GEOMETRY

INTRODUCTION

Line : A line has length. It has neither width nor thickness. It can be extended indefinitely in both directions.

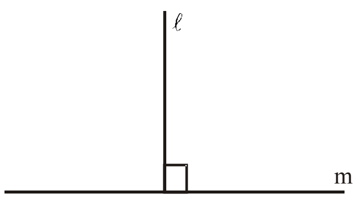
Ray : A line with one end point is called a ray. The end point is called the origin.

Line segment : A line with two end points is called a segment.

Parallel lines : Two lines, which lie in a plane and do not intersect, are called parallel lines. The distance between two parallel lines is constant.

We denote it by PQ || AB.

Perpendicular lines : Two lines, which lie in a plane and intersect each other at right angles are called perpendicular lines.

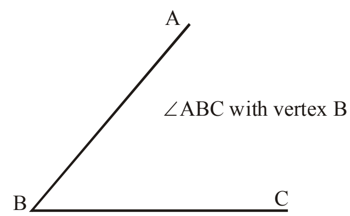


We denote it by $\ell \perp m$.

PROPERTIES

- Three or more points are said to be collinear if they lie on a line, otherwise they are said to be non-collinear.
- Two or more lines are said to be coplanar if they lie in the same plane, otherwise they are said to be non-coplanar.
- A line, which intersects two or more given coplanar lines in distinct points, is called a transversal of the given lines.
- A line which is perpendicular to a line segment, i.e., intersect at 90° and passes through the mid point of the segment is called the perpendicular bisector of the segment.
- Every point on the perpendicular bisector of a segment is equidistant from the two endpoints of the segment.
- If two lines are perpendicular to the same line, they are parallel to each other.
- Lines which are parallel to the same line are parallel to each other.

Angles : An angle is the union of two non-collinear rays with a common origin. The common origin is called the vertex and the two rays are the sides of the angle.

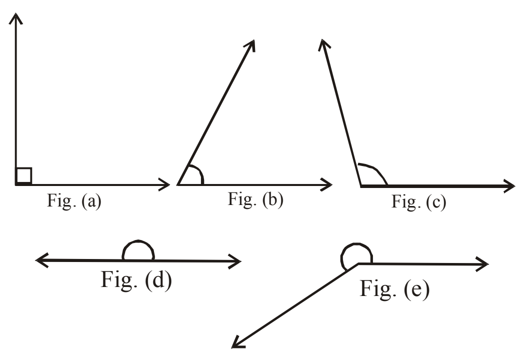


Congruent angles : Two angles are said to be congruent, denoted by ≅ , if their measures are equal.

Bisector of an angle : A ray is said to be the bisector of an angle if it divides the interior of the angle into two angles of equal measure.

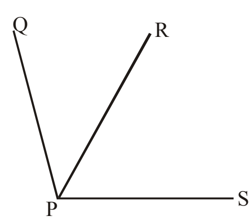
TYPES OF ANGLE

1. A right angle is an angle of 90° as shown in [fig. (a)].
2. An angle less than 90° is called an acute angle [fig. (b)].
3. An angle greater than 90° but less than 180° is called an obtuse angle [fig (c)].
4. An angle of 180° is a straight line [fig. (d)].
5. An angle greater than 180° but less than 360° is called a reflex angle [fig.(e)].



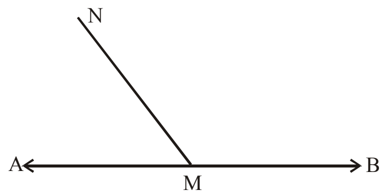
PAIRS OF ANGLES

Adjacent angles : Two angles are called adjacent angles if they have a common side and their interiors are disjoint.



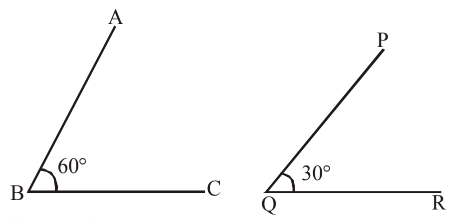
$\angle$ QPR is adjacent to $\angle$RPS

Linear Pair : Two angles are said to form a linear pair if they have a common side and their other two sides are opposite rays. The sum of the measures of the angles is 180°.



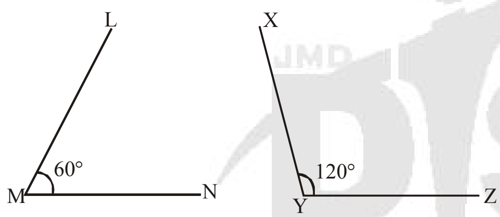
$\angle AMN + \angle BMN = 180°$

Complementary angles : Two angles whose sum is 90°, are complementary, each one is the complement of the other.



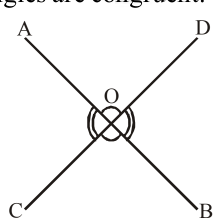
$\angle ABC + \angle PQR = 90°$

Supplementary angles : Two angles whose sum is 180° are supplementary, each one is the supplement of the other.



$\angle LMN + \angle XYZ = 60° + 120° = 180°$

Vertically Opposite angles : Two angles are called vertically opposite angles if their sides form two pairs of opposite rays. Vertically opposite angles are congruent.



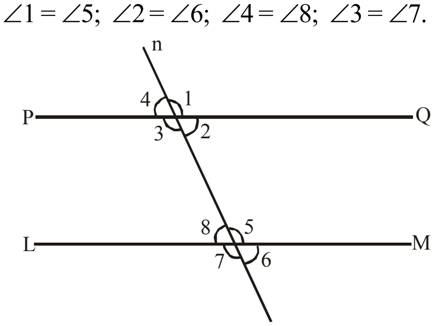
$\angle AOD = \angle COB$ and $\angle AOC = \angle BOD$

Corresponding angles : Here, PQ || LM and n is transversal.

Then, $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$ and $\angle 4$ and $\angle 8$ are corresponding angles.

When two lines are intersected by a transversal, they form four pairs of corresponding angles.

The pairs of corresponding angles thus formed are congruent. i.e. $\angle 1 = \angle 5$; $\angle 2 = \angle 6$; $\angle 4 = \angle 8$; $\angle 3 = \angle 7$.



Alternate angles : In the above figure, $\angle 3$ and $\angle 5$, $\angle 2$ and $\angle 8$ are Alternative angles.

When two lines are itnersected by a transversal, they form two pairs of alternate angles.

The pairs of alternate angles thus formed are congruent. i.e.

$\angle 3 = \angle 5$ and $\angle 2 = \angle 8$

Interior angles : In the above figure, $\angle 2$ and $\angle 5$, $\angle 3$ and $\angle 8$ are Interior angles.

When two lines are intersected by a transversal, they form two pairs of interior angles.

The pairs of interior angles thus formed are supplementary. i.e.

$\angle 2 + \angle 5 = \angle 3 + \angle 8 = 180°$

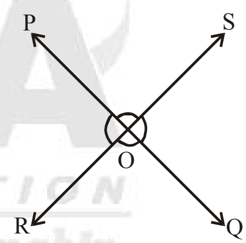
Example 1 :

In figure given below, lines PQ and RS intersect each other at point O. If $\angle POR : \angle ROQ = 5 : 7$, find all the angles.

Solution :

$\angle POR + \angle ROQ = 180°$ (Linear pair of angles)

But $\angle POR : \angle ROQ = 5 : 7$ (Given)



$\therefore \angle POR = \frac{5}{12} \times 180° = 75°$

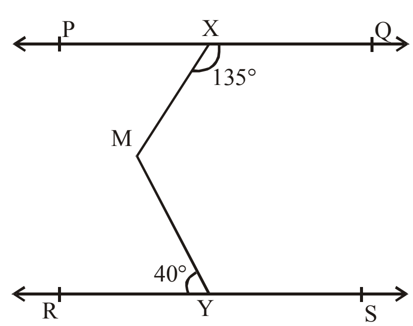
Similarly, $\angle ROQ = \frac{7}{12} \times 180° = 105°$

Now, $\angle POS = \angle ROQ = 105°$ (Vertically opposite angles)

and $\angle SOQ = \angle POR = 75°$ (Vertically opposite angles)

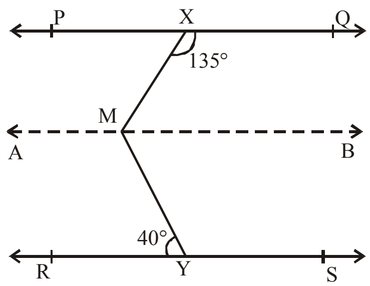
Example 2 :

In fig. if PQ || RS, $\angle MXQ = 135°$ and $\angle MYR = 40°$, find $\angle XMY$.



Solution :

Here, we need to draw a line AB parallel to line PQ, through point M as shown in figure.



Now, AB || PQ and PQ || RS $\Rightarrow$ AB || RS

Now, $\angle QXM + \angle XMB = 180°$

($\because$ AB || PQ, interior angles on the same side of the transversal)

But $\angle QXM = 135° \Rightarrow 135° + \angle XMB = 180°$

$\therefore \quad \angle XMB = 45°$ ....(i)

Now, $\angle BMY = \angle MYR$ ($\because$ AB || RS, alternate angles)

$\therefore \quad \angle BMY = 40°$ ....(ii)

Adding (i) and (ii), we get

$\angle XMB + \angle BMY = 45° + 40°$

i.e. $\angle XMY = 85°$

Example 3 :

An angle is twice its complement. Find the angle.

Solution :

If the complement is x, the angle = 2x

$2x + x = 90°$

$\Rightarrow 3x = 90° \Rightarrow x = 30°$

$\therefore$ The angle is $2 \times 30° = 60°$

Example 4 :

The supplement of an angle is one-fifth of itself. Determine the angle and its supplement.

Solution :

Let the measure of the angle be x°. Then the measure of its supplementary angle is 180° – x°.

It is given that $180 - x = \frac{1}{5}x$

$\Rightarrow 5(180° - x) = x$

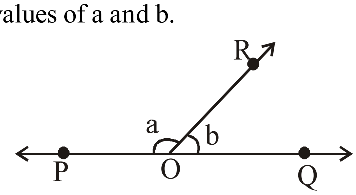
$\Rightarrow 900 - 5x = x \Rightarrow 900 = 5x + x$

$\Rightarrow 900 = 6x \Rightarrow 6x = 900 \Rightarrow x = \frac{900}{6} = 150$

Supplementary angle is 180° – 150° = 30°

Example 5 :

In figure, $\angle POR$ and $\angle QOP$ form a linear pair. If a – b = 80°, find the values of a and b.



Solution :

$\because \quad \angle POR$ and $\angle QOR$ for a linear pair

$\therefore \quad \angle POR + \angle QOR = 180°$ (Linear pair axiom)

or $a + b = 180°$ .... (i)

But $a - b = 80°$ .... (ii) [Given]

Adding eqs. (i) and (ii), we get

$2a = 260° \quad \therefore \quad a = \frac{260}{2} = 130°$

Substituting the value of a in (1), we get

$130° + b = 180°$

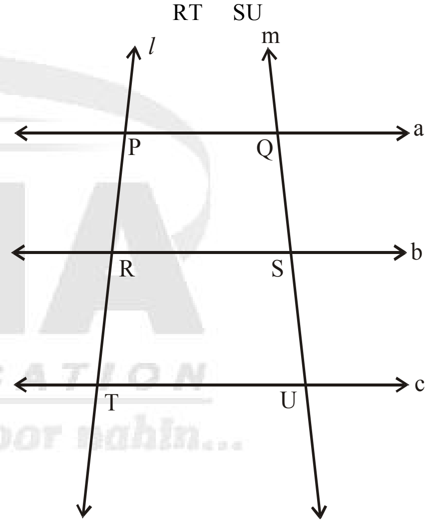
$b = 180° - 130° = 50°$

PROPORTIONALITY THEOREM

The ratio of intercepts made by three parallel lines on a transversal is equal to the ratio of the corresponding intercepts made on any other transversal by the same parallel lines.

