or BF) and DH (Fig.9.20.3e)
(v) Supported on edges AE, BF, CG and DH (Fig.9.20.3f)

Cantilevered landing and intermediate supports (Figs.9.20.3d, e and f) are helpful to induce negative moments near the supports which reduce the positive moment and thereby the depth of slab becomes economic.

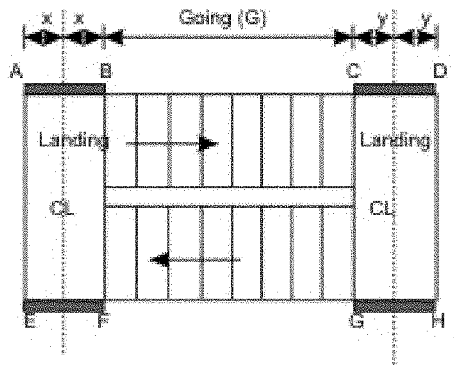

Fig. 9.20.4(a): Beams at two ends of landings

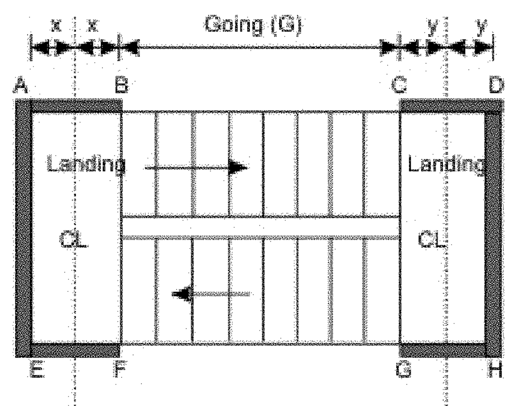

Fig. 9.20.4(b): Beams at three ends of landings

Fig. 9.20.4: Staircases (spanning longitudinally) and landings (spanning transversely)

In the case of two flight stair, sometimes the flight is supported between the landings which span transversely (Figs.9.20.4a and b). It is worth mentioning that some of the above mentioned structural systems are statically determinate while others are statically indeterminate where deformation conditions have to taken into account for the analysis.

Longitudinal spanning of stair slab is also possible with other configurations including single flight, open-well helicoidal and free-standing staircases.

(B) Stair slab spanning transversely

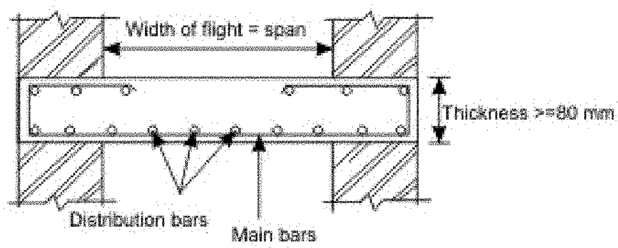

Fig. 9.20.5(a): Slabs supported between two stringer beams or walls

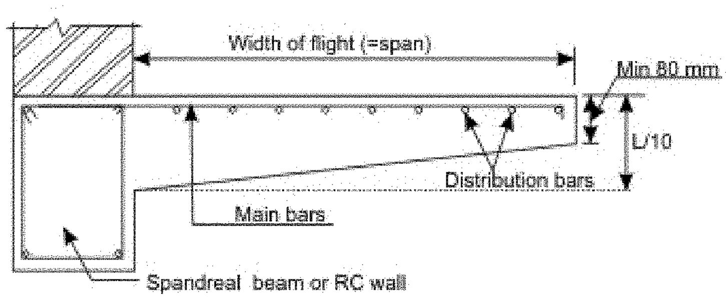

Fig. 9.20.5(b): Cantilever slab from a spandreal beam or wall

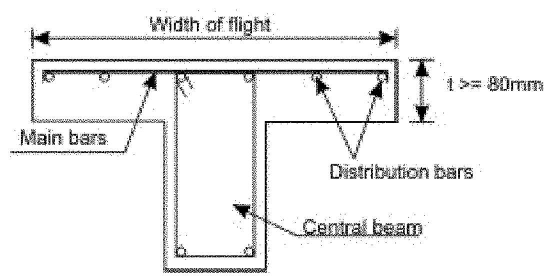

Fig. 9.20.5(c): Doubly cantilever slab from a central beam

Fig. 9.20.5: Transversely spanning staircases

Here, either the waist slabs or the slab components of isolated tread-slab and trade-riser units are supported on their sides or are cantilevers along the width direction from a central beam. The slabs thus bend in a transverse vertical plane. The following are the different arrangements:

(i) Slab supported between two stringer beams or walls (Fig.9.20.5a)
(ii) Cantilever slabs from a spandreal beam or wall (Fig.9.20.5b)
(iii) Doubly cantilever slabs from a central beam (Fig.9.20.5c)

9.20.6 Effective Span of Stairs

The stipulations of clause 33 of IS 456 are given below as a ready reference regarding the determination of effective span of stair. Three different cases are given to determine the effective span of stairs without stringer beams.

(i) The horizontal centre-to-centre distance of beams should be considered as the effective span when the slab is supported at top and bottom risers by beams spanning parallel with the risers.

(ii) The horizontal distance equal to the going of the stairs plus at each end either half the width of the landing or one meter, whichever is smaller when the stair slab is spanning on to the edge of a landing slab which spans parallel with the risers. See Table 9.1 for the effective span for this type of staircases shown in Fig.9.20.3a.

Table 9.1 Effective span of stairs shown in Fig.9.20.3a

Sl. No.	x	y	Effective span in metres
1	< 1 m	< 1 m	$G + x + y$
2	< 1 m	≥ 1 m	$G + x + 1$
3	≥ 1 m	< 1 m	$G + y + 1$
4	≥ 1 m	≥ 1 m	$G + 1 + 1$

Note: G = Going, as shown in Fig. 9.20.3a

9.20.7 Distribution of Loadings on Stairs

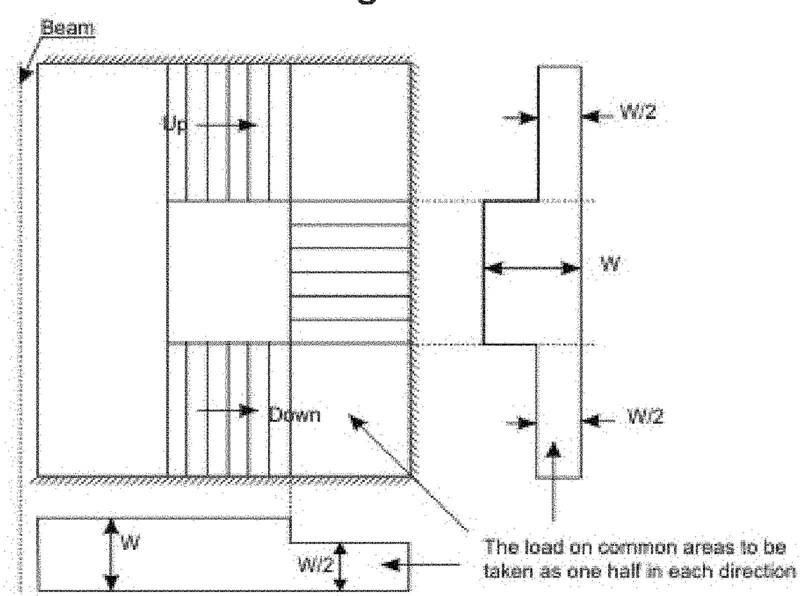

Fig. 9.20.6: Loadings on open-well staircases

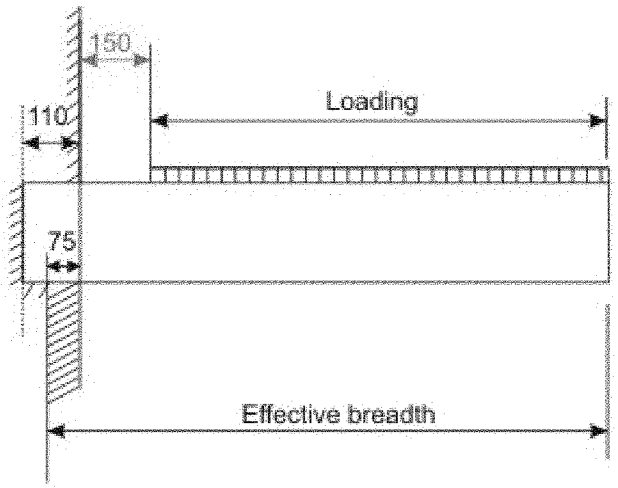

Fig. 9.20.7: Loading on staircases built into walls

Figure 9.20.6 shows one open-well stair where spans partly cross at right angle. The load in such stairs on areas common to any two such spans should be taken as fifty per cent in each direction as shown in Fig.9.20.7. Moreover, one 150 mm strip may be deducted from the loaded area and the effective breadth of the section is increased by 75 mm for the design where flights or landings are embedded into walls for a length of at least 110 mm and are designed to span in the direction of the flight (Fig.9.20.7).

9.20.8 Structural Analysis

Most of the structural systems of stair spanning longitudinally or transversely are standard problems of structural analysis, either statically determinate or indeterminate. Accordingly, they can be analysed by methods of analysis suitable for a particular system. However, the rigorous analysis is difficult and involved for a trade-riser type or free standing staircase where the slab is repeatedly folded. This type of staircase has drawn special attraction due to its aesthetic appeal and, therefore, simplified analysis for this type of staircase spanning longitudinally is explained below. It is worth mentioning that certain idealizations are made in the actual structures for the applicability of the simplified analysis. The designs based on the simplified analysis have been found to satisfy the practical needs.

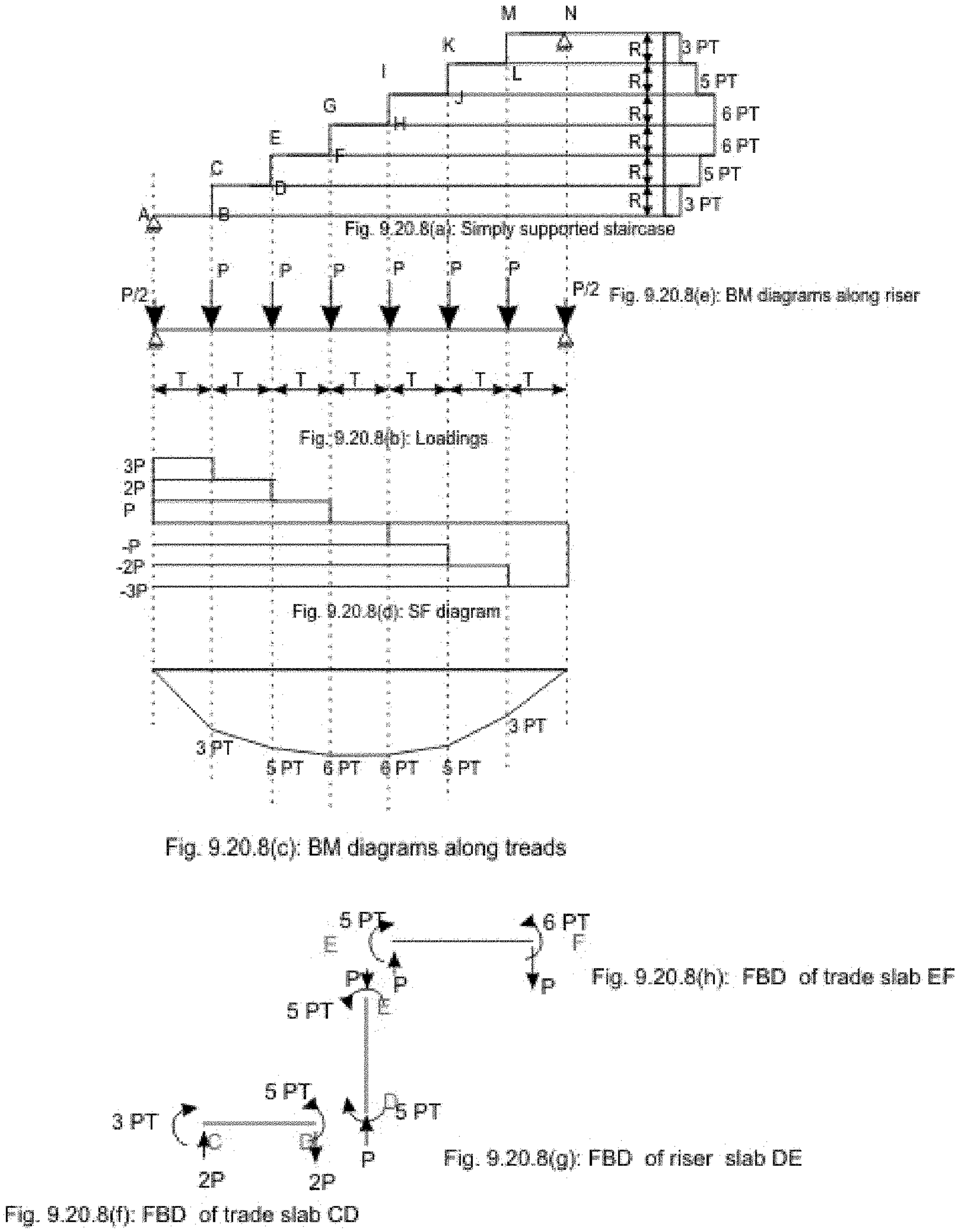

Fig. 9.20.8: Structural analysis of simply supported trade-riser staircase

Figure 9.20.8a shows the simply supported trade-riser staircase. The uniformly distributed loads are assumed to act at the riser levels (Fig.9.20.8b). The bending moment and shear force diagrams along the treads and the bending moment diagram along the risers are shown in Figs.9.20.8c, d and e, respectively. The free body diagrams of CD, DE and EF are shown in Figs.9.20.8f, g and h, respectively. It is seen that the trade slabs are subjected to varying bending moments and constant shear force (Fig.9.20.8f). On the other hand the riser slabs are subjected to a constant bending moment and axial force (either compressive or tensile). The assumption is that the riser and trade slabs are rigidly connected. It has been observed that both trade and riser slabs may be designed for bending moment alone as the shear stresses in trade slabs and axial forces in riser slabs are comparatively low. The slab thickness of the trade and risers should be kept the same and equal to span/25 for simply supported and span/30 for continuous stairs.

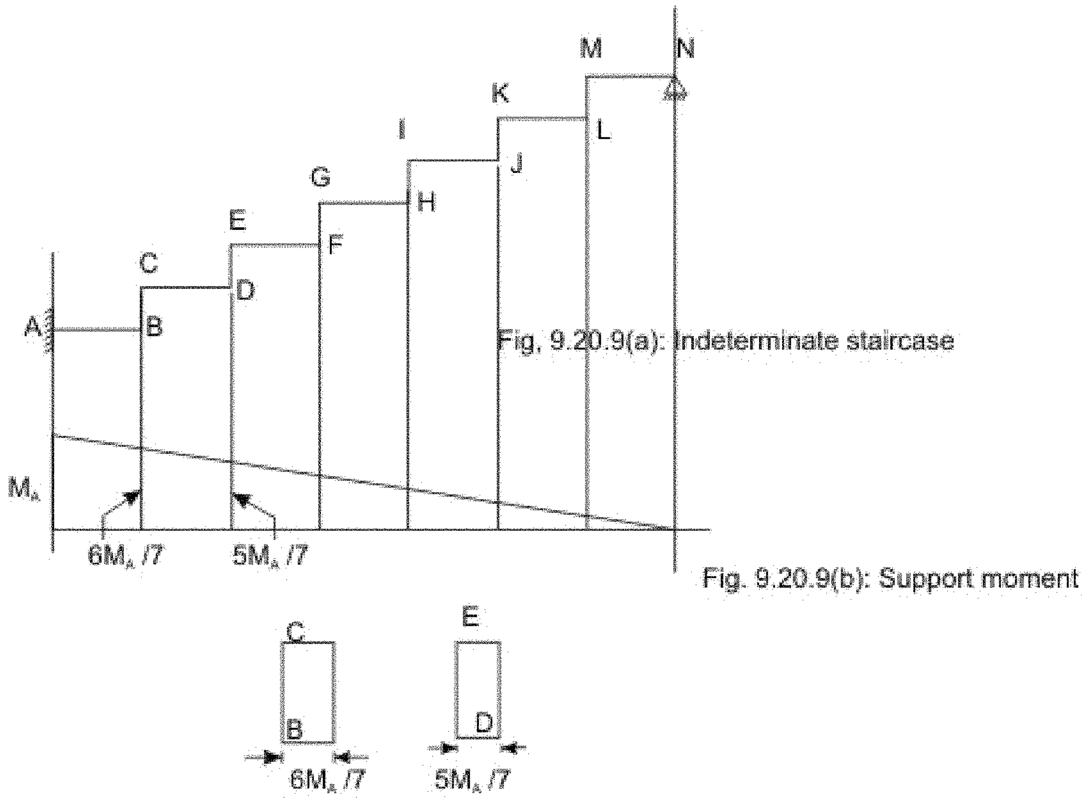

Fig. 9.20.9(c): FBD of BC Fig. 9.20.9(d): FBD of DE

Fig. 9.20.9: Structural analysis of an indeterminate trade-riser staircase

Figure 9.20.9a shows an indeterminate trade-riser staircase. Here, the analysis can be done by adding the effect of the support moment M_A (Fig.9.20.9b) with the results of earlier simply supported case. However, the value of M_A can be determined using the moment-area method. The free body diagrams of two vertical risers BC and DE are show in Figs.9.20.9c and d, respectively.

9.20.9 Illustrative Examples

Two typical examples of waist-slab and trade-riser types spanning longitudinally are taken up here to illustrate the design.

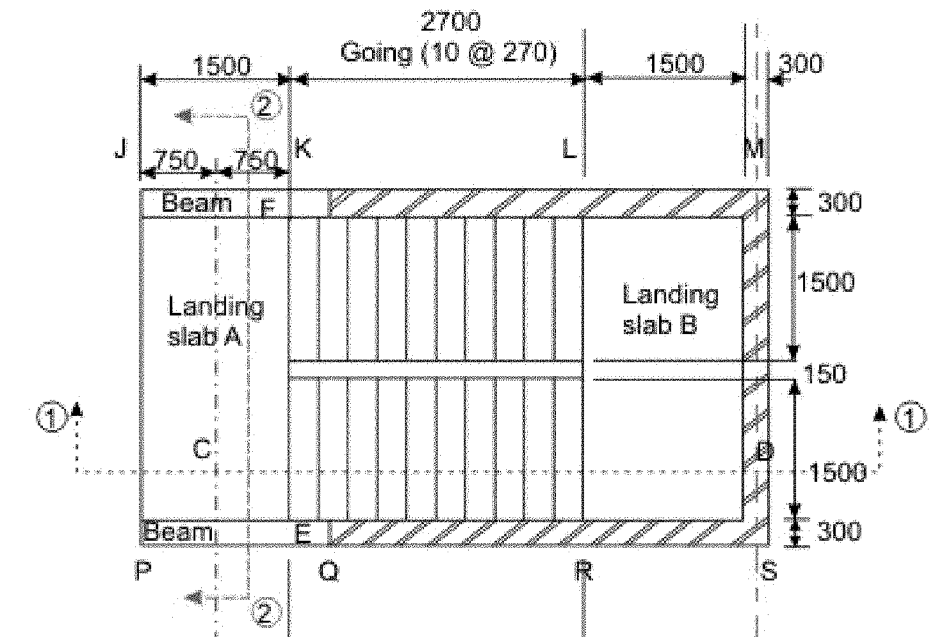

Fig. 9.20.10: Example 9.1

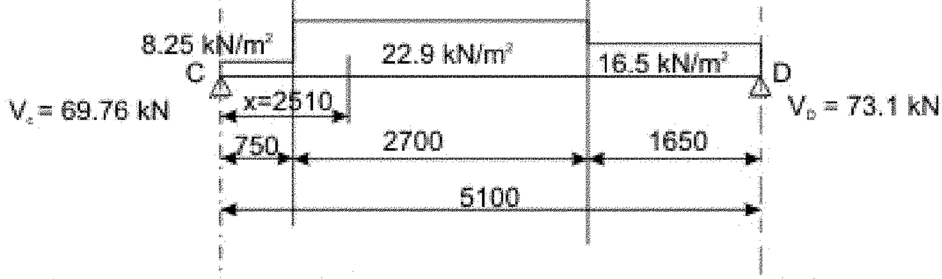

Fig. 9.20.11: Calculation of loads, sec 1-1 of Example 9.1, (Fig. 9.20.10)

Example 9.1:

Design the waist-slab type of the staircase of Fig.9.20.10. Landing slab A is supported on beams along JK and PQ, while the waist-slab and landing slab B are spanning longitudinally as shown in Fig.9.20.10. The finish loads and live loads are 1 kN/m^2 and 5 kN/m^2, respectively. Use riser R = 160 mm, trade T = 270 mm, concrete grade = M 20 and steel grade = Fe 415.