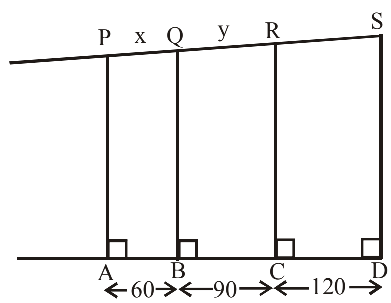
If line a || b || c, and lines *l* and *m* are two transversals, then

$$\frac{PR}{RT} = \frac{QS}{SU}$$



Example 6 :

In the figure, if PS = 360, find PQ, QR and RS.



Solution :

PA, QB, RC and SD are perpendicular to AD. Hence, they are parallel. So the intercepts are proportional.

$\therefore \quad \frac{AB}{BD} = \frac{PQ}{QS} \qquad \Rightarrow \frac{60}{210} = \frac{x}{360 - x}$

$\Rightarrow \frac{2}{7} = \frac{x}{360 - x} \qquad \Rightarrow x = \frac{720}{9} = 80$

$\therefore \quad PQ = 80$

So, QS = 360 – 80 = 280

Again, $\frac{BC}{CD} = \frac{QR}{RS}$

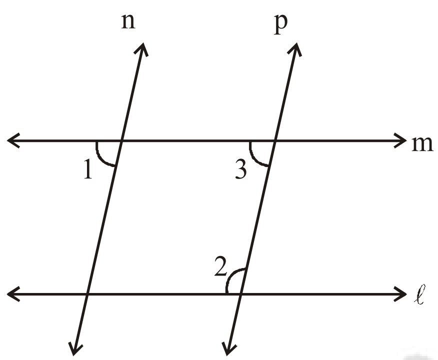
$\therefore \quad \frac{90}{120} = \frac{y}{280 - y} \qquad \Rightarrow \quad \frac{3}{4} = \frac{y}{280 - y}$

$\Rightarrow \quad y = 120$

$\therefore \quad QR = 120$ and $SR = 280 - 120 = 160$

Example 7 :

In figure if $\ell \parallel m$, $n \parallel p$ and $\angle 1 = 85°$ find $\angle 2$.



Solution :

$\because$ n || p and m is transversal

$\therefore \quad \angle 1 = \angle 3 = 85°$ (Corresponding angles)

Also, $m \parallel \ell$ & p is transversal

$\therefore \quad \angle 2 + \angle 3 = 180°$ ($\therefore$ Consecutive interior angles)

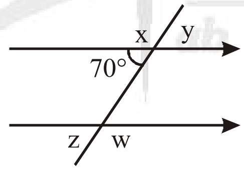
$\Rightarrow \quad \angle 2 + 85° = 180°$

$\Rightarrow \quad \angle 2 = 180° - 85°$

$\Rightarrow \quad \angle 2 = 95°$

Example 8 :

From the adjoining diagrams, calculate $\angle x$, $\angle y$, $\angle z$ and $\angle w$.



Solution :

$\angle y = 70°$

$\angle x + 70 = 180°$

..... (vertical opp. angle)

$\therefore \quad \angle x = 180 - 70 = 110°$

.... (adjacent angles on a st. line or linear pair)

$\angle z = 70°$ .....(corresponding angles)

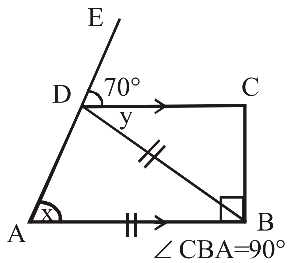
$\angle z + \angle w = 180°$ ..... (adjacent angles on a st. line or linear pair)

$\therefore \quad 70 + \angle w = 180°$

$\therefore \quad \angle w = 180° - 170° = 110°$

Example 9 :

From the adjoining diagram Find (i) $\angle x$ (ii) $\angle y$



Solution :

$\angle x = \angle EDC = 70°$ (corresponding angles)

Now, $\angle ADB = x = 70°$ [AD = DB]

In Δ ABD,

$\angle ABD = 180 - \angle x - \angle x$

$= 180 - 70 - 70 = 40°$

$\Rightarrow \quad \angle BDC = \angle ABD = 40°$ (alternate angles)

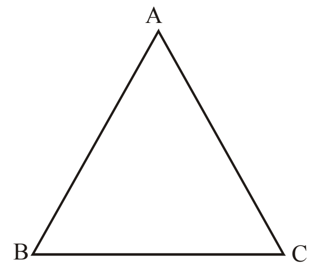
$\Rightarrow \quad \angle y = 40°$

TRIANGLES

The plane figure bounded by the union of three lines, which join three non-collinear points, is called a triangle. A triangle is denoted by the symbol Δ.

The three non-collinear points, are called the vertices of the triangle.

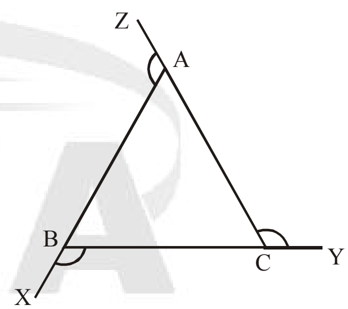
In ΔABC, A, B and C are the vertices of the triangle; AB, BC, CA are the three sides, and $\angle A$, $\angle B$, $\angle C$ are the three angles.



Sum of interior angles : The sum of the three interior angles of a triangle is 180°.

$\angle A + \angle B + \angle C = 180°$

Exterior angles and interior angles



(i) The measure of an exterior angle is equal to the sum of the measures of the two interior opposite angles of the triangle.

$\therefore \angle ACY = \angle ABC + \angle BAC$

$\angle CBX = \angle BAC + \angle BCA$ and $\angle BAZ = \angle ABC + \angle ACB$

(ii) The sum of an interior angle and adjacent exterior angle is 180°.

i.e. $\angle ACB + \angle ACY = 180°$

$\angle ABC + \angle CBX = 180°$ and $\angle BAC + \angle BAZ = 180°$

Example 10 :

If the ratio of three angles of a triangle is 1 : 2 : 3, find the angles.

Solution :

Ratio of the three angles of a Δ = 1 : 2 : 3

Let the angles be x, 2x and 3x.

$\therefore \quad x + 2x + 3x = 180°$

$\therefore \quad 6x = 180° \qquad \therefore \quad x = 30°$

Hence the first angle = x = 30°

The second angle = 2x = 60°

The third angle = 3x = 90°

CLASSIFICATION OF TRIANGLES

Based on sides :

Scalene triangle : A triangle in which none of the three sides is equal is called a scalene triangle.

Isosceles triangle : A triangle in which at least two sides are equal is called an isosceles triangle.

Equilateral triangle : A triangle in which all the three sides are equal is called an equilateral triangle. In an equilateral triangle, all the angles are congruent and equal to 60°.

Based on angles :

Right triangle : If any one angle of a triangle is a right angle, i.e., 90° then the triangle is a right-angled triangle.

Acute triangle : If all the three angles of a triangle are acute, i.e., less than 90°, then the triangle is an acute angled triangle.

Obtuse triangle : If any one angle of a triangle is obtuse, i.e., greater than 90°, then the triangle is an obtuse-angled triangle.

SOME BASIC DEFINITIONS

1. **Altitude (height) of a triangle :** The perpendicular drawn from the vertex of a triangle to the opposite side is called an altitude of the triangle.
2. **Median of a triangle :** The line drawn from a vertex of a triangle to the opposite side such that it bisects the side, is called the median of the triangle.
   - A median bisects the area of the triangle.
3. **Orthocentre :** The point of intersection of the three altitudes of a triangle is called the orthocentre. The angle made by any side at the orthocentre = 180°– the opposite angle to the side.
4. **Centroid :** The point of intersection of the three medians of a triangle is called the centroid. The centroid divides each median in the ratio 2 : 1.
5. **Circumcentre :** The point of intersection of the perpendicular bisectors of the sides of a triangle is called the circumcentre.
6. **Incentre :** The point of intersection of the angle bisectors of a triangle is called the incentre.
   (i) Angle bisector divides the opposite sides in the ratio of remaining sides

   Example : $\frac{BD}{DC} = \frac{AB}{AC} = \frac{c}{b}$

   (ii) Incentre divides the angle bisectors in the ratio $(b + c) : a$, $(c + a) : b$ and $(a + b) : c$

CONGRUENCY OF TRIANGLES

Two triangles are congruent if the sides and angles of one triangle are equal to the corresponding sides and angles of the other triangle.

(i) **SAS Congruence rule :** Two triangles are congruent if two sides and the included angle of one triangle are equal to the sides and the included angle of the other triangle.

(ii) **ASA Congruence rule :** Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of other triangle.

(iii) **AAS Congruence rule :** Two triangles are congruent if any two pairs of angles and one pair of corresponding sides are equal.

(iv) **SSS Congruence rule :** If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.

(v) **RHS Congruence rule :** If in two right triangles, the hypotenuse and one side of the triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.

SIMILARITY OF TRIANGLES

For a given correspondence between two triangles, if the corresponding angles are congruent and their corresponding sides are in proportion, then the two triangles are said to be similar. Similarlity is denoted by ~.

(i) **AAA Similarlity :** For a given correspondence between two triangles, if the two angles of one triangle are congruent to the corresponding two angles of the other triangle, then the two triangles are similar.

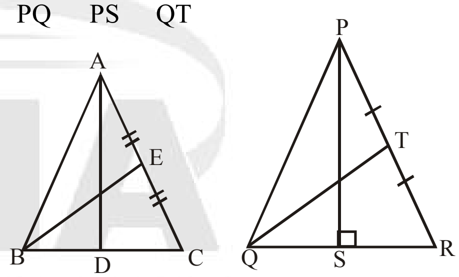
(ii) **SSS Similarity :** If the corresponding sides of two triangles are proportional, their corresponding angles are equal and hence the triangles are similar.

(iii) **SAS Similarity :** If one angle of a triangle is equal to one angle of the other and the sides including these angles are proportional, the triangles are similar.

PROPERTIES OF SIMILAR TRIANGLES

1. If two triangles are similar,
   Ratio of sides = Ratio of height = Ratio of Median = Ratio of angle bisectors = Ratio of inradii = Ratio of circumradii.
   If $\Delta ABC \sim \Delta PQR$

   $$\frac{AB}{PQ} = \frac{AD}{PS} = \frac{BE}{QT}$$

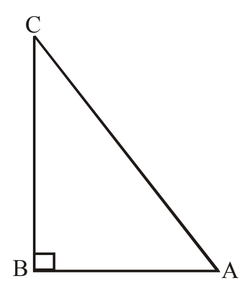


The ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding sides.
If $\Delta ABC \sim \Delta PQR$, then

$$\frac{Ar(\Delta ABC)}{Ar(\Delta PQR)} = \frac{(AB)^2}{(PQ)^2} = \frac{(BC)^2}{(QR)^2} = \frac{(AC)^2}{(PR)^2}$$

PYTHAGORAS THEOREM

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



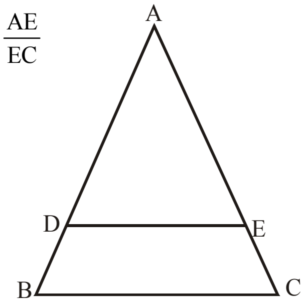
If a right triangle ABC right angled at B. Then,
By Pythagoras theorem, $AC^2 = AB^2 + BC^2$

BASIC PROPORTIONALITY THEOREM (BPT)

If a line is drawn parallel to one side of a triangle, to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

If ΔABC in which a line parallel to BC intersects AB to D and AC at E. Then,

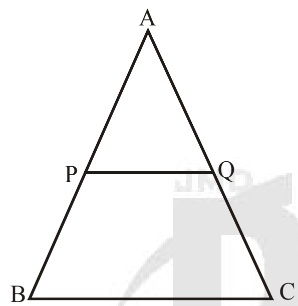
By BPT, $\frac{AD}{DB} = \frac{AE}{EC}$



MID-POINT THEOREM

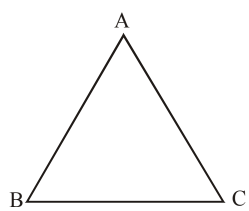
The line segment joining the mid-points of two sides of a triangle is parallel to the third side and half of it.

In ΔABC, if P and Q are the mid-points of AB and AC respectively then $PQ \parallel BC$ and $PQ = \frac{1}{2}BC$



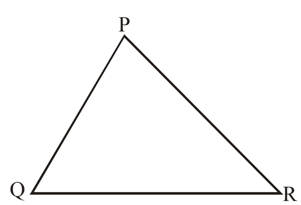
INEQUALITIES IN A TRIANGLE

(i) If two sides of a triangle are unequals, the angle opposite to the longer side is larger. Conversely,
In any triangle, the side opposite to the larger angle is longer.