Solution:

With R = 160 mm and T = 270 mm, the inclined length of each step = $\{(160)^2 + (270)^2\}^{½}$ = 313.85 mm.

(A) Design of going and landing slab B

Step 1: Effective span and depth of slab

The effective span (cls. 33.1b and c) = 750 + 2700 + 1500 + 150 = 5100 mm. The depth of waist slab = 5100/20 = 255 mm. Let us assume total depth of 250 mm and effective depth = 250 – 20 – 6 = 224 mm (assuming cover = 20 mm and diameter of main reinforcing bar = 12 mm). The depth of landing slab is assumed as 200 mm and effective depth = 200 – 20 – 6 = 174 mm.

Step 2: Calculation of loads (Fig.9.20.11, sec. 1-1)

(i) Loads on going (on projected plan area)

(a) Self-weight of waist-slab = 25(0.25)(313.85)/270 = 7.265 kN/m^2

(b) Self-weight of steps = 25(0.5)(0.16) = 2.0 kN/m^2

(c) Finishes (given) = 1.0 kN/m^2

(d) Live loads (given) = 5.0 kN/m^2

Total = 15.265 kN/m^2

Total factored loads = 1.5(15.265) = 22.9 kN/m^2

(ii) Loads on landing slab A (50% of estimated loads)

(a) Self-weight of landing slab = 25(0.2) = 5 kN/m^2

(b) Finishes (given) = 1 kN/m^2

(c) Live loads (given) = 5 kN/m^2

Total = 11 kN/m^2

Factored loads on landing slab A = 0.5(1.5)(11) = 8.25 kN/m^2

(iii) Factored loads on landing slab B = (1.5)(11) = 16.5 kN/m^2

The loads are drawn in Fig.9.20.11.

Step 3: Bending moment and shear force (Fig. 9.20.11)

Total loads for 1.5 m width of flight = 1.5{8.25(0.75) + 22.9(2.7) + 16.5(1.65)}

= 142.86 kN

V_C = 1.5{8.25(0.75)(5.1 – 0.375) + 22.9(2.7)(5.1 – 0.75 – 1.35)

+ 16.5(1.65)(1.65)(0.5)}/5.1 = 69.76 kN

V_D = 142.86 – 69.76 = 73.1 kN

The distance x from the left where shear force is zero is obtained from:

x = {69.76 – 1.5(8.25)(0.75) + 1.5(22.9)(0.75)}/(1.5)(22.9) = 2.51 m

The maximum bending moment at x = 2.51 m is

= 69.76(2.51) – (1.5)(8.25)(0.75)(2.51 – 0.375)

- (1.5)(22.9)(2.51 – 0.75)(2.51 – 0.75)(0.5) = 102.08 kNm.

For the landing slab B, the bending moment at a distance of 1.65 m from D

= 73.1(1.65) – 1.5(16.5)(1.65)(1.65)(0.5) = 86.92 kNm

Step 4: Checking of depth of slab

From the maximum moment, we get d = $\{102080/2(2.76)\}^{½}$ = 135.98 mm < 224 mm for waist-slab and < 174 mm for landing slabs. Hence, both the depths of 250 mm and 200 mm for waist-slab and landing slab are more than adequate for bending.

For the waist-slab, τ_v = 73100/1500(224) = 0.217 N/mm^2. For the waist-slab of depth 250 mm, k = 1.1 (cl. 40.2.1.1 of IS 456) and from Table 19 of IS 456, τ_c = 1.1(0.28) = 0.308 N/mm^2. Table 20 of IS 456, $\tau_{c\max}$ = 2.8 N/mm^2. Since $\tau_v < \tau_c < \tau_{c\max}$, the depth of waist-slab as 250 mm is safe for shear.

For the landing slab, τ_v = 73100/1500(174) = 0.28 N/mm^2. For the landing slab of depth 200 mm, k = 1.2 (cl. 40.2.1.1 of IS 456) and from Table 19 of IS 456, τ_c = 1.2(0.28) = 0.336 N/mm^2 and from Table 20 of IS 456, $\tau_{c\max}$ = 2.8 N/mm^2. Here also $\tau_v < \tau_c < \tau_{c\max}$, so the depth of landing slab as 200 mm is safe for shear.

Step 5: Determination of areas of steel reinforcement

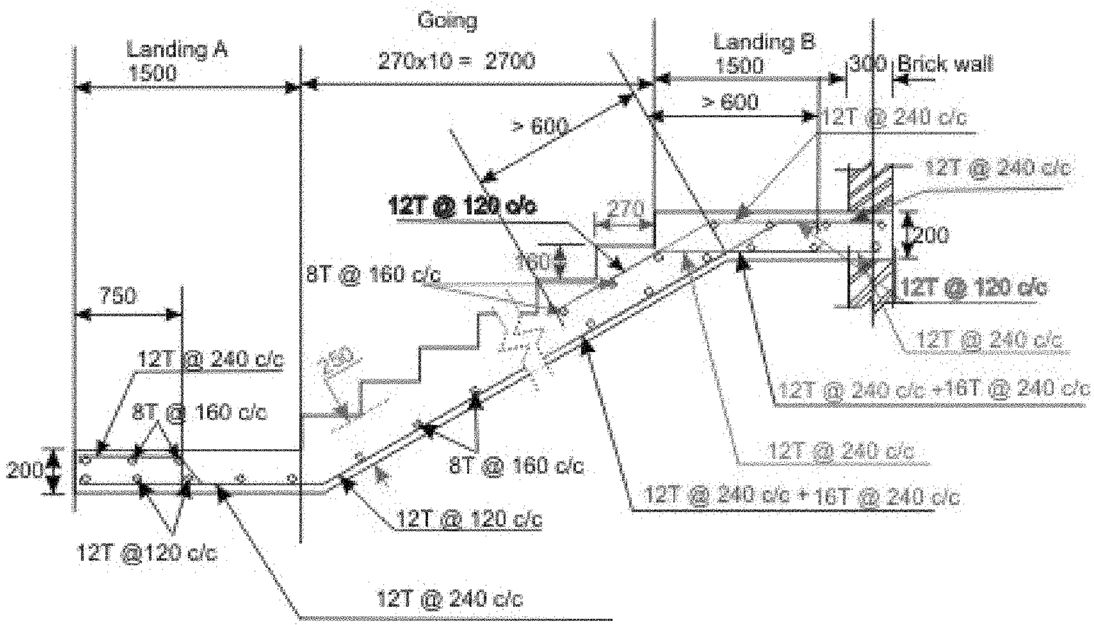

Fig. 9.20.12: Reinforcing bars of Example 9.1, sec 1-1 of Fig. 9.20.10

(i) Waist-slab: M_u/bd^2 = 102080/(1.5)224(224) = 1.356 N/mm^2. Table 2 of SP-16 gives p = 0.411.

The area of steel = 0.411(1000)(224)/(100) = 920.64 mm^2. Provide 12 mm diameter @ 120 mm c/c (= 942 mm^2/m).

(ii) Landing slab B: M_u/bd^2 at a distance of 1.65 m from V_D (Fig. 9.20.11) = 86920/(1.5)(174)(174) = 1.91 N/mm^2. Table 2 of SP-16 gives: p = 0.606. The area of steel = 0.606(1000) (174)/100 = 1054 mm^2/m. Provide 16 mm diameter @ 240 mm c/c and 12 mm dia. @ 240 mm c/c (1309 mm^2) at the bottom of landing slab B of which 16 mm bars will be terminated at a distance of 500 mm from the end and will continue up to a distance of 1000 mm at the bottom of waist slab (Fig. 9.20.12).

Distribution steel: The same distribution steel is provided for both the slabs as calculated for the waist-slab. The amount is = 0.12(250) (1000)/100 = 300 mm^2/m. Provide 8 mm diameter @ 160 mm c/c (= 314 mm^2/m).

Step 6: Checking of development length and diameter of main bars

Development length of 12 mm diameter bars = 47(12) = 564 mm, say 600 mm and the same of 16 mm dia. Bars = 47(16) = 752 mm, say 800 mm.

(i) For waist-slab

M_1 for 12 mm diameter @ 120 mm c/c (= 942 mm^2) = 942(102.08)/920.64 = 104.44 kNm. With V (shear force) = 73.1 kN, the diameter of main bars $\leq$ {1.3(104440)/73.1}/47 $\leq$ 39.5 mm. Hence, 12 mm diameter is o.k.

(ii) For landing-slab B

M_1 for 16 mm diameter @ 120 mm c/c (= 1675 mm^2) = 1675(102.08)/1650.88 = 103.57 kNm. With V (shear force) = 73.1 kN, the diameter of main bars $\leq$ {1.3(103570)/73.1}/47 = 39.18 mm. Hence, 16 mm diameter is o.k.

The reinforcing bars are shown in Fig.9.20.12 (sec. 1-1).

(B) Design of landing slab A

Step 1: Effective span and depth of slab

The effective span is lesser of (i) (1500 + 1500 + 150 + 174), and (ii) (1500 + 1500 + 150 + 300) = 3324 mm. The depth of landing slab = 3324/20 = 166 mm, < 200 mm already assumed. So, the depth is 200 mm.

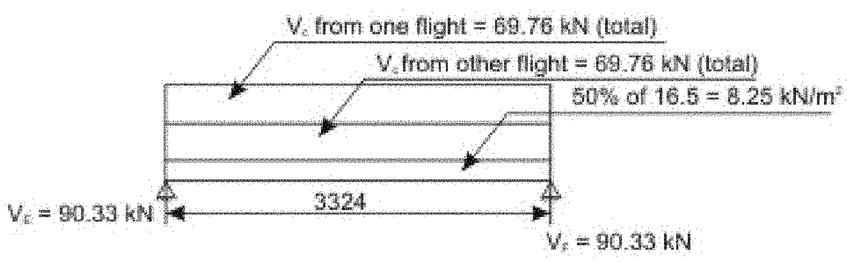

Fig. 9.20.13: Calculation of loads, sec 2-2 of Example 9.1, (Fig. 9.20.10)

Step 2: Calculation of loads (Fig.9.20.13)

The following are the loads:

(i) Factored load on landing slab A(see Step 2 of A @ 50%) = 8.25 kN/m^2

(ii) Factored reaction V_C (see Step 3 of A) = 69.76 kN as the total load of one flight

(iii) Factored reaction V_C from the other flight = 69.76 kN

Thus, the total load on landing slab A

= (8.25)(1.5)(3.324) + 69.76 + 69.76 = 180.65 kN

Due to symmetry of loadings, $V_E = V_F = 90.33$ kN. The bending moment is maximum at the centre line of EF.

Step 3: Bending moment and shear force (width = 1500 mm)

Maximum bending moment = (180.65)(3.324)/8 = 75.06 kNm

Maximum shear force = 0.5(180.65) = 90.33 kN

Step 4: Checking of depth of slab

In Step 3 of A, it has been observed that 135.98 mm is the required depth for bending moment = 102.08 kNm. So, the depth of 200 mm is safe for this bending moment of 75.06 kNm. However, a check is needed for shear force.

$$\tau_v = 90330/1500(174) = 0.347 \text{ N/mm}^2 > 0.336 \text{ N/mm}^2$$

The above value of τ_c = 0.336 N/mm^2 for landing slab of depth 200 mm has been obtained in Step 4 of A. However, here τ_c is for the minimum tensile steel in the slab. The checking of depth for shear shall be done after determining the area of tensile steel as the value of τ_v is marginally higher.

Step 5: Determination of areas of steel reinforcement

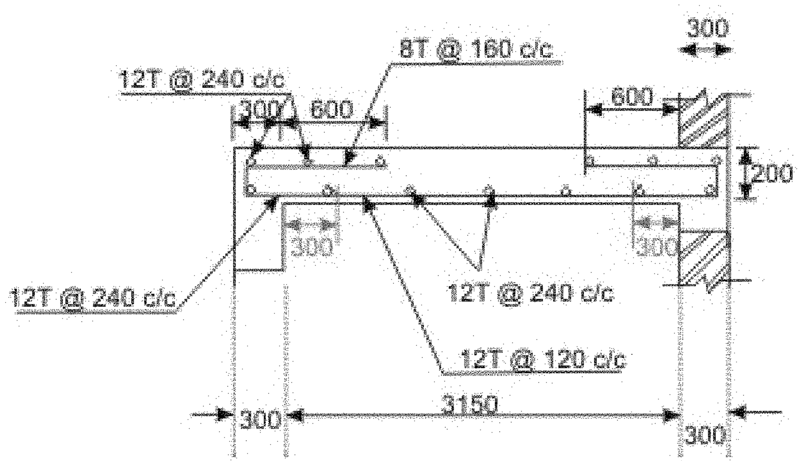

Fig. 9.20.14: Reinforcing bars of Example 9.1, sec 2-2 of Fig. 9.20.10

For M_u/bd^2 = 75060/(1.5)(174)(174) = 1.65 N/mm^2, Table 2 of SP-16 gives p = 0.512.

The area of steel = (0.512)(1000)(174)/100 = 890.88 mm^2/m. Provide 12 mm diameter @ 120 mm c/c (= 942 mm^2/m). With this area of steel p = 942(100)/1000(174) = 0.541.

Distribution steel = The same as in Step 5 of A i.e., 8 mm diameter @ 160 mm c/c.

Step 6: Checking of depth for shear

Table 19 and cl. 40.2.1.1 gives: τ_c = (1.2)(0.493) = 0.5916 N/mm^2. τ_v = 0.347 N/mm^2 (see Step 3 of B) is now less than τ_c (= 0.5916 N/mm^2). Since, $\tau_v < \tau_c < \tau_{c\max}$, the depth of 200 mm is safe for shear.

The reinforcing bars are shown in Fig. 9.20.14.

Example 9.2:

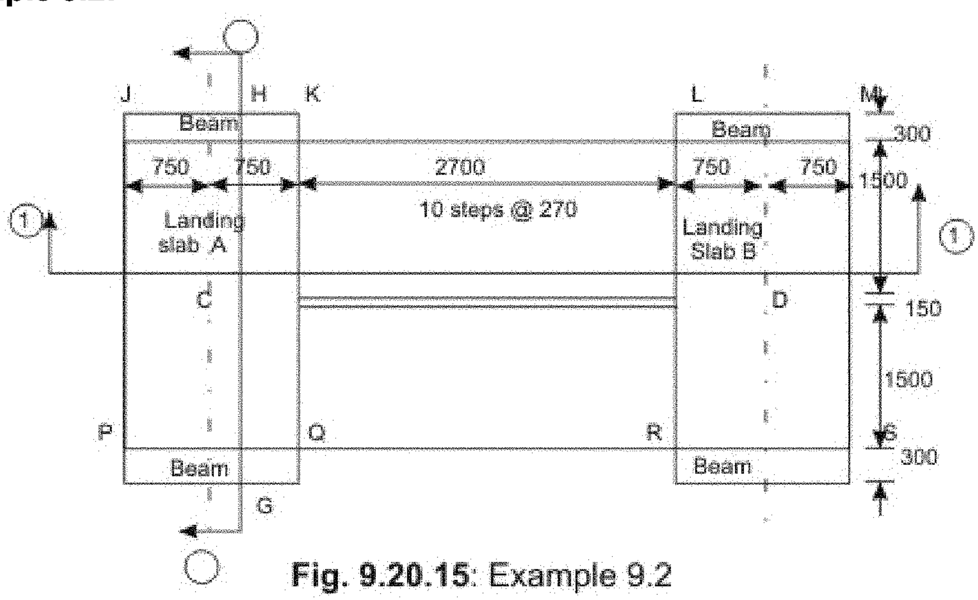

Fig. 9.20.15: Example 9.2

Design a trade-riser staircase shown in Fig.9.20.15 spanning longitudinally. Landing slabs are supported on beams spanning transversely. The dimensions of riser and trade are 160 mm and 270 mm, respectively. The finish loads and live loads are 1 kN/m^2 and 5 kN/m^2, respectively. Use M 20 and Fe 415.

Solution:

The distribution of loads on landings common to two spans perpendicular to each other shall be done as per cl. 33.2 of IS 456 (50% in each direction), since the going is supported on landing slabs which span transversely. The effective span in the longitudinal direction shall be taken as the distance between two centre lines of landings.

(A) Design of going

Step 1: Effective span and depth of slab

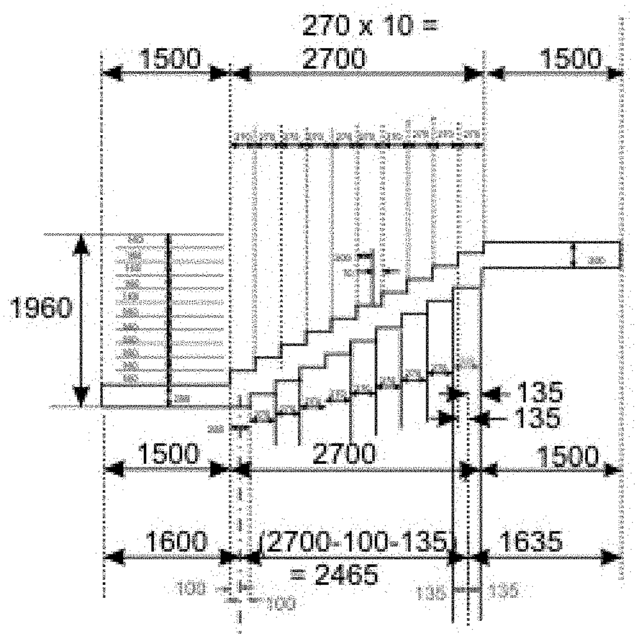

Fig. 9.20.16: Arrangement of loadings and Going of Example 9.2

Figure 9.20.16 shows the arrangement of the landings and going. The effective span is 4200 mm. Assume the thickness of trade-riser slab = 4200/25 = 168 mm, say 200 mm. The thickness of landing slab is also assumed as 200 mm.

Step 2: Calculation of loads (Fig. 9.20.17)

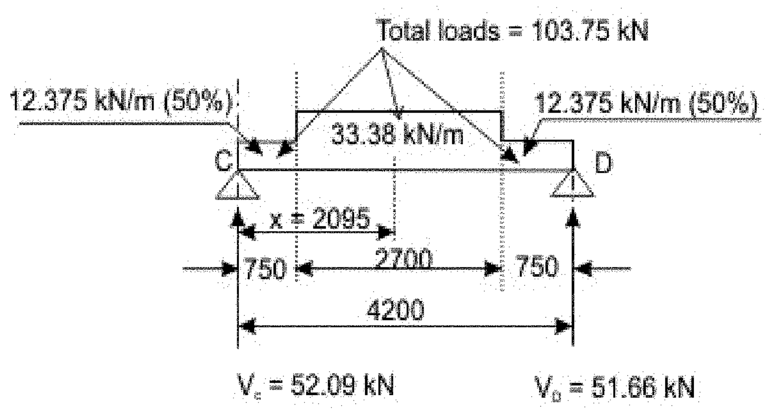

Fig. 9.20.17: Loads of section 1-1, Fig. 9.20.15, (Example 9.2)

The total loads including self-weight, finish and live loads on projected area of going (1500 mm x 2465 mm) is first determined to estimate the total factored loads per metre run.

(i) Self-weight of going

(a) Nine units of (0.2)(0.36)(1.5) @ 25(9) = 24.3 kN

(b) One unit of (0.27)(0.36)(1.5) @ 25(1) = 3.645 kN

(c) Nine units of (0.07)(0.2)(1.5) @ 25(9) = 4.725 kN

(ii) Finish loads @ 1 kN/m^2 = (1.5)(2.465)(1) = 3.6975 kN

(iii) Live loads @ 5 kN/m^2 = (1.5)(2.465)(5) = 18.4875 kN

Total = 54.855 kN

Factored loads per metre run = 1.5(54.855)/2.465 = 33.38 kN/m

(iv) Self-weight of landing slabs per metre run = 1.5(0.2)(25) = 7.5 kN/m

(v) Live loads on landings = (1.5)(5) = 7.5 kN/m

(vi) Finish loads on landings = (1.5)(1) = 1.5 kN/m

Total = 16.5 kN/m

Factored loads = 1.5(16.5) = 24.75 kN/m

Due to common area of landings only 50 per cent of this load should be considered. So, the loads = 12.375 kN/m. The loads are shown in Fig.9.20.17.