If $AB > AC$ then $\angle C > \angle B$

(ii) The sum of any two side of a triangle is greater than the third side.



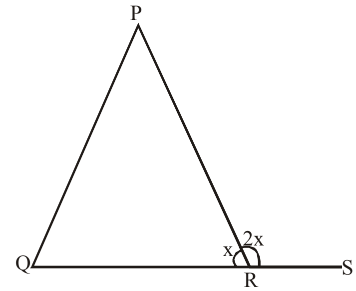
$PQ + PR > QR$; $PQ + QR > PR$ and $QR + PR > PQ$

Example 10 :

The interior and its adjacent exterior angle of a triangle are in the ratio 1 : 2. What is the sum of the other two angles of the triangle ?

Solution :

If the interior angle is x, exterior angle is 2x.



$\because$ $x + 2x = 180°$

$\Rightarrow$ $3x = 180°$

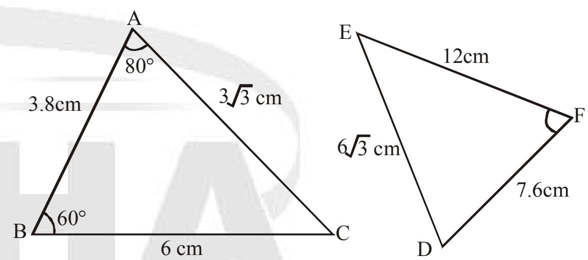
$\Rightarrow$ $x = 60°$

$\therefore$ Exterior angle $= 120°$

Hence sum of the other two angles of triangle = 120°
(Exterior angle is the sum of two opposite interior angles)

Example 11 :

In figure, find $\angle F$.



Solution :

In triangles ABC and DEF, we have

$\frac{AB}{DF} = \frac{3.8}{7.6} = \frac{1}{2}$

Similarly, $\frac{BC}{FE} = \frac{6}{12} = \frac{1}{2}$ and $\frac{AC}{DE} = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}$, i.e.,

in the two triangles, sides are proportional.

$\therefore$ $\Delta ABC \sim \Delta DEF$ (by SSS Similarity)

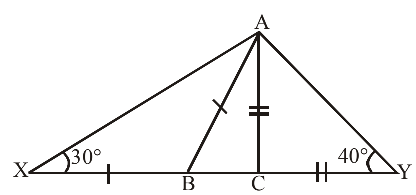
$\therefore$ $\angle B = \angle F$ (Corresponding angles are equal)

But $\angle B = 60°$ (Given)

$\therefore$ $\angle F = 60°$

Example 12 :

In the given figure, find $\angle BAC$ and $\angle XAY$.



Solution :

$\angle AXB = \angle XAB = 30°$ ($\because$ BX = BA)

$\angle ABC = 30° + 30° = 60°$ (Exterior angle)

$\angle CYA = \angle YAC = 40°$ ($\because$ CY = CA)

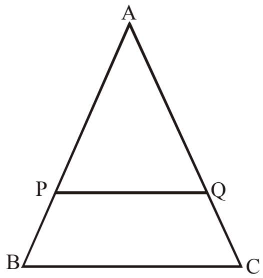
$\angle ACB = 40° + 40° = 80°$ (Exterior angle)

$\angle BAC = 180° - (60° + 80°) = 40°$ (Sum of all angles of a triangle is 180°.

$\angle XAY = 180 - (30 + 40) = 110°$

Example 13 :

In the fig., PQ || BC, AQ = 4 cm, PQ = 6 cm and BC = 9 cm. Find QC.



Solution :

By BPT, $\frac{AQ}{QC} = \frac{PQ}{BC}$

$\frac{4}{QC} = \frac{6}{9} \Rightarrow QC = 6$ cm

Example 14 :

Of the triangles with sides 11, 5, 9 or with sides 6, 10, 8; which is a right triangle ?

Solution :

(Longest side)$^2 = 11^2 = 121$;

$5^2 + 9^2 = 25 + 81 = 106$

$\therefore \quad 11^2 \neq 5^2 + 9^2$

So, it is not a right triangle.

Again, (longest side)$^2 = (10)^2 = 100$;

$6^2 + 8^2 = 36 + 64 = 100$

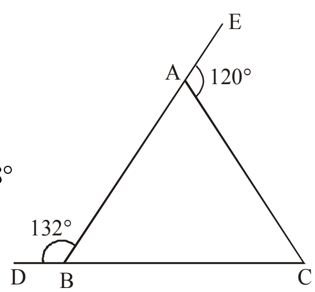
$10^2 = 6^2 + 8^2$

$\therefore$ It is a right triangle.

Example 15 :

In figure, $\angle DBA = 132°$ and $\angle EAC = 120°$.

Show that AB > AC.



Solution :

As DBC is a straight line,

$132° + \angle ABC = 180°$

$\Rightarrow \angle ABC = 180° - 132° = 48°$

For ΔABC,

$\angle EAC$ is an exterior angle

$120° = \angle ABC + \angle BCA$

(ext. $\angle$ = sum of two opp. interior $\angle$ s)

$\Rightarrow 120° = 48° + \angle BCA$

$\Rightarrow \angle BCA = 120° - 48° = 72°$

Thus, we find that $\angle BCA > \angle ABC$

$\Rightarrow$ AB > AC (side opposite to greater angle is greater)

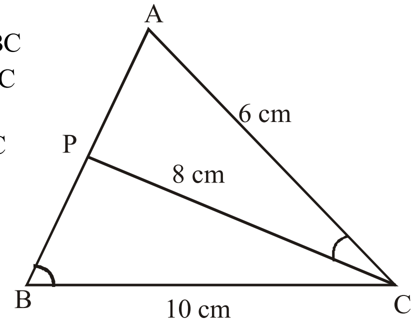
Example 16 :

From the adjoining diagram, calculate

(i) AB (ii) AP

(iii) ar ΔAPC : ar ΔABC

Solution :



In ΔAPC and ΔABC

$\angle ACP = \angle ABC$

$\angle A = \angle A$

$\Rightarrow \Delta ACP \sim \Delta ABC$

$\Rightarrow \frac{AP}{AC} = \frac{PC}{BC} = \frac{AC}{AB}$

$\therefore \frac{AP}{6} = \frac{8}{10} = \frac{6}{AB}$

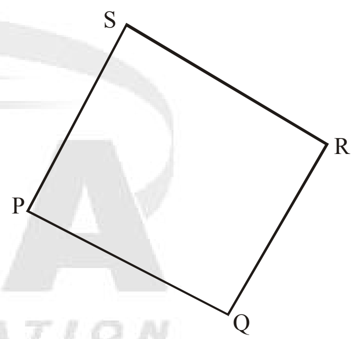
$\Rightarrow AP = 6 \times \frac{8}{10} = 4.8$ and $AB = \frac{60}{8} = 7.5$

$\Rightarrow$ AP = 4.8 cm and AB = 7.5 cm

$$\frac{\Delta ACP}{\Delta ABC} = \frac{CP^2}{BC^2} = \frac{8^2}{10^2} = 0.64$$

QUADRILATERALS

A figure formed by joining four points is called a quadrilateral. A quadrilateral has four sides, four angles and four vertices.

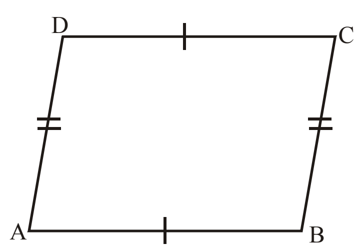


In quadrilateral PQRS, PQ, QR, RS and SP are the four sides; P, Q, R and S are four vertices and $\angle P$, $\angle Q$, $\angle R$ and $\angle S$ are the four angles.

- The sum of the angles of a qudrilateral is 360°.
  $\angle P + \angle Q + \angle R + \angle S = 360°$

TYPES OF QUADRILATERALS :

1. **Parallelogram :** A quadrilateral whose opposite sides are parallel is called parallelogram.



Properties :

(i) Opposite sides are parallel and equal.

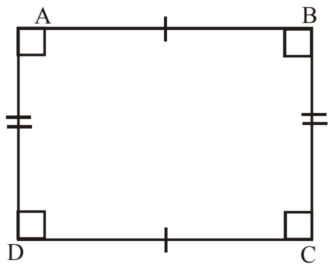
(ii) Opposite angles are equal.

(iii) Diagonals bisect each other.

(iv) Sum of any two adjacent angles is 180°.

(v) Each diagonal divides the parallelogram into two triangles of equal area.

2. **Rectangle :** A parallelogram, in which each angle is a right angle, i.e., 90° is called a rectangle.



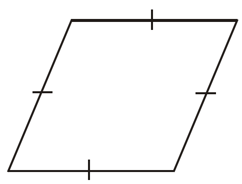
Properties :

(i) Opposite sides are parallel and equal.

(ii) Each angle is equal to 90°.

(iii) Diagonals are equal and bisect each other.

3. **Rhombus :** A parallelogram in which all sides are congruent (or equal) is called a rhombus.



Properties :

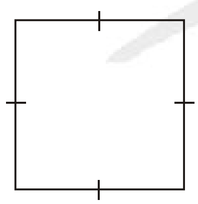
(i) Opposite sides are parallel.

(ii) All sides are equal.

(iii) Opposite angles are equal.

(iv) Diagonals bisect each other at right angle.

4. **Square :** A rectangle in which all sides are equal is called a square.



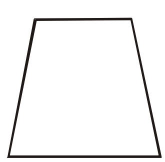
Properties :

(i) All sides are equal and opposite sides are parallel.

(ii) All angles are 90°.

(iii) The diagonals are equal and bisect each other at right angle.

5. **Trapezium :** A quadrilateral is called a trapezium if two of the opposite sides are parallel but the other two sides are not parallel.



Properties :

(i) The segment joining the mid-points of the non-parallel sides is called the median of the trapezium.

Median $= \frac{1}{2} \times$ **sum of the parallel sides**

Example 17 :

The angle of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

Solution :

Let the angles of quadrilateral are 3x, 5x, 9x, 13x.

$\therefore$ $3x + 5x + 9x + 13x = 360°$

(Sum of the angles of quadrilateral)

$\Rightarrow$ $30x = 360°$

$\Rightarrow$ $x = 12°$

Hence angles of quadrilateral are :

$3x = 3 \times 12° = 36°$

$5x = 5 \times 12° = 60°$

$9x = 9 \times 12° = 108°$

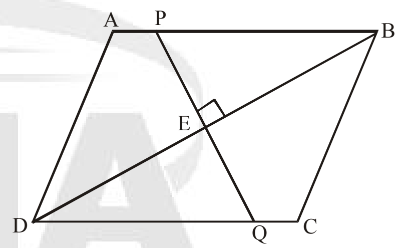
$13x = 13 \times 12° = 156°$

Example 18 :

ABCD is a parallelogram. E is the mid point of the diagonal DB. DQ = 10 cm, DB = 16 cm. Find PQ.

Solution :

$\angle EDQ = \angle EBP$ (Alternate angles)



$\therefore$ $\angle DEQ = \angle BEP$ (opposite angles)

$\therefore$ $\Delta DEQ \cong \Delta BEP$ (By ASA congruency)

$\therefore$ $PE = EQ$

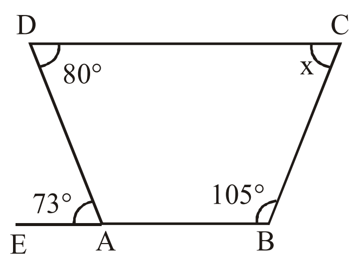
$(EQ)^2 = (DQ)^2 - (DE)^2$

$= 10^2 - 8^2 = 100 - 64 = 36$

$\therefore$ EQ = 6 cm and PQ = 12 cm.

Example 19 :

Use the information given in figure to calculate the value of x.