Step 3: Bending moment and shear force

Total factored loads = 33.38(2.465) + 12.375(0.85 + 0.885) = 103.75 kN

$$V_C = \{12.375(0.85)(4.2 - 0.425) + 33.38(2.465)(0.885 + 1.2325)$$

$$+ 12.375(0.885)(0.885)(0.5)\}/4.2 = 52.09 \text{ kN}$$

$$V_D = 103.75 - 52.09 = 51.66 \text{ kN}$$

The distance x from the left support where shear force is zero is now determined:

$52.09 - 12.375(0.85) - 32.38(x - 0.85) = 0$

or $x = \{52.09 - 12.375(0.85) + 33.38(0.85)\}/33.38 = 2.095$ m

Maximum factored bending moment at $x = 2.095$ m is

$52.09(2.095) - 12.375(0.85)(2.095 - 0.425) - 33.38(2.095 - 0.85)(2.095 - 0.85)(0.5)$

$= 65.69$ kNm

Step 4: Checking of depth of slab

From the maximum bending moment, we have

$d = \{(65690)/(1.5)(2.76)\}^{½} = 125.97 \text{ mm} < 174 \text{ mm}$

From the shear force $V_u = V_A$, we get $\tau_v = 52090/(1500)(174) = 0.199$ N/mm^2. From cl. 40.2.1.1 and Table 19 of IS 456, we have $\tau_c = (1.2)(0.28) = 0.336$ N/mm^2. Table 20 of IS 456 gives $\tau_{c\max} = 2.8$ N/mm^2. Since, $\tau_v < \tau_c < \tau_{c\max}$, the depth of 200 mm is accepted.

Step 5: Determination of areas of steel reinforcement

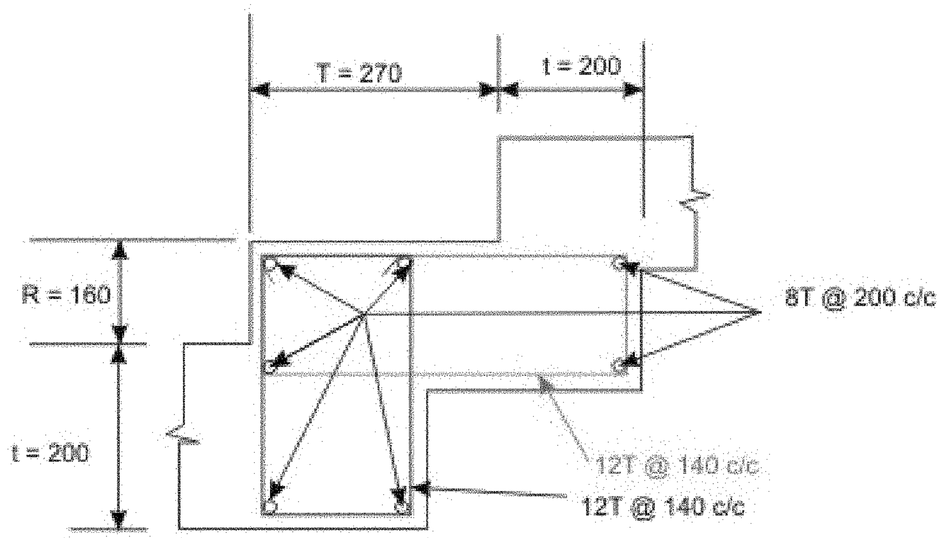

Fig. 9.20.18: Reinforcing bars - Example 9.2

$M_u/bd^2 = 65.69(10^6)/(1500)(174)(174) = 1.446$ N/mm^2

Table 2 of SP-16 gives, p = 0.4416, to have A_{st} = 0.4416(1000)(174)/100 = 768.384 mm^2/m. Provide 12 mm diameter bars @ 140 mm c/c (= 808 mm^2) in form of closed ties (Fig.9.20.18).

Distribution bars: Area of distribution bars = 0.12(1000)(200)/100 = 240 mm^2/m.

Provide 8 mm diameter bars @ 200 mm c/c. The reinforcing bars are shown in Fig. 9.20.18.

(B) Design of landing slab

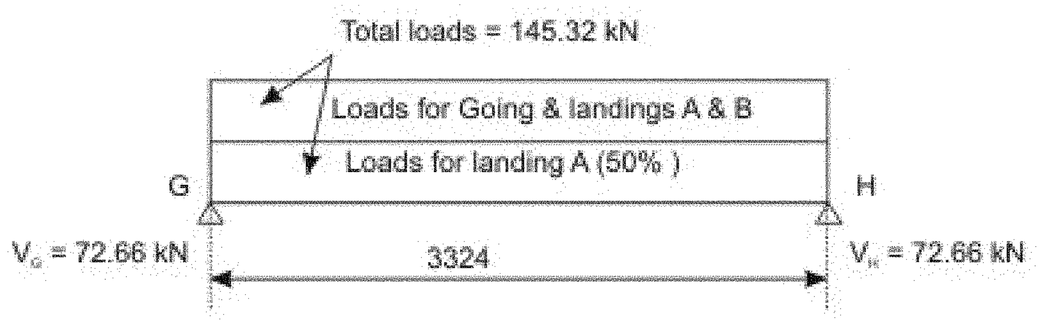

Fig. 9.20.19: Loads and reactions (Example 9.2)

Step 1: Effective span and depth of slab

With total depth D = 200 mm and effective depth d = 174 mm, the effective span (cl. 22.2a) = lesser of (1500 + 150 + 1500 + 174) and (1500 + 150 + 1500 + 300) = 3324 mm.

Step 2: Calculation of loads (Fig.9.20.19)

(i) Factored load of landing slab A = 50% of Step 2 (iv to vi) @ 12.375 kN/m = 12.375(3.324 = 41.1345 kN

(ii) Factored reaction V_C from one flight (see Step 3) = 52.09 kN

(iii) Factored reaction V_C from other flight = 52.09 kN

Total factored load = 145.32 kN. Due to symmetry of loads, V_G = V_H = 72.66 kN. The bending moment is maximum at the centre line of GH.

Step 3: Bending moment and shear force (width b = 1500 mm)

Maximum bending moment = 145.32(3.324)/8 = 60.38 kNm

Maximum shear force $V_G = V_H = 145.32/2 = 72.66$ kN

Step 4: Checking of depth of slab

From bending moment: $d = \{60380/(1.5)(2.76)\}^{1/2} = 120.77$ mm < 174 mm. Hence o.k.

From shear force: $\tau_v = 72660/(1500)(174) = 0.278$ N/mm^2

From Step 4 of A: $\tau_c = 0.336$ N/mm^2, $\tau_{c\max} = 2.8$ N/mm^2. Hence, the depth is o.k. for shear also.

Step 5: Determination of areas of steel reinforcement

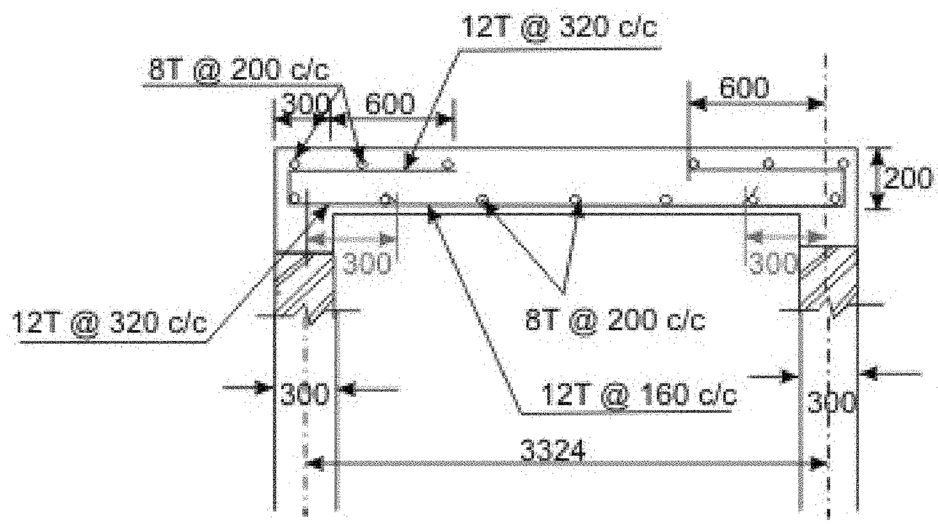

Fig. 9.20.20: Reinforcing bars of Example 9.2

$M_u/bd^2 = 60380/(1.5)(174)(174) = 1.33$ N/mm^2

Table 2 of SP-16 gives, $p = 0.4022$. So, $A_{st} = 0.4022(1000)(174)/100 = 699.828$ mm^2. Provide 12 mm diameter bars @ 160 mm c/c (= 707 mm^2).

Distribution steel area = (0.12/100)(1000)(200) = 240 mm^2

Provide 8 mm diameter @ 200 mm c/c (= 250 mm^2).

Step 6: Checking of development length

The moment M_1 for 12 mm @ 160 mm c/c (707 mm^2) = (707/699.828)60.38 = 60.998 kNm.

The shear force V = 72.66 kN. The diameter of the bar should be less/equal to {(1.3)(60.998)(10^6)/72.66(10^3)}/47 = 23.2 mm. Hence 12 mm diameter bars are o.k.

Use L_d = 47(12) = 564 mm, = 600 mm (say). The reinforcing bars are shown in Fig. 9.20.20.

9.20.10 Practice Questions and Problems with Answers

Q.1: Name five types of staircases based on geometrical configurations.

A.1: See sec. 9.20.2.

Q.2: Draw a typical flight and show: (a) trade, (b) nosing, (c) riser, (d) waist and (e) going.

A.2: Figures 9.20.2a, b and c (sec. 9.20.3)

Q.3: Mention four general considerations for the design of a staircase.

A.3: See sec. 9.20.4.

Q.4: Draw schematic diagrams of different types of staircases based on different structural systems.

A.4: Figures 9.20.3a, b, c, d, e; Figs.9.20.4a and b; Figs.9.20.5a, b and c (sec.9.20.5).

Q.5: Explain the method of determining the effective spans of stairs.

A.5: See sec. 9.20.6 (Table 9.1 also).

Q.6: Explain the distribution of loadings of open-well stairs and for those where the landings are embedded in walls.

A.6: See sec. 9.20.7.

Q.7: Illustrate the simplified analysis of longitudinally spanning free-standing staircases.

A.7: See sec. 9.20.8.

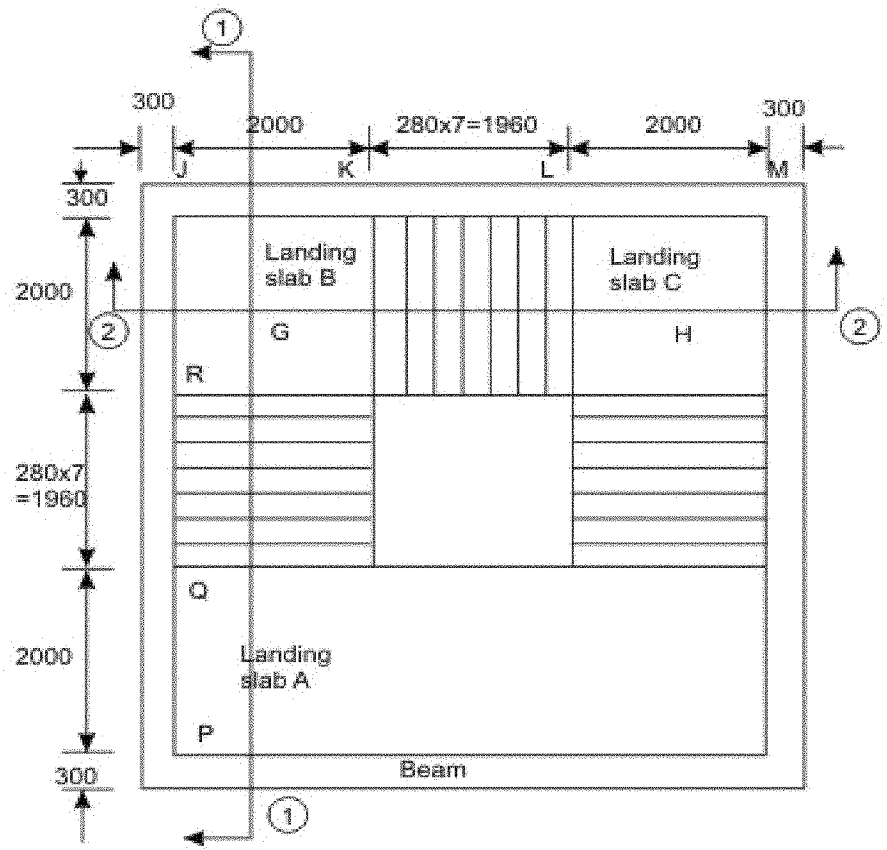

Fig. 9.20.21: Example Q.8

Q.8: Design the open-well staircase of Fig.9.20.21. The dimensions of risers and trades are 160 mm and 270 mm, respectively. The finish loads and live loads are 1 kN/m^2 and 5 kN/m^2, respectively. Landing A has a beam at the edge while other landings (B and C) have brick walls. Use concrete of grade M 20 and steel of grade Fe 415.

A.8:

Solution:

In this case landing slab A is spanning longitudinally along sec. 11 of Fig.9.20.21. Landing slab B is common to spans of sec. 11 and sec. 22, crossing at right angles. Distribution of loads on landing slab B shall be made 50 per cent in each direction (cl. 33.2 of IS 456). The effective span for sec. 11 shall be from the centre line of edge beam to centre line of brick wall, while the effective span for sec. 22 shall be from the centre line of landing slab B to centre line of landing slab C (cl. 33.1b of IS 456).

(A) Design of landing slab A and going (sec. 11 of Fig.9.20.21)

Step 1: Effective span and depth of slab

The effective span = 150 + 2000 + 1960 + 1000 = 5110 mm. The depth of waist slab is assumed as 5110/20 = 255.5 mm, say 250 mm. The effective depth = 250 – 20 – 6 = 224 mm. The landing slab is also assumed to have a total depth of 250 mm and effective depth of 224 mm.

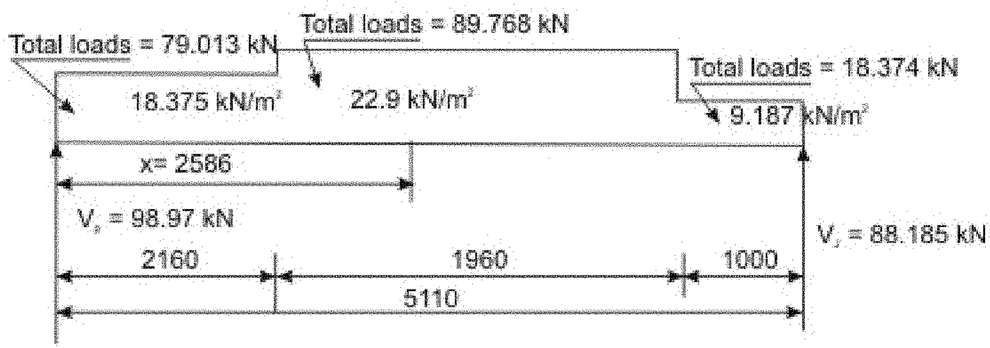

Fig 9.20.22: Calculation of loads sec 1-1 of Fig. 9.20.21, Example 9.8

Step 2: Calculation of loads (Fig.9.20.22)

(i) Loads on going (on projected plan area)

(a) Self weight of waist slab = 25(0.25)(313.85/270) = 7.265 kN/m^2

(b) Self weight of steps = 25(0.5)(0.16) = 2.0 kN/m^2

(c) Finish loads (given) = 1.0 kN/m^2

(d) Live loads (given) = 5.0 kN/m^2

Total = 15.265 kN/m^2

So, the factored loads = 1.5(15.265) = 22.9 kN/m^2

(ii) Landing slab A

(a) Self weight of slab = 25(0.25) = 6.25 kN/m^2

(b) Finish loads = 1.00 kN/m^2

(c) Live loads = 5.00 kN/m^2

Total = 12.25 kN/m^2

Factored loads = 1.5(12.25) = 18.375 kN/m^2

(iii) Landing slab B = 50 per cent of loads of landing slab A = 9.187 kN/m^2

The total loads of (i), (ii) and (iii) are shown in Fig.9.22.

Total loads (i) going $= 22.9(1.96)(2) = 89.768$ kN

Total loads (ii) landing slab A $= 18.375(2.15)(2) = 79.013$ kN

Total loads (iii) landing slab B $= 9.187(1.0)(2) = 18.374$ kN

Total loads $= 187.155$ kN

The loads are shown in Fig. 9.20.22.

Step 3: Bending moment and shear force (width = 2.0 m, Fig. 9.20.22)

$V_P = \{79.013(5.11 - 1.075) + 89.768(5.11 - 3.13) + 18.374(0.5)\}/5.11$

$= 98.97$ kN

$V_J = 187.155 - 98.97 = 88.185$ kN

The distance x where the shear force is zero is obtained from:

$98.97 - 79.013 - 22.9(2)(x - 2.15) = 0$

or $x = 2.15 + (98.97 - 79.013)/22.9(2) = 2.586$ m

Maximum bending moment at $x = 2.586$ m (width = 2 m)

$= 98.97(2.586) - 79.013 - (22.9)(2)(0.436)(0.436)(0.5) = 161.013$ kNm

Maximum shear force $= 98.97$ kN

Step 4: Checking of depth

From the maximum moment $d = \{161.013(10^3)/2(2.76)\}^{1/2} = 170.8$ mm < 224 mm. Hence o.k.

From the maximum shear force, $\tau_v = 98970/2000(224) = 0.221$ N/mm^2. For the depth of slab as 250 mm, $k = 1.1$(cl. 40.2.1.1 of IS 456) and $\tau_c = 1.1(0.28) = 0.308$ N/mm^2 (Table 19 of IS 456). $\tau_{c\max} = 2.8$ N/mm^2 (Table 20 of IS 456). Since, $\tau_v < \tau_c < \tau_{c\max}$, the depth of slab as 250 mm is safe.

Step 5: Determination of areas of steel reinforcement

M_u/bd^2 = 161.013(10^3)/2(224)(224) = 1.60 N/mm^2. Table 2 of SP-16 gives p = 0.494, to have A_{st} = 0.494(1000)(224)/100 = 1106.56 mm^2/m. Provide 12 mm diameter bars @ 100 mm c/c (= 1131 mm^2/m) both for landings and waist slab.

Distribution reinforcement = 0.12(1000)(250)/100 = 300 mm^2/m. Provide 8 mm diameter @ 160 mm c/c (= 314 mm^2).

Step 6: Checking of development length

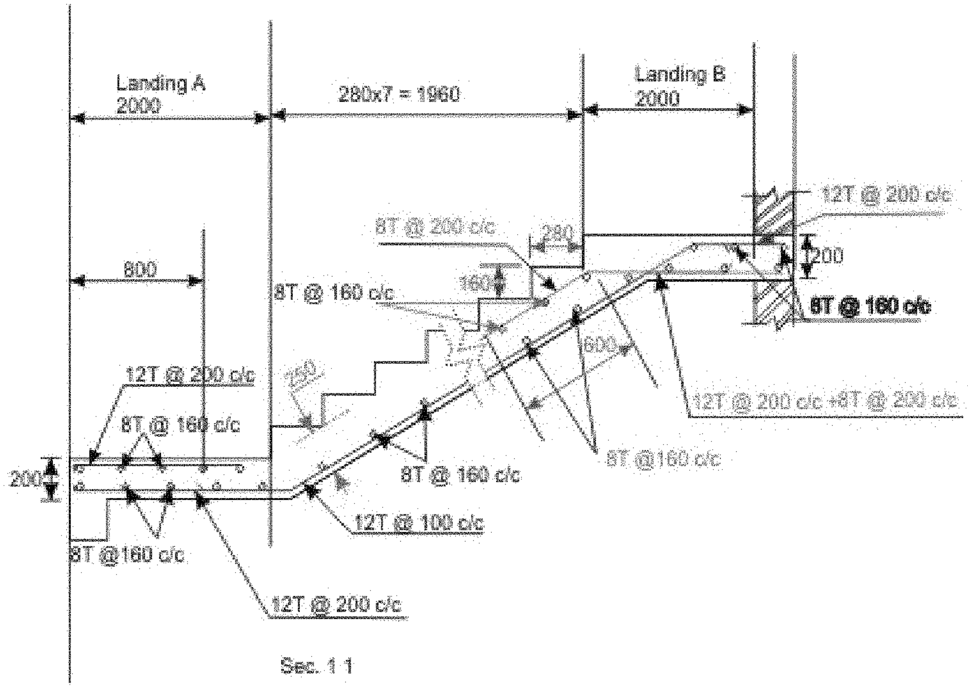

Fig. 9.20.23: Reinforcing bars, sec 1-1 of Fig 9.20.21, Example Q.8

Development length of 12 mm diameter bars 7(12) = 564 mm. Provide L_d = 600 mm.