Solution :

Since, EAB is a straight line

$\therefore$ $\angle DAE + \angle DAB = 180°$

$\Rightarrow$ $73° + \angle DAB = 180°$

i.e., $\angle DAB = 180° - 73° = 107°$

Since, the sum of the angles of quadrilateral ABCD is 360°

$\therefore$ $107° + 105° + x + 80° = 360°$

$\Rightarrow$ $292° + x = 360°$

and, $x = 360° - 292° = 68°$

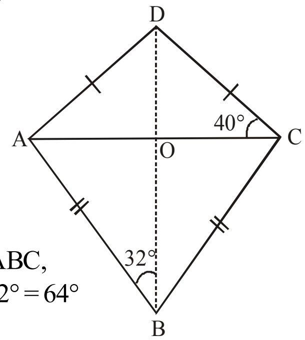
Example 20 :

In the adjoining kite, diagonals intersect at O. If $\angle ABO = 32°$ and $\angle OCD = 40°$, find

(i) $\angle ABC$

(ii) $\angle ADC$

(iii) $\angle BAD$



Solution :

Given, ABCD is a kite.

(i) As diagonal BD bisects $\angle ABC$,

$\angle ABC = 2\angle ABO = 2 \times 32° = 64°$

(ii) $\angle DOC = 90°$

[diagonals intersect at right angles]

$\angle ODC + 40° + 90° = 180°$ [sum of angles in Δ OCD]

$\Rightarrow \angle ODC = 180° - 40° - 90° = 50°$

As diagonal BD bisects $\angle ADC$,

$\angle ADC = 2\angle ODC = 2 \times 50° = 100°$

(iii) As diagonal BD bisects $\angle ABC$

$\angle OBC = \angle ABO = 32°$

$\angle BOC = 90°$ [diagonals intersect at right angles]

$\angle OCB + 90° + 32° = 180°$ [sum of angles in Δ OBC]

$\Rightarrow \angle OCB = 180° - 90° - 32° = 58°$

$\angle BCD = \angle OCD + \angle OCB = 40° + 58° = 98°$

$\therefore \angle BAD = \angle BCD = 98°$ [In kite ABCD, $\angle A = \angle C$)

POLYGON

A plane figure formed by three or more non-collinear points joined by line segments is called a polygon.

A polygon with 3 sides is called a triangle.

A polygon with 4 sides is called a quadrilateral.

A polygon with 5 sides is called a pentagon.

A polygon with 6 sides is called a hexagon.

A polygon with 7 sides is called a heptagon.

A polygon with 8 sides is called an octagon.

A polygon with 9 sides is called a nonagon.

A polygon with 10 sides is called a decagon.

Regular polygon : A polygon in which all its sides and angles are equal, is called a regular polygon.

Sum of all interior angles of a regular polygon of side n is given by (2n – 4) 90°.

Hence, angle of a regular polygon $= \dfrac{(2n-4)90°}{n}$

Sum of an interior angle and its adjacent exterior angle is 180°.

Sum of all exterior angles of a polygon taken in order is 360°.

Example 21 :

The sum of the measures of the angles of regular polygon is 2160°. How many sides does it have?

Solution :

Sum of all angles = 90° (2n – 4)

$\Rightarrow 2160 = 90\,(2n - 4)$

$2n = 24 + 4$

$\therefore \quad n = 14$

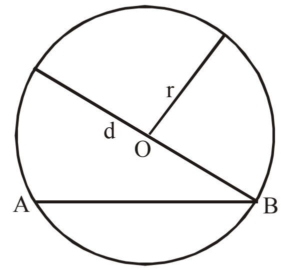
Hence the polygon has 14 sides.

CIRCLE

The collection of all the points in a plane, which are at a fixed distance from a fixed point in the plane, is called a circle.

The fixed point is called the centre of the circle and the fixed distance is called the radius (r).

Chord : A chord is a segment whose endpoints lie on the circle. AB is a chord in the figure.

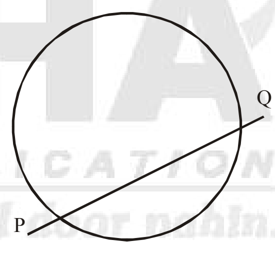


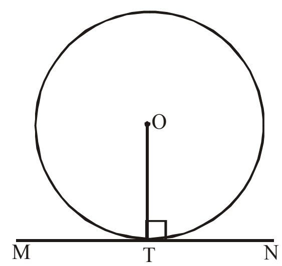
Diameter : The chord, which passes through the centre of the circle, is called the diameter (d) of the circle. The length of the diameter of a circle is twice the radius of the circle.

$$d = 2r$$

Secant : A secant is a line, which intersects the circle in two distinct points.

Tangent : Tangent is a line in the plane of a circle and having one and only one point common with the circle. The common point is called the point of contact.



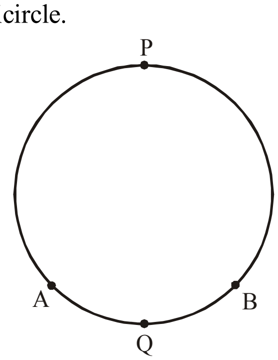


PQ is a secant

MN is a tangent. T is the point of contact.

Semicircle : Half of a circle cut off by a diameter is called the semicircle. The measure of a semicircle is 180°.

Arc : A piece of a circle between two points is called an arc. A minor arc is an arc less than the semicircle and a major arc is an arc greater than a semicircle.

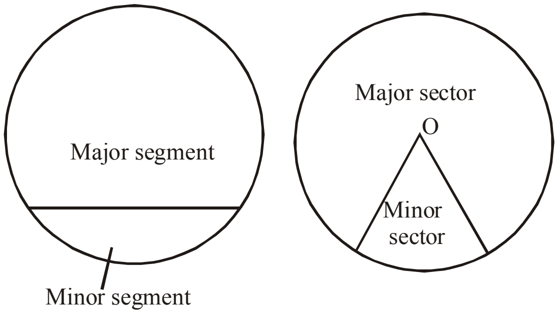


$\overset{\frown}{AQB}$ is a minor arc and $\overset{\frown}{APB}$ is a major arc.

Circumference : The length of the complete circle is called its circumference (C).

$$C = 2\pi r$$

Segment : The region between a chord and either of its arcs is called a segment.



Sector : The region between an arc and the two radii, joining the centre to the endpoints of the arc is called a sector.

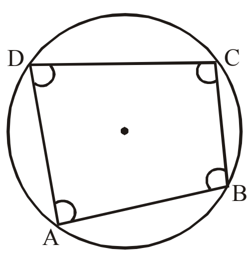
REMEMBER

- ★ Equal chords of a circle subtend equal angles at the centre.
- ★ The perpendicular from the centre of a circle to a chord bisects the chord.
- ★ Equal chords of a circle are equidistant from the centre.
- ★ The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
- ★ Angles in the same segment of a circle are equal.
- ★ Angle in a semicircle is a right angle.
- ★ The tangent at any point of a circle is perpendicular to the radius through the point of contact.
- ★ The length of tangents drawn from an external point to a circle are equal.

CYCLIC QUADRILATERAL

If all the four vertices of a quadrilateral lies on a circle then the quadrilateral is said to be cyclic quadrilateral.

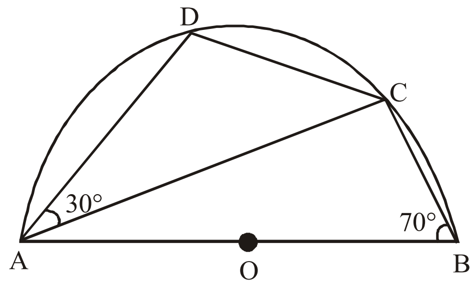
- The sum of either pair of the opposite angles of a cyclic quadrilateral is 180°.
  i.e. $\angle A + \angle C = 180°$
  $\angle B + \angle D = 180°$



- Conversely, if the sum of any pair of opposite angles of quadrilateral is 180°, then the quadrilateral must be cyclic.

Example 22 :

In the adjoining figure, C and D are points on a semi-circle described on AB as diameter. If $\angle ABC = 70°$ and $\angle CAD = 30°$, calculate $\angle BAC$ and $\angle ACD$.



Solution :

$\angle ACB = 90°$ [Angle in a semi-circle]

In ΔABC, $\angle BAC + \angle ACB + \angle ABC = 180°$ [Sum of the $\angle$ s of Δ is 180°]

$\Rightarrow \angle BAC + 90° + 70° = 180°$

$\Rightarrow \angle BAC = (180° - 160°) = 20°$

Now, ABCD being a cyclic quadrilateral, we have

$\angle ABC + \angle ADC = 180°$

(Opposite $\angle$ s of a cyclic quad. are supplementary]

$\Rightarrow 70° + \angle ADC = 180°$

$\Rightarrow \angle ADC = (180° - 70°) = 110°$

Now, in ΔADC, we have

$\angle CAD + \angle ADC + \angle ACD = 180°$

(Sum of the $\angle$ s of a Δ is 180°)

$\Rightarrow 30° + 110° + CD = 180°$

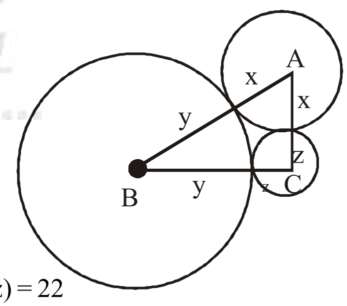
$\Rightarrow \angle ACD = (180° - 140°) = 40°$

Hence, $\angle BAC = 20°$ and $\angle ACD = 40°$

Example 23 :

With the vertices of Δ ABC as centres, three circles are described, each touching the other two externally. If the sides of the triangle are 9 cm, 7 cm and 6 cm. find the radii of the circles.

Solution :



Let AB = 9 cm, BC = 7 cm and CA = 6 cm

Let x, y, z be the radii of circles with centres A, B, C respectively.

Then, $x + y = 9$, $y + z = 7$ and $z + x = 6$

Adding, we get $2(x + y + z) = 22$

$\Rightarrow x + y + z = 11$

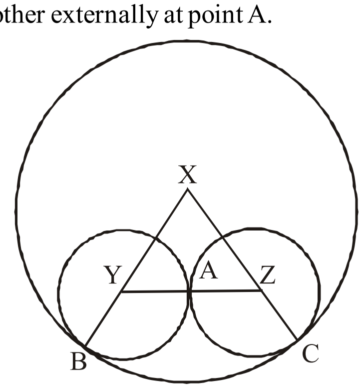
$\therefore x = [(x + y + z) - (y + z)] = (11 - 7)$ cm = 4 cm.

Similarly, $y = (11 - 6)$ cm = 5 cm and $z = (11 - 9)$cm = 2 cm.

Hence, the radii of circles with centres A, B, C are 4 cm, 5 cm, and 2cm respectively.

Example 24 :

In the adjoining figure, 2 circles with centres Y and Z touch each other externally at point A.



Another circle, with centre X, touches the other 2 circles internally at Band C. If XY = 6cm, YZ = 9 cm and ZX = 7 cm, then find the radii of the circles.

Solution :

Let X, Y, Z be the radii of the circle, centres X, Y, Z respectively YAZ, XYB, XZC are straight lines (Contact of circles)

$XY = X - Y = 6$ ..... (1)

$XZ = X - Z = 7$ ..... (2)

$YZ = Y + Z = 9$ ..... (3)

$\Rightarrow$ (1) + (2) + (3)

$2X = 22 \Rightarrow X = 11, Y = 5, Z = 4$

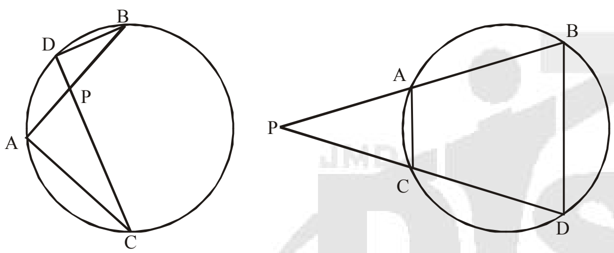
The radius of the circle, centre X, is 11 cm.

The radius of the circle, centre Y, is 5 cm.

The radius of the circle, centre Z, is 4 cm.

SOME IMPORTANT THEOREMS

I. If two chords of a circle intersect inside or outside the circle, then the rectangle formed by the two parts of one chord is equal in area to the rectangle formed by the two parts of the other.