For the slabs M_1 for 12 mm diameter @ 100 mm c/c = (1131)(161.013)/1106.56 = 164.57 kNm. Shear force = 98.97 kN. Hence, 47 $\phi \le$ 1.3(164.57)/98.97 $\le$ 2161.67 mm or the diameter of main bar $\phi \le$ 45.99 mm. Hence, 12 mm diameter is o.k. The reinforcing bars are shown in Fig.9.20.23.

(B) Design of landing slabs B and C and going (sec. 22 of Fig.9.20.21)

Step 1: Effective span and depth of slab

The effective span from the centre line of landing slab B to the centre line of landing slab C = 1000 + 1960 + 1000 = 3960 mm. The depths of waist slab and landing slabs are maintained as 250 mm like those of sec. 11.

Step 2: Calculation of loads (Fig.9.20.24)

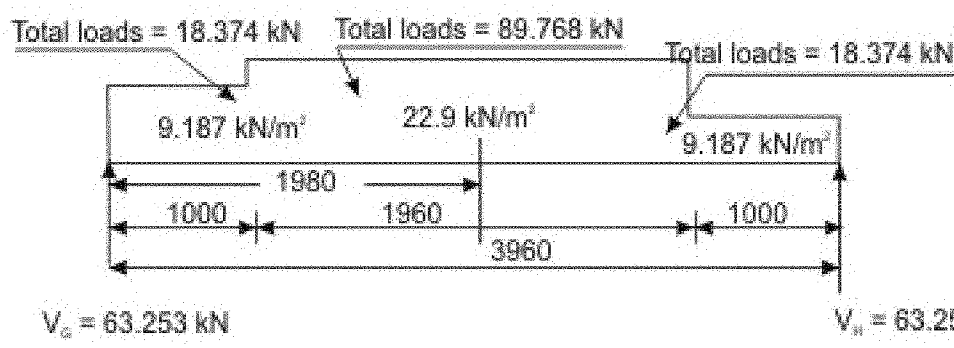

Fig 9.20.24: Calculation of loads, sec 2-2 of Fig. 9.20.21, Example Q.8

(i) Loads on going (Step 2(i) of A) = 22.9 kN/m^2

(ii) Loads on landing slab B (Step 2(iii)) = 9.187 kN/m^2

(iii) Loads on landing slab C (Step 2(iii)) = 9.187 kN/m^2

Total factored loads are:

(i) Going = 22.9(1.96)(2) = 89.768 kN

(ii) Landing slab A = 9.187(1.0)(2) = 18.374 kN

(iii) Landing slab B = 9.187(1.0)(2) = 18.374 kN

Total = 126.506 kN

The loads are shown in Fig.9.20.24.

Step 3: Bending moment and shear force (width = 2.0 m, Fig.9.20.24)

The total load is 126.506 kN and symmetrically placed to give $V_G = V_H =$ 63.253 kN. The maximum bending moment at $x = 1.98$ m (centre line of the span 3.96 m = 63.253(1.98) – 18.374(1.98 – 0.5) – 22.9(2)(0.98)(0.98)(0.5) = 76.05 kNm. Maximum shear force = 63.253 kN.

Since the maximum bending moment and shear force are less than those of the other section (maximum moment = 161.013 kNm and maximum shear force = 98.97 kN), the depth of 250 mm here is o.k. Accordingly, the amount of reinforcing bars are determined.

Step 4: Determination of areas of steel reinforcement

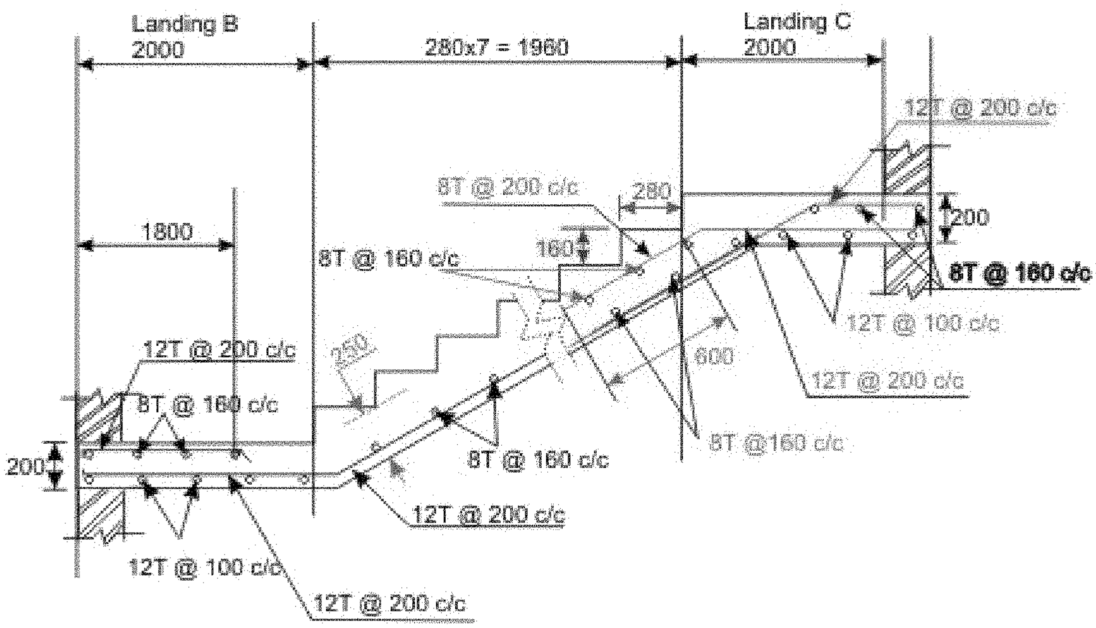

Fig. 9.20.25: Reinforcing bars, sec 2-2 of Fig 9.20.21, Example Q.8

M_u/bd^2 = 76.05(10^3)/2(224)(224) = 0.76 N/mm^2. Table 2 of SP-16 gives p = 0.221. The area of steel = (0.221)(1000)(224)/100 = 495.04 mm^2. Providing 12 mm diameter @ 220 mm c/c gives 514 mm^2, however let us provide 12 mm diameter @ 200 mm c/c (565 mm^2) as it is easy to detail with 12 mm diameter @ 100 mm c/c for the other section. Distribution bars are same as for sec. 11 i.e., 8 mm diameter @ 160 mm c/c.

Step 5: Checking of development length

For the slab reinforcement 12 mm dia. @ 200 mm c/c, M_1 = (565)(76.05)/495.04 = 86.80 kNm, V = 63.25 kN. So, the diameter of main bar ϕ $\leq$ {(1.3)(86.80)(10_3)/(63.25)}/47, i.e., $\leq$ 37.96 mm. Hence, 12 mm diameter bars are o.k. Distribution steel shall remain the same as in sec. 11, i.e., 8 mm diameter @ 160 mm c/c.

The reinforcing bars are shown in Fig.9.20.25. Figures 9.20.23 and 9.20.25 show the reinforcing bars considered separately. However, it is worth mentioning that the common areas (landing B and C) will have the bars of larger areas of either section eliminating the lower bars of other section.

9.20.11 References

1. Reinforced Concrete Limit State Design, 6th Edition, by Ashok K. Jain, Nem Chand & Bros, Roorkee, 2002.

2. Limit State Design of Reinforced Concrete, 2nd Edition, by P.C.Varghese, Prentice-Hall of India Pvt. Ltd., New Delhi, 2002.
3. Advanced Reinforced Concrete Design, by P.C.Varghese, Prentice-Hall of India Pvt. Ltd., New Delhi, 2001.
4. Reinforced Concrete Design, 2nd Edition, by S.Unnikrishna Pillai and Devdas Menon, Tata McGraw-Hill Publishing Company Limited, New Delhi, 2003.
5. Limit State Design of Reinforced Concrete Structures, by P.Dayaratnam, Oxford & I.B.H. Publishing Company Pvt. Ltd., New Delhi, 2004.
6. Reinforced Concrete Design, 1st Revised Edition, by S.N.Sinha, Tata McGraw-Hill Publishing Company. New Delhi, 1990.
7. Reinforced Concrete, 6th Edition, by S.K.Mallick and A.P.Gupta, Oxford & IBH Publishing Co. Pvt. Ltd. New Delhi, 1996.
8. Behaviour, Analysis & Design of Reinforced Concrete Structural Elements, by I.C.Syal and R.K.Ummat, A.H.Wheeler & Co. Ltd., Allahabad, 1989.
9. Reinforced Concrete Structures, 3rd Edition, by I.C.Syal and A.K.Goel, A.H.Wheeler & Co. Ltd., Allahabad, 1992.
10. Textbook of R.C.C, by G.S.Birdie and J.S.Birdie, Wiley Eastern Limited, New Delhi, 1993.
11. Design of Concrete Structures, 13th Edition, by Arthur H. Nilson, David Darwin and Charles W. Dolan, Tata McGraw-Hill Publishing Company Limited, New Delhi, 2004.
12. Concrete Technology, by A.M.Neville and J.J.Brooks, ELBS with Longman, 1994.
13. Properties of Concrete, 4th Edition, 1st Indian reprint, by A.M.Neville, Longman, 2000.
14. Reinforced Concrete Designer's Handbook, 10th Edition, by C.E.Reynolds and J.C.Steedman, E & FN SPON, London, 1997.
15. Indian Standard Plain and Reinforced Concrete – Code of Practice (4th Revision), IS 456: 2000, BIS, New Delhi.
16. Design Aids for Reinforced Concrete to IS: 456 – 1978, BIS, New Delhi.

9.20.12 Test 20 with Solutions

Maximum Marks = 50, Maximum Time = 1 hour

Answer all questions.

TQ.1: Draw a typical flight and show: (a) trade, (b) nosing, (c) riser, (d) waist and (e) going.
(5 marks)

A.TQ.1: Figures 9.20.2a, b and c (sec. 9.20.3)

TQ.2: Draw schematic diagrams of different types of staircases based on different structural systems.
(10 marks)

A.TQ.2: Figures 9.20.3a, b, c, d, e; Figs.9.20.4a and b; Figs.9.20.5a, b and c (sec.9.20.5).

TQ.3: Illustrate the simplified analysis of longitudinally spanning free-standing staircases.
(10 marks)

A.TQ.3: See sec. 9.20.8.

TQ.4: Design the staircase of illustrative example 9.1 of Fig.9.20.10 if supported on beams along KQ and LR only making both the landings A and B as cantilevers. Use the finish loads = 1 kN/m^2, live loads = 5 kn/m^2, riser R = 160 mm, trade T = 270 mm, grade of concrete = M 20 and grade of steel = Fe 415.

(25 marks)

A.TQ.4:

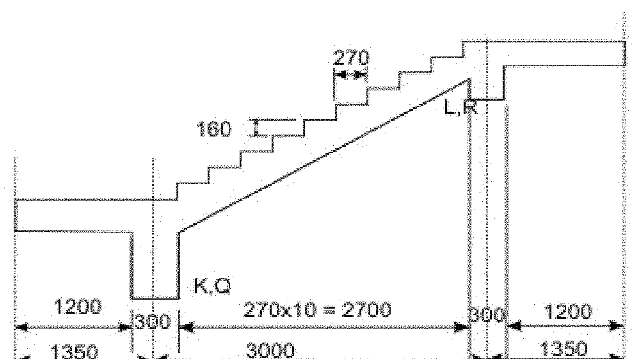

Fig. 9.20.26: General arrangement of staircase (TQ.4)
(sec 1-1 of Fig. 9.20.10)

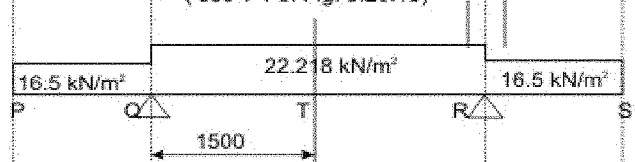

Fig. 9.20.27: Calculation of loads (TQ.4)

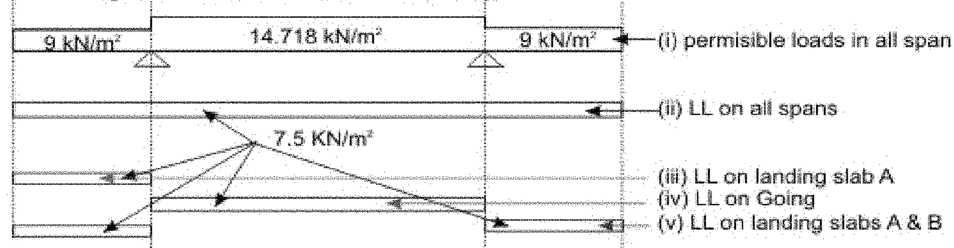

Fig. 9.20.28: Five cases of loadings (TQ.4)

Solution:

The general arrangement is shown in Fig.9.20.26. With R = 160 mm and T = 270 mm, the inclined length of each step = $\{(160)^2 + (270)^2\}^{1/2}$ = 313.85 mm. The structural arrangement is that the going is supported from beams along KQ and LR and the landings A and B are cantilevers.

Step 1: Effective span and depth of slab

As per cl. 33.1a of IS 456, the effective span of going = 3000 mm and as per cl. 22.2c of IS 456, the effective length of cantilever landing slabs = 1350 mm. The depth of waist slab and landing is kept at 200 mm (greater of 3000/20 and 1350/7). The effective depth = 200 – 20 – 6 = 174 mm.

Step 2: Calculation of loads

(i) Loads on going (on projected plan area)

(a) Self weight of waist-slab = 25(0.20)(313.85)/270 = 6.812 kN/m^2

(b) Self weight of steps = 25(0.5)(0.16) = 2.0 kN/m^2

(c) Finishes (given) = 1.0 kN/m^2

(d) Live loads (given) = 5.0 kN/m^2

Total: 14.812 kN/m^2

Total factored loads = 1.5(14.812) = 22.218 kN/m^2

(ii) Loads on landing slabs A and B

(a) Self weight of landing slabs = 25(0.2) = 5 kN/m^2

(b) Finishes (given) = 1 kN/m^2

(c) Live loads (given) = 5 kN/m^2

Total: 11 kN/m^2

Total factored loads = 1.5(11) = 16.5 kN/m^2. The total loads are shown in Fig.9.20.27.

Step 3: Bending moments and shear forces

Here, there are two types of loads: (i) permanent loads consisting of self-weights of slabs and finishes for landings and self-weights of slab, finishes and steps for going, and (ii) live loads. While the permanent loads will be acting everywhere all the time, the live loads can have several cases. Accordingly, five different cases are listed below. The design moments and shear forces will be considered taking into account of the values in each of the cases,. The different cases are (Fig.9.20.28):

(i) Permanent loads on going and landing slabs

(ii) Live loads on going and landing slabs

(iii) Live loads on landing slab A only

(iv) Live loads on going only

(v) Live loads on landing slabs A and B only

The results of V_Q, V_R, negative bending moment at Q and positive bending moment at T (Fig.9.20.27) are summarized in Table 9.2. It is seen from Table 9.2 that the design moments and shear forces are as follows:

(a) Positive bending moment = 25.195 kNm at T for load cases (i) and (iv).

(b) Negative bending moment = -22.553 kNm at Q for load cases (i) and (ii).

(c) Maximum shear force = 83.4 kN at Q and R for load cases (i) and (ii).

Table 9.2 Values of reaction forces and bending moments for different cases of loadings (Example: TQ.4, Figs. 9.20.26 to 9.20.28)

Case	V_Q (kN)	V_R (kN)	Negative moment at Q (kNm)	Positive moment at T (kNm)
Permanent loads on going and landings	+51.34	+51.34	-12.302	+12.535
Live loads on going and landings	+32.06	+32.06	-10.251	+2.404
Live loads on landing A only	+18.605	-3.417	-10.251	-5.13
Live loads on	+16.875	+16.875	0	+2.66

going only				
Live loads on landings A and B only	+15.1875	+15.1875	-10.251	-10.251
Critical positive *M* (i) + (iv)	+68.215	+68.215	-12.302	+25.195
Critical negative *M* (i) + (ii)	+83.40	+83.40	-22.553	+14.939

Step 4: Checking of depth of slab

The depth is checked for the positive moment of 25.195 kNm as the two depths are the same. The effective depth of slab $d = \{25.195(10^6)/1500(2.76)\}^{1/2}$ = 78 mm < 174 mm. Hence o.k.

The nominal shear stress τ_v = 83400/1500(174) = 0.3195 N/mm^2. Using the value of k = 1.2 (cl. 40.2.11 of IS 456) and from Table 19 of IS 456, we get τ_c = 1.2(0.28) = 0.336 N/mm^2 and $\tau_{c\max}$ = 2.8 N/mm^2 from Table 20 of IS 456. Since, $\tau_v < \tau_c < \tau_{c\max}$, the depth of 200 m is safe against shear.

Step 5: Determination of areas of steel reinforcement

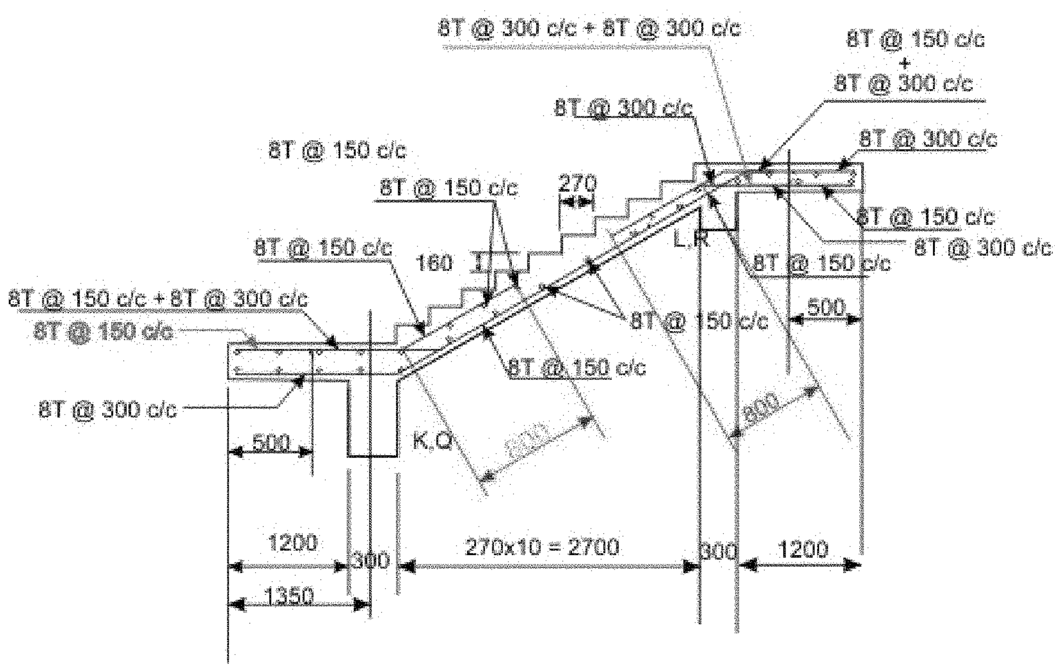

Fig. 9.20.29: Reinforcing bars of Example TQ.4

(i) Waist slab: M_u/bd^2 = 25195/1.5(174)(174) = 0.555 N/mm^2. Table 2 of SP-16 gives: p = 0.165. Accordingly, A_{st} = 0.165(1000)(174)/100 = 287.1 mm^2/m. Provide 8 mm dia. bars @ 150 mm c/c (= 335 mm^2).

(ii) Landing slab: Since the difference of positive and negative bending moments is not much, same reinforcement bars i.e., 8 mm diameter @ 150 mm c/c is used as positive and negative steel bars of waist and landing slabs.

Distribution bars: 0.12(200)(1000)/100 = 240 mm^2/m. The same bar i.e., 8 mm diameter @ 150 mm c/c can be used as distribution bar. The extra amount is useful to take care of change of ending moments due to different cases of loadings.

The reinforcement bars are shown in Fig.9.20.29.

9.20.13 Summary of this Lesson

This lesson explains the different types of staircases based on geometrical considerations. The different terminologies commonly used in a typical flight are mentioned. Important guidelines to be considered at the planning stage of the staircase are discussed. The classification of staircases based on structural system is explained. The distribution of loadings, determination of effective spans and selection of preliminary discussions of trade, riser and depth of slabs are illustrated. Simplified analysis procedures of staircases including free-standing staircase are explained. Several numerical problems are solved in the illustrative examples, practice problem and test, which will help to understand the applications of all the guidelines and the design of different types of staircases.