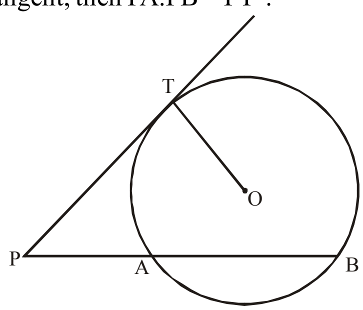


(i) (ii)

Two chords AB and CD of a circle such that they intersect each other at a point P lying inside (fig. (i)) or outside (fig. (ii)) the circle.

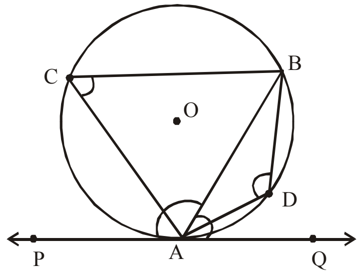
$$PA.PB = PC.PD$$

II. If PAB is a secant to a circle intersecting it at A and B, and PT is a tangent, then $PA.PB = PT^2$.



III. **Alternate segment theorem :**

If a line touches a circle and from the point of contact a chord is drawn, the angle which this chord makes with the given line are equal respectively to the angles formed in the corresponding alternate segments.



PQ is a tangent to a circle with centre O at a point A, AB is chord and C,D are points in the two segments of the circle formed by the chord AB. Then,

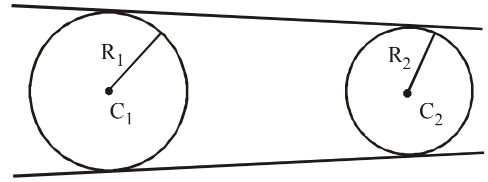
$\angle BAQ = \angle ACB$

$\angle BAP = \angle ADB$

COMMON TANGENTS FOR A PAIR OF CIRCLE

(A) Length of direct common tangent

$$L_1 = \sqrt{(C_1C_2)^2 - (R_1 - R_2)^2}$$

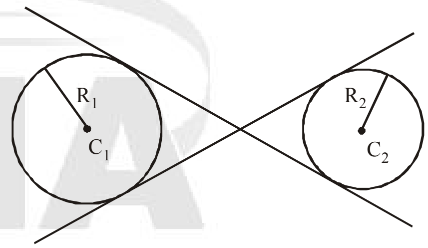


where C_1C_2 = Distance between the centres

(B) Length of transverse common tangent

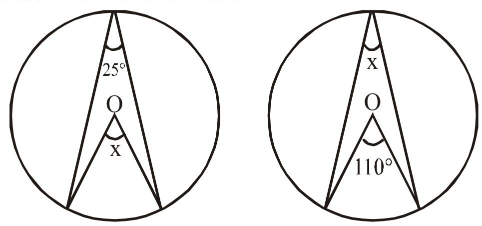
$$L_2 = \sqrt{(C_1C_2)^2 - (R_1 + R_2)^2}\ ;$$

where C_1C_2 = Distance between the centres, and R_1 and R_2 be the radii of the two circles.

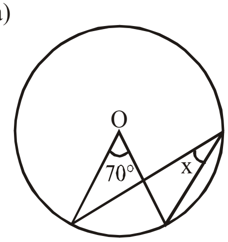


Example 25 :

Find the angle marked as x in each of the following figures where O is the centre of the circle.



(a) (b)



(c)

Solution :

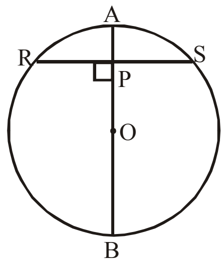
We know that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

(a) $x = 2 \times 25° = 50°$ (b) $x = \frac{1}{2} \times 110° = 55°$

(c) $x = \frac{1}{2} \times 70° = 35°$

Example 26 :

In the figure, RS = 12 cm and radius of the circle is 10 cm. Find PB.



Solution :

$RP = PS = 6$ cm

$OS^2 = PO^2 + PS^2$

$10^2 = PO^2 + 6^2$

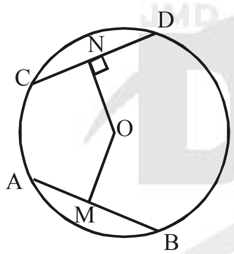
$PO^2 = 100 - 36 = 64$

$PO = 8$ cm

$\therefore \quad PB = PO + OB = 8 + 10 = 18$ cm

Example 27 :

In the figure, AB = 16 cm, CD = 12 cm and OM = 6 cm. Find ON.



Solution :

$MB = \frac{1}{2} \times AB = 8$ cm (perpendicular from the centre of the circle bisects the chord)

$OB^2 = OM^2 + MB^2$

$\Rightarrow OB^2 = 6^2 + 8^2 = 36 + 64 = 100$

$\Rightarrow OB = 10$ cm

$OB = OD = 10$cm (Radii)

$OD^2 = ON^2 + ND^2$

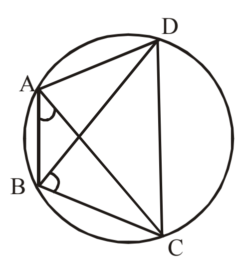
$10^2 = ON^2 + 6^2$

$\therefore \quad ON^2 = 100 - 36 = 64$

Hence $ON = 8$ cm

Example 28 :

In figure, ABCD is a cyclic quadrilateral in which AC and BD are its diagonals. If $\angle DBC = 55°$ and $\angle BAC = 45°$, find $\angle BCD$.



Solution :

$\angle CAD = \angle DBC = 55°$ (Angles in the same segment)

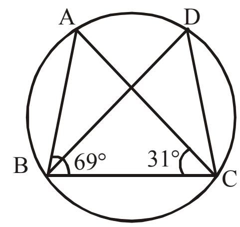
$\therefore \quad \angle DAB = \angle CAD + \angle BAC = 55° + 45° = 100°$

But $\angle DAB + \angle BCD = 180°$ (Opposite angles of a cyclic quadrilateral)

$\Rightarrow \angle BCD = 180° - 100 = 80°$

Example 29 :

In figure, $\angle ABC = 69°$, $\angle ACB = 31°$, find $\angle BDC$.



Solution :

In ΔABC,

$\angle ABC + \angle ACB + \angle BAC = 180°$

$\Rightarrow \quad 69° + 31° + \angle BAC = 180°$

$\Rightarrow \quad \angle BAC = 180° - 100°$

$\therefore \quad \angle BAC = 80°$

But $\angle BAC = \angle BDC$

(Angles in the same segment of a circle are equal)

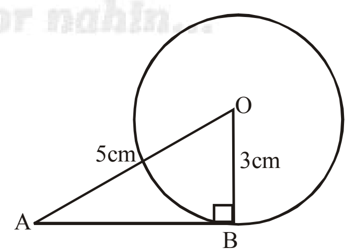
Hence $\angle BDC = 80°$

Example 30 :

Find the length of the tangent from a point which is at a distance of 5 cm from the centre of the circle of radius 3 cm.

Solution :

Let AB be the tangent. ΔABO is a right triangle at B.



By pythagoras theorem,

$OA^2 = AB^2 + BO^2$

$\Rightarrow \quad 5^2 = AB^2 + 3^2$

$\Rightarrow \quad 25 = AB^2 + 9$

$\Rightarrow \quad AB^2 = 25 - 9 = 16$

$\therefore \quad AB = 4$

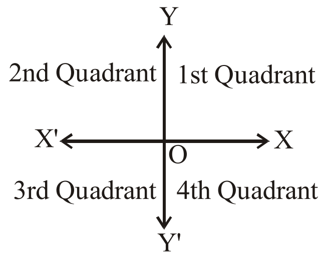
Hence, length of the tangent is 4 cm.

COORDINATE GEOMETRY

The Cartesian Co-ordinate System : Let X'OX and YOY' be two perpendicular straight lines meeting at fixed point O then X'OX is called X axis Y'OY is called the axis of y or y axis point 'O' is called the origin. X axis is known as **abscissa** and y - axis is known as **ordinate.**

Distance Formula: The distance between two points whose co-ordinates are given : $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance from origin : $\sqrt{(x-0)^2+(y-0)^2}$



Section Formula : $x = \dfrac{m_1x_2+m_2x_1}{m_1+m_2}$

(Internally division) $y = \dfrac{m_1y_2+m_2y_1}{m_1+m_2}$

These points divides the line segment in the ratio $m_1 : m_2$.

TRIANGLE

Suppose ABC be a triangle such that the coordinates of its vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$. Then, area of the triangle

$$= \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$$

Centroid of triangle : The coordinates of the centroid are

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Example 31 :

Find the distance between the point P (a cosα, a sinα) and Q (a cosβ, a sinβ).

Solution :

$d^2 = (a\cos\alpha - a\cos\beta)^2 + (a\sin\alpha - a\sin\beta)^2$

$= a^2(\cos\alpha - \cos\beta)^2 + a^2(\sin\alpha - \sin\beta)^2$

$$= a^2\left\{2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}\right\}^2 + a^2\left\{2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}\right\}^2$$

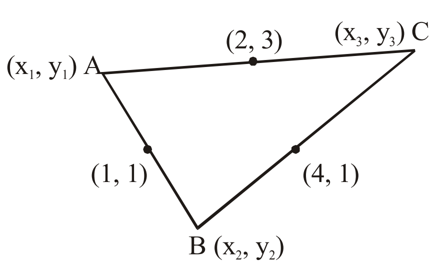
$$= 4a^2\sin^2\frac{\alpha-\beta}{2}\left\{\sin^2\frac{\alpha+\beta}{2}+\cos^2\frac{\alpha+\beta}{2}\right\}$$

$$= 4a^2\sin^2\frac{\alpha-\beta}{2} \quad \Rightarrow d = 2a\sin\frac{\alpha-\beta}{2}$$

Example 32 :

The coordinates of mid-points of the sides of a triangle are (1, 1), (2, 3) and (4, 1). Find the coordinates of the centroid.

Solution :



Let the coordinates of the vertices be $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$.

Then, we have

$x_1+x_2=2,\ x_2+x_3=8,\ x_3+x_1=4$

and, $y_1+y_2=2,\ y_2+y_3=2,\ y_3+y_1=6$

From the above equations, we have

$x_1+x_2+x_3=7$ and $y_1+y_2+y_3=5$

Solving together, we have $x_1=-1, x_2=3, x_3=5$

and $y_1=3, y_2=-1, y_3=3$

Therefore the coordinates of the vertices are (–1, 3), (3, –1) and (5, 3).

Hence, the centroid is $\left(\dfrac{-1+3+5}{3}, \dfrac{3-1+3}{3}\right)$ i.e. $\left(\dfrac{7}{3}, \dfrac{5}{3}\right)$.

Alternatively:

The coordinates of the centroid of the triangle formed by joining the mid points of the sides of the triangle are coincident

$\therefore$ The centroid has coordinates $\left(\dfrac{1+2+4}{3}, \dfrac{1+3+1}{3}\right)$

i.e. $\left(\dfrac{7}{3}, \dfrac{5}{3}\right)$.

Example 33 :

If distance between the point (x, 2) and (3, 4) is 2, then the value of x =

Solution :

$$2 = \sqrt{(x-3)^2+(2-4)^2} \Rightarrow 2 = \sqrt{(x-3)^2+4}$$

Squaring both sides

$$4 = (x-3)^2+4 \Rightarrow x-3=0 \Rightarrow x=3$$

Example 34 :

Find the co-ordinates of a point which divides the line segment joining each of the following points in the given ratio :

(a) (2, 3) and (7, 8) in the ratio 2 : 3 internally

(b) (–1, 4) and (0, –3) in the ratio 1 : 4 internally.

Solution :

(a) Let A(2, 3) and B(7, 8) be the given points.

Let P(x, y) divide AB in the ratio 2 : 3 internally.

Using section formula, we have,

$$x = \frac{2\times7+3\times2}{2+3} = \frac{20}{5} = 4$$

and $$y = \frac{2\times8+3\times3}{2+3} = \frac{25}{5} = 5$$

$\therefore$ P(4, 5) divides AB in the ratio 2 : 3 internally.

(b) Let A (–1, 4) and B (0, –3) be the given points.

Let P(x, y) divide AB in the ratio 1 : 4 internally

Using section formula, we have

$$x = \frac{1\times0+4\times(-1)}{1+4} = -\frac{4}{5}$$

and $$y = \frac{1\times(-3)+4\times4}{1+4} = \frac{13}{5}$$

$\therefore$ $P\left(-\dfrac{4}{5}, \dfrac{13}{5}\right)$ divides AB in the ratio 1 : 4 internally.

Example 35 :

Find the mid-point of the line-segment joining two points (3, 4) and (5, 12).

Solution :

Let A(3, 4) and B(5, 12) be the given points.

Let C(x, y) be the mid-point of AB. Using mid-point formula,

we have, $x = \frac{3+5}{2} = 4$ and $y = \frac{4+12}{2} = 8$

$\therefore$ C(4, 8) are the co-ordinates of the mid-point of the line segment joining two points (3, 4) and (5, 12).

Example 36 :

The co-ordinates of the mid-point of a line segment are (2, 3). If co-ordinates of one of the end points of the line segment are (6, 5), find the co-ordiants of the other end point.

Solution :

Let other the end point be A(x, y)

It is given that C (2, 3) is the mid point

$\therefore$ We can write, $2 = \frac{x+6}{2}$ and $3 = \frac{y+5}{2}$

or $4 = x + 6$ or $6 = y + 5$

or $x = -2$ or $y = 1$

$\therefore$ A (–2, 1) be the co-ordinates of the other end point.

Example 37 :

The area of a triangle is 5. Two of its vertices are (2, 1) and (3, –2). The third vertex lies on y = x + 3. Find the third vertex.

Solution :

Let the third vertex be (x_3, y_3), area of triangle

$= \frac{1}{2}|[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]|$

As $x_1 = 2, y_1 = 1, x_2 = 3, y_2 = -2$, Area of $\Delta = 5$

$\Rightarrow 5 = \frac{1}{2}|2(-2 - y_3) + 3(y_3 - 1) + x_3(1+2)|$

$\Rightarrow 10 = |3x_3 + y_3 - 7| \Rightarrow 3x_3 + y_3 - 7 = \pm 10$

Taking positive sign,

$3x_3 + y_3 - 7 = 10 \Rightarrow 3x_3 + y_3 = 17$ .... (i)

Taking negative sign

$3x_3 + y_3 - 7 = -10 \Rightarrow 3x_3 + y_3 = -3$ .... (ii)

Given that $(x_3, -y_3)$ lies on $y = x + 3$

So, $-x_3 + y_3 = 3$ .... (iii)

Solving eqs. (i) and (iii), $x_3 = \frac{7}{2}$, $y_3 = \frac{13}{2}$

Solving eqs. (ii) and (iii), $x_3 = \frac{-3}{2}$, $y_3 = \frac{3}{2}$.

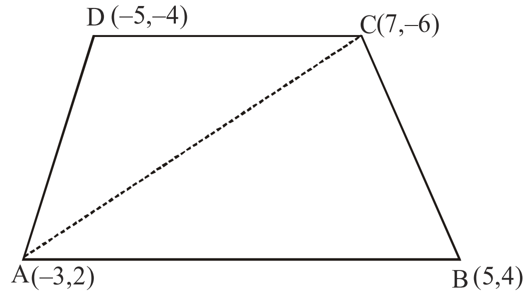
So the third vertex are $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(\frac{-3}{2}, \frac{3}{2}\right)$

Example 38 :

Find the area of quadrilateral whose vertices, taken in order, are A (–3, 2), B(5, 4), C (7, – 6) and D (–5, –4).

Solution :

Area of quadrilateral = Area of Δ ABC + Area of Δ ACD



So, Area of Δ ABC $= \frac{1}{2}|(-3)(4+6) + 5(-6-2) + 7(2-4)|$

$= \frac{1}{2}|-30 - 40 - 14| = \frac{1}{2}|-84| = 42$ sq. units

So, Area of Δ ACD

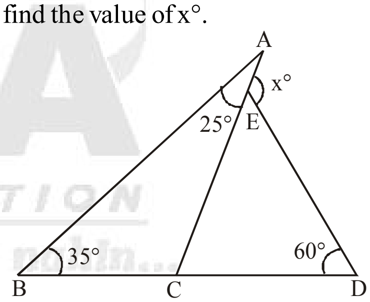
$= \frac{1}{2}|-3(-6+4) + 7(-4-2) + (-5)(2+6)|$

$= \frac{1}{2}|+6 - 42 - 40| = \frac{1}{2}|-76| = 38$ sq. units

So, Area of quadrilateral ABCD = 42 + 38 = 80 sq. units.

Example 39 :

In the figure, find the value of x°.



Solution :

In the Δ ABC, $\angle A + \angle B + \angle ACB = 180°$

$\Rightarrow 25° + 35° + \angle ACB = 180°$

$\Rightarrow \angle ACB = 120°$

Now, $\angle ACB + \angle ACD = 180°$ (linear pair)

or $120° + \angle ACD = 180°$

or $\angle ACD = 60° = \angle ECD$

Again in the Δ CDE, CE is produced to A.

Hence, $\angle AED = \angle ECD + \angle EDC$

$\Rightarrow x = 60° + 60° = 120°$

Example 40 :

Find the equation of the circle whose diameter is the line joining the points (– 4, 3) and (12, – 1). Find the intercept made by it on the y-axis.

Solution :

The equation of the required circle is

$(x + 4)(x - 12) + (y - 3)(y + 1) = 0$

On the y-axis, x = 0

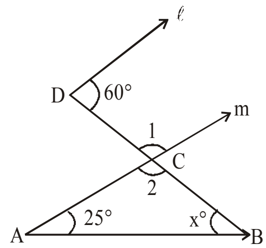
$\Rightarrow -48 + y^2 - 2y - 3 = 0 \Rightarrow y^2 - 2y - 51 = 0$

$\Rightarrow y = 1 \pm \sqrt{52}$

Hence the intercept on the y-axis = $2\sqrt{52} = 4\sqrt{13}$

Example 41 :

In figure, if $\ell \parallel m$, then find the value of x.



Solution :

As $\ell \parallel m$ and DC is transversal

$\therefore \quad \angle D + \angle 1 = 180°$

$60° + \angle 1 = 180°$

$\angle 1 = 120°$

Here, $\angle 2 = \angle 1 = 120°$

(vertically opposite angles)

In the Δ ABC

$\angle A + \angle B + \angle C = 180°$

$25° + x° + 120° = 180°$

or $\quad x = 35°$

Example 42 :

M and N are points on the sides PQ and PR respectively of a Δ PQR. For each of the following cases state whether MN is parallel to QR :

(a) PM = 4, QM = 4.5, PN = 4, NR = 4.5

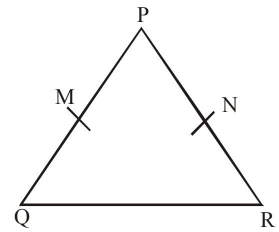
(b) PQ = 1.28, PR = 2.56, PM = 0.16, PN = 0.32

Solution :

(a) The triangle PQR is isosceles

$\Rightarrow$ MN || QR by converse of Proportionally theorem

(b) Again by converse of proportionally theorem, MN || QR



Example 43 :

The point A divides the join the points (– 5, 1) and (3, 5) in the ratio k : 1 and coordinates of points B and C are (1, 5) and (7, –2) respectively. If the area of Δ ABC be 2 units, then find the value (s) of k.

Solution :

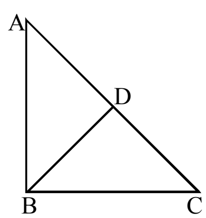
$A \equiv \left(\frac{3k-5}{k+1}, \frac{5k+1}{k+1}\right)$, Area of Δ ABC = 2 units

$\Rightarrow \frac{1}{2}\left[\frac{3k-5}{k+1}(5+2) + 1\left(-2 - \frac{5k+1}{k+1}\right) + 7\left(\frac{5k+1}{k+1} - 5\right)\right] = \pm 2$

$\Rightarrow 14k - 66 = \pm 4\,(k+1) \Rightarrow k = 7$ or $31/9$

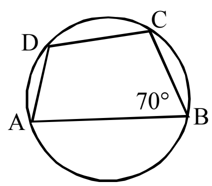
EXERCISE

1. In triangle ABC, angle B is a right angle. If (AC) is 6 cm, and D is the mid-point of side AC. The length of BD is



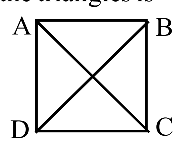
(a) 4 cm (b) $\sqrt{6}$cm
(c) 3 cm (d) 3.5 cm

2. AB is diameter of the circle and the points C and D are on the circumference such that $\angle CAD = 30°$. What is the measure of $\angle ACD$?



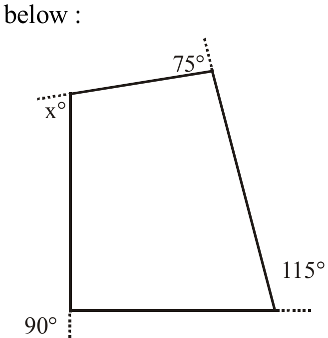
(a) 40° (b) 50°
(c) 30° (d) 90°

3. ABCD is a square of area 4, which is divided into four non overlapping triangles as shown in the fig. Then the sum of the perimeters of the triangles is



(a) $8(2+\sqrt{2})$ (b) $8(1+\sqrt{2})$
(c) $4(1+\sqrt{2})$ (d) $4(2+\sqrt{2})$

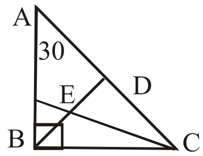
4. The sides of a quadrilateral are extended to make the angles as shown below :



What is the value of x ?

(a) 100 (b) 90
(c) 80 (d) 75

5. AB ⊥ BC and BD ⊥ AC. And CE bisects the angle C. $\angle A = 30°$. The, what is $\angle CED$.

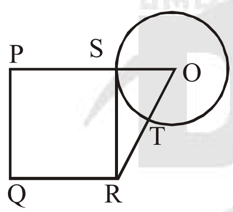


(a) 30° (b) 60°
(c) 45° (d) 65°

6. Instead of walking along two adjacent sides of a rectangular field, a boy took a short cut along the diagonal and saved a distance equal to half the longer side. Then the ratio of the shorter side to the longer side is
(a) 1/2 (b) 2/3
(c) 1/4 (d) 3/4

7. In a triangle ABC, points P, Q and R are the mid-points of the sides AB, BC and CA respectively. If the area of the triangle ABC is 20 sq. units, find the area of the triangle PQR
(a) 10 sq. units (b) 5.3 sq. units
(c) 5 sq. units (d) None of these

8. PQRS is a square. SR is a tangent (at point S) to the circle with centre O and TR = OS. Then, the ratio of area of the circle to the area of the square is



(a) $\pi/3$ (b) $11/7$
(c) $3/\pi$ (d) $7/11$

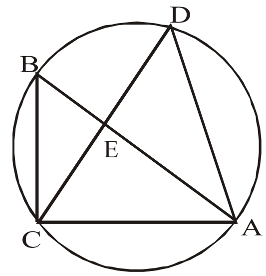
9. Two circles touch each other internally. Their radii are 2 cm and 3 cm. The biggest chord of the outer circle which is outside the inner circle is of length
(a) $2\sqrt{2}$ cm (b) $3\sqrt{2}$ cm
(c) $2\sqrt{3}$ cm (d) $4\sqrt{2}$ cm

10. A triangle and a parallelogram are constructed on the same base such that their areas are equal. If the altitude of the parallelogram is 100 m, then the altitude of the triangle is :
(a) 100 m (b) 200 m
(c) $100\sqrt{2}$ m (d) $10\sqrt{2}$ m

11. The sum of the interior angles of a polygon is 1620°. The number of sides of the polygon are :
(a) 9 (b) 11
(c) 15 (d) 12

12. From a circular sheet of paper with a radius of 20 cm, four circles of radius 5cm each are cut out. What is the ratio of the uncut to the cut portion?
(a) 1 : 3 (b) 4 : 1
(c) 3 : 1 (d) 4 : 3

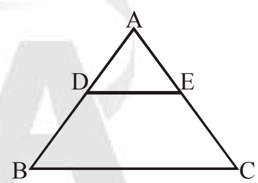
13. In the adjoining the figure, points A, B, C and D lie on the circle. AD = 24 and BC = 12. What is the ratio of the area of the triangle CBE to that of the triangle ADE



(a) 1 : 4 (b) 1 : 2
(c) 1 : 3 (d) Insufficient data

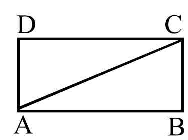
14. Find the co-ordinates of the point which divides the line segment joining the points (4, –1) and (–2, 4) internally in the ratio 3 : 5
(a) $\left(\frac{6}{4}, \frac{7}{2}\right)$ (b) $\left(\frac{4}{7}, \frac{8}{7}\right)$
(c) $\left(\frac{7}{4}, \frac{7}{8}\right)$ (d) $\left(\frac{7}{12}, \frac{8}{4}\right)$

15. In ΔABC, DE || BC and $\frac{AD}{DB} = \frac{3}{5}$. If AC = 5.6 cm, find AE.



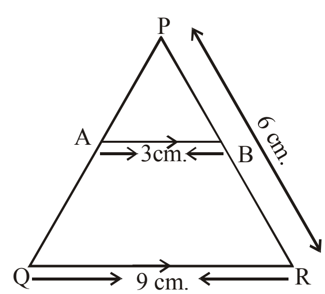
(a) 2.1 cm (b) 3.1 cm
(c) 1.2 cm (d) 2.3 cm

16. In the adjoining figure, AC + AB = 5 AD and AC – AD = 8. Then the area of the rectangle ABCD is



(a) 36 (b) 50
(c) 60 (d) Cannot be answered

17. In the given fig. AB || QR, find the length of PB.



(a) 3 cm (b) 2 cm
(c) 4 cm (d) 6 cm

18. In ΔABC, AD is the bisector of $\angle A$ if AC = 4.2 cm., DC = 6 cm., BC = 10 cm., find AB.
(a) 2.8 cm (b) 2.7 cm
(c) 3.4 cm (d) 2.6 cm

19. Two circles of radii 10 cm. 8 cm. intersect and length of the common chord is 12 cm. Find the distance between their centres.
(a) 13.8 cm (b) 13.29 cm
(c) 13.2 cm (d) 12.19 cm

20. ABCD is a cyclic quadrilateral in which $BC \parallel AD$, $\angle ADC = 110°$ and $\angle BAC = 50°$ find $\angle DAC$
(a) 60° (b) 45°
(c) 90° (d) 120°

21. The length of a ladder is exactly equal to the height of the wall it is resting against. If lower end of the ladder is kept on a stool of height 3 m and the stool is kept 9 m away from the wall the upper end of the ladder coincides with the tip of the wall. Then, the height of the wall is
(a) 12 m (b) 15 m
(c) 18 m (d) 11 m

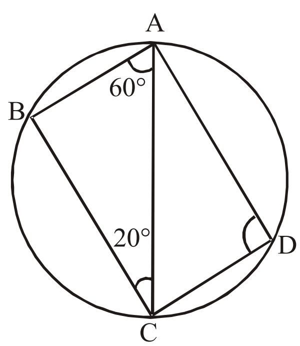
22. In a triangle ABC, the internal bisector of the angle A meets BC at D. If AB = 4, AC = 3 and $\angle A = 60°$, then the length of AD is
(a) $2\sqrt{3}$ (b) $\frac{12\sqrt{3}}{7}$
(c) $15\sqrt{\frac{3}{8}}$ (d) $6\sqrt{\frac{3}{7}}$

23. In a quadrilateral $\angle ABCD$, $\angle B = 90°$ and $AD^2 = AB^2 + BC^2 + CD^2$, then $\angle ACD$ is equal to :
(a) 90° (b) 60°
(c) 30° (d) None of these

24. How many sides a regular polygon has with its sum of interior angles eight times its sum of exterior angles?
(a) 16 (b) 24
(c) 18 (d) 30

25. The Co-ordinates of the centroid of the triangle ABC are (6, 1). If two vertices A and B are (3, 2) and (11, 4) find the third vertex
(a) (4, –3) (b) (2, 1)
(c) (2, 4) (d) (3, 3)

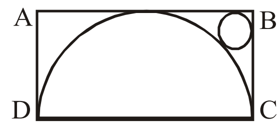
26. In given fig, if $\angle BAC = 60°$ and $\angle BCA = 20°$ find $\angle ADC$



(a) 60° (b) 45°
(c) 80° (d) 90°

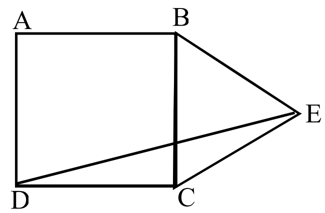
27. In a triangle ABC, the lengths of the sides AB, AC and BC are 3, 5 and 6 cm, respectively. If a point D on BC is drawn such that the line AD bisects the angle A internally, then what is the length of BD ?
(a) 2 cm (b) 2.25 cm
(c) 2.5 cm (d) 3 cm

28. The figure shows a rectangle ABCD with a semi-circle and a circle inscribed inside it as shown. What is the ratio of the area of the circle to that of the semi-circle?



(a) $(\sqrt{2}-1)^2$ (b) $2(\sqrt{2}-1)^2$
(c) $(\sqrt{2}-1)^2/2$ (d) None of these

29. If ABCD is a square and BCE is an equilateral triangle, what is the measure of the angle DEC?

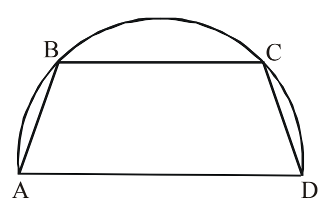


(a) 15° (b) 30°
(c) 20° (d) 45°

30. AB and CD two chords of a circle such that AB = 6 cm CD = 12 cm. And $AB \parallel CD$. The distance between AB and CD is 3 cm. Find the radius of the circle.
(a) $3\sqrt{5}$ (b) $2\sqrt{5}$
(c) $3\sqrt{4}$ (d) $5\sqrt{3}$

31. ABCD is a square, F is the mid-point of AB and E is a point on BC such that BE is one-third of BC. If area of $\Delta FBE = 108$ m^2, then the length of AC is :
(a) 63 m (b) $36\sqrt{2}$ m
(c) $63\sqrt{2}$ m (d) $72\sqrt{2}$ m

32. On a semicircle with diameter AD, chord BC is parallel to the diameter. Further, each of the chords AB and CD has length 2, while AD has length 8. What is the length of BC?



(a) 7.5 (b) 7
(c) 7.75 (d) None of the above

33. The line x + y = 4 divides the line joining the points (–1, 1) and (5, 7) in the ratio
(a) 2 : 1 (b) 1 : 2
(c) 1 : 2 externally (d) None of these

34. If the three vertices of a rectangle taken in order are the points (2, –2), (8, 4) and (5, 7). The coordinates of the fourth vertex is
(a) (1, 1) (b) (1, –1)
(c) (–1, 1) (d) None of these

35. The centroid of a triangle, whose vertices are (2, 1), (5, 2)

and (3, 4) is

(a) $\left(\frac{8}{3}, \frac{7}{3}\right)$ (b) $\left(\frac{10}{3}, \frac{7}{3}\right)$

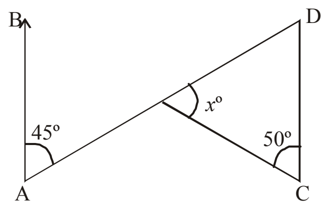
(c) $\left(-\frac{10}{3}, \frac{7}{3}\right)$ (d) $\left(\frac{10}{3}, -\frac{7}{3}\right)$

36. If O be the origin and if the coordinates of any two points Q_1 and Q_2 be (x_1, y_1) and (x_2, y_2) respectively, then $OQ_1.OQ_2 \cos Q_1OQ_2 =$

(a) $x_1x_2 - y_1y_2$ (b) $x_1y_1 - x_2y_2$

(c) $x_1x_2 + y_1y_2$ (d) $x_1y_1 + x_2y_2$

37. In the given figure, AB ‖ CD, $\angle BAE = 45°$, $\angle DCE = 50°$ and $\angle CED = x$, then find the value of x.



(a) 85° (b) 95°

(c) 60° (d) 20°

38. If the coordinates of the points A, B, C be (4, 4), (3, – 2) and (3, – 16) respectively, then the area of the triangle ABC is:

(a) 27 (b) 15

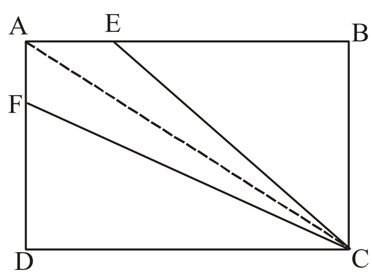
(c) 18 (d) 7

39. Arc ADC is a semicircle and DB ⊥ AC. If AB = 9 and BC = 4, find DB.

(a) 6 (b) 8

(c) 10 (d) 12

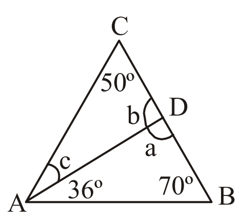
40. In the given figure given below, E is the mid-point of AB and F is the midpoint of AD. if the area of FAEC is 13, what is the area of ABCD ?



(a) 19.5 (b) 26

(c) 39 (d) None of these

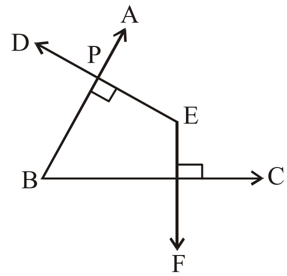
41. Given the adjoining figure. Find a, b, c



(a) 74°, 106°, 20° (b) 90°, 20°, 24°

(c) 60°, 30°, 24° (d) 106°, 24°, 74°

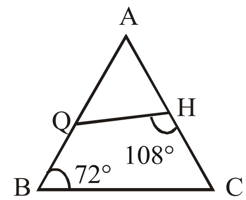
42. In the given figure, ∠ ABC and ∠ DEF are two angles such that BA ⊥ ED and EF ⊥ BC, then find value of ∠ ABC + ∠ DEF.



(a) 120° (b) 180°

(c) 150° (d) 210°

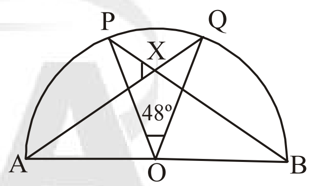
43. In the figure AG = 9, AB = 12, AH = 6, Find HC.



(a) 18 (b) 12

(c) 16 (d) 6

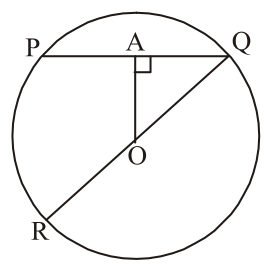
44. In the figure given below, AB is a diametre of the semicircle APQB, centre O, ∠ POQ = 48° cuts BP at X, calculate ∠AXP.



(a) 50° (b) 55°

(c) 66° (d) 40°

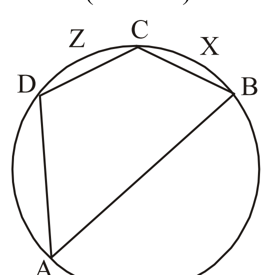
45. OA is perpendicular to the chord PQ of a circle with centre O. If QR is a diametre, AQ = 4 cm, OQ = 5 cm, then PR is equal to



(a) 6 cm (b) 4 cm.

(c) 8 cm (d) 10 cm

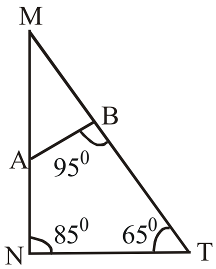
46. In the cyclic quadrilateral ABCD BCD =120° , m(arc DZC) = 7°, find DAB and m (arc CXB).



(a) 60°, 70° (b) 60°, 40°

(c) 60°, 50° (d) 60°,60°

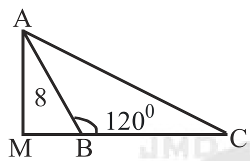
47. In the figure, if $\frac{NT}{AB} = \frac{9}{5}$ and if MB = 10, find MN.



(a) 5 (b) 4
(c) 28 (d) 18

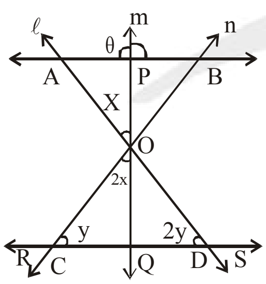
48. The perimeter of the triangle whose vertices are (– 1,4), (–4, –2), (3, –4), will be :
(a) 38 (b) 16
(c) 42 (d) None of the above

49. In the figure, AB = 8, BC = 7 m, $\angle ABC = 120^0$. Find AC.



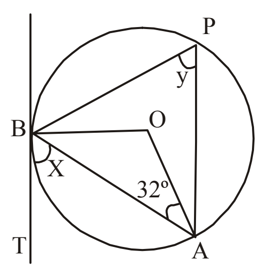
(a) 11 (b) 12
(c) 13 (d) 14

50. Give that segment AB and CD are parallel, if lines ℓ, m and n intersect at point O. Find the ratio of θ to $\angle ODS$



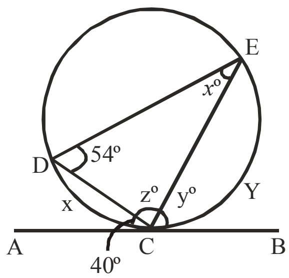
(a) 2 : 3 (b) 3 : 2
(c) 3 : 4 (d) Data insufficient

51. In the given figure, AB is chord of the circle with centre O, BT is tangent to the circle. The values of x and y are



(a) 52°, 52° (b) 58°, 52°
(c) 58°, 58° (d) 60°, 64°

52. In the given figure, m $\angle EDC = 54^\circ$. m $\angle DCA = 40^\circ$. Find x, y and z.



(a) $20^\circ, 27^\circ, 86^\circ$ (b) $40^\circ, 54^\circ, 86^\circ$
(c) $20^\circ, 27^\circ, 43^\circ$ (d) $40^\circ, 54^\circ, 43^\circ$

53. The distance between two parallel chords of length 8 cm each in a circle of diameter 10 cm is
(a) 6 cm (b) 7 cm
(c) 8 cm (d) 5.5 cm

54. In a quadrilateral ABCD, the bisectors of $\angle A$ and $\angle B$ meet at O. If $\angle C = 70°$ and $\angle D = 130°$, then measure of $\angle AOB$ is
(a) 40° (b) 60°
(c) 80° (d) 100°

55. In ΔABC, E and D are points on sides AB and AC respectively such that $\angle ABC = \angle ADE$. If AE = 3 cm, AD = 2 cm and EB = 2 cm, then length of DC is
(a) 4 cm (b) 4.5 cm
(c) 5.0 cm (d) 5.5 cm

56. In a circle with centre O, AB is a chord, and AP is a tangent to the circle. If $\angle AOB = 140°$, then the measure of $\angle PAB$ is
(a) 35° (b) 55°
(c) 70° (d) 75°

57. In ΔABC, $\angle A < \angle B$. The altitude to the base divides vertex angle C into two parts C_1 and C_2, with C_2 adjacent to BC. Then
(a) $C_1 + C_2 = A + B$ (b) $C_1 - C_2 = A - B$
(c) $C_1 - C_2 = B - A$ (d) $C_1 + C_2 = B - A$

58. If O is the in-centre of ΔABC; if $\angle BOC = 120°$, then the measure of $\angle BAC$ is
(a) 30° (b) 60°
(c) 150° (d) 75°

59. Two parallel chords of a circle of diameter 20 cm are 12 cm and 16 cm long. If the chords are in the same side of the centre, then the distance between them is
(a) 28 cm (b) 2 cm
(c) 4 cm (d) 8 cm

60. In a Δ ABC, $\frac{AB}{AC} = \frac{BD}{DC}$, $\angle B = 70°$ and $\angle C = 50°$, then $\angle BAD = ?$
(a) 60° (b) 20°
(c) 30° (d) 50°

61. In a Δ ABC, AD, BE and CF are three medians. The perimeter of ΔABC is always
(a) equal to $\left(\overline{AD} + \overline{BE} + \overline{CF}\right)$
(b) greater than $\left(\overline{AD} + \overline{BE} + \overline{CF}\right)$
(c) less than $\left(\overline{AD} + \overline{BE} + \overline{CF}\right)$
(d) None of these

62. In a ΔABC, $\overline{AD}, \overline{BE}$ and $\overline{CF}$ are three medians. Then the ratio $(\overline{AD}+\overline{BE}+\overline{CF}):(\overline{AB}+\overline{AC}+\overline{BC})$ is
(a) equal to $\frac{3}{4}$ (b) less than $\frac{3}{4}$
(c) greater than $\frac{3}{4}$ (d) equal to $\frac{1}{2}$

63. Two circles with radii 25 cm and 9 cm touch each other externally. The length of the direct common tangent is
(a) 34 cm (b) 30 cm (c) 36 cm (d) 32 cm

64. If AB = 5 cm, AC = 12 and $AB \perp AC$ then the radius of the circumcircle of ΔABC is
(a) 6.5 cm (b) 6 cm (c) 5 cm (d) 7 cm

65. The radius of the circumcircle of the triangle made by x-axis, y-axis and $4x + 3y = 12$ is
(a) 2 unit (b) 2.5 unit (c) 3 unit (d) 4 unit

66. The length of the circum-radius of a triangle having sides of lengths 12 cm, 16 cm and 20 cm is
(a) 15 cm (b) 10 cm (c) 18 cm (d) 16 cm

67. If D is the mid-point of the side BC of ΔABC and the area of ΔABD is 16 cm^2, then the area of ΔABC is
(a) 16 cm^2 (b) 24 cm^2 (c) 32 cm^2 (d) 48 cm^2

68. ABC is a triangle. The medians CD and BE intersect each other at O. Then $\Delta ODE : \Delta ABC$ is
(a) 1 : 3 (b) 1 : 4 (c) 1 : 6 (d) 1 : 12

69. If P, R, T are the area of a parallelogram, a rhombus and a triangle standing on the same base and between the same parallels, which of the following is true?
(a) $R < P < T$ (b) $P > R > T$
(c) $R = P = T$ (d) $R = P = 2T$

70. AB is a diameter of the circumcircle of ΔAPB; N is the foot of the perpendicular drawn from the point P on AB. If $AP = 8$ cm and $BP = 6$ cm, then the length of BN is
(a) 3.6 cm (b) 3 cm (c) 3.4 cm (d) 3.5 cm

71. Two circles with same radius r intersect each other and one passes through the centre of the other. Then the length of the common chord is
(a) r (b) $\sqrt{3}r$ (c) $\frac{\sqrt{3}}{2}r$ (d) $\sqrt{5}r$

72. The bisector of $\angle A$ of ΔABC cuts BC at D and the circumcircle of the triangle at E. Then
(a) $AB : AC = BD : DC$ (b) $AD : AC = AE : AB$
(c) $AB : AD = AC : AE$ (d) $AB : AD = AE : AC$

73. Two circles intersect each other at P and Q. PA and PB are two diameters. Then $\angle AQB$ is
(a) 120° (b) 135° (c) 160° (d) 180°

74. O is the centre of the circle passing through the points A, B and C such that $\angle BAO = 30°$, $\angle BCO = 40°$ and $\angle AOC = x°$. What is the value of x ?
(a) 70° (b) 140° (c) 210° (d) 280°

75. A and B are centres of the two circles whose radii are 5 cm and 2 cm respectively. The direct common tangents to the circles meet AB extended at P. Then P divides AB.
(a) externally in the ratio 5 : 2
(b) internally in the ratio 2 : 5
(c) internally in the ratio 5 : 2
(d) externally in the ratio 7 : 2

76. A, B, P are three points on a circle having centre O. If $\angle OAP = 25°$ and $\angle OBP = 35°$, then the measure of $\angle AOB$ is
(a) 120° (b) 60° (c) 75° (d) 150°

77. Side $\overline{BC}$ of ΔABC is produced to D. If $\angle ACD = 140°$ and $\angle ABC = 3\angle BAC$, then find $\angle A$.
(a) 55° (b) 45° (c) 40° (d) 35°

78. The length of tangent (upto the point of contact) drawn from an external point P to a circle of radius 5 cm is 12 cm. The distance of P from the centre of the circle is
(a) 11 cm (b) 12 cm (c) 13 cm (d) 14 cm

79. ABCD is a cyclic quadrilateral, AB is a diameter of the circle. If $\angle ACD = 50°$, the value of $\angle BAD$ is
(a) 30° (b) 40° (c) 50° (d) 60°

80. Two circles of equal radii touch externally at a point P. From a point T on the tangent at P, tangents TQ and TR are drawn to the circles with points of contact Q and R respectively. The relation of TQ and TR is
(a) TQ < TR (b) TQ > TR
(c) TQ = 2TR (d) TQ = TR

81. When two circles touch externally, the number of common tangents are
(a) 4 (b) 3 (c) 2 (d) 1

82. D and E are the mid-points of AB and AC of ΔABC. If $\angle A = 80°$, $\angle C = 35°$, then $\angle EDB$ is equal to
(a) 100° (b) 115° (c) 120° (d) 125°

83. If the inradius of a triangle with perimeter 32 cm is 6 cm, then the area of the triangle in sq. cm is
(a) 48 (b) 100 (c) 64 (d) 96

84. If two circles of radii 9 cm and 4 cm touch externally, then the length of a common tangent is
(a) 5 cm (b) 7 cm
(c) 8 cm (d) 12 cm

ANSWER KEY

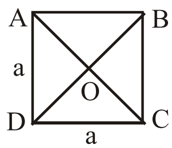
1	(c)	**12**	(c)	**23**	(a)	**34**	(c)	**45**	(a)	**56**	(c)	**67**	(c)	**78**	(c)
2	(a)	**13**	(a)	**24**	(c)	**35**	(b)	**46**	(c)	**57**	(c)	**68**	(d)	**79**	(b)
3	(b)	**14**	(c)	**25**	(a)	**36**	(c)	**47**	(d)	**58**	(b)	**69**	(d)	**80**	(d)
4	(c)	**15**	(a)	**26**	(c)	**37**	(a)	**48**	(d)	**59**	(b)	**70**	(a)	**81**	(b)
5	(b)	**16**	(c)	**27**	(b)	**38**	(d)	**49**	(c)	**60**	(c)	**71**	(b)	**82**	(b)
6	(d)	**17**	(b)	**28**	(d)	**39**	(a)	**50**	(c).	**61**	(b)	**72**	(d)	**83**	(d)
7	(c)	**18**	(a)	**29**	(a)	**40**	(b)	**51**	(c)	**62**	(c)	**73**	(d)	**84**	(d)
8	(a)	**19**	(b)	**30**	(a)	**41**	(a)	**52**	(b)	**63**	(b)	**74**	(b)		
9	(d)	**20**	(a)	**31**	(b)	**42**	(b)	**53**	(a)	**64**	(a)	**75**	(a)		
10	(b)	**21**	(b)	**32**	(b)	**43**	(b)	**54**	(d)	**65**	(b)	**76**	(a)		
11	(b)	**22**	(b)	**33**	(b)	**44**	(c)	**55**	(d)	**66**	(b)	**77**	(d)		

HINTS & SOLUTIONS

1. (c) In a right angled Δ, the length of the median is $\frac{1}{2}$ the length of the hypotenuse. Hence $BD = \frac{1}{2}AC = 3cm.$

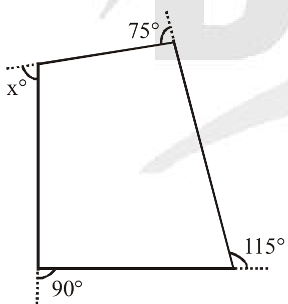
2. (a) $\angle D = 180 - \angle B = 180 - 70 = 110°$
$\therefore \angle ACD = 180 - \angle D - \angle CAD$
$180 - 110 - 30 = 40°$

3. (b)



ABCD is square $a^2 = 4 \Rightarrow a = 2$
$AC = BD = 2\sqrt{2}$
perimeters of four triangles
$= AB + BC + CD + DA + 2(AC + BD)$
$= 8 + 2(2\sqrt{2} + 2\sqrt{2}) = 8(1 + \sqrt{2})$

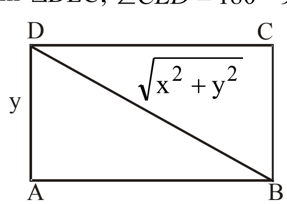
4. (c) Sum of all the exterior angles of a polygon taken in order is 360°.



i.e. $x + 90 + 115 + 75 = 360$
or $x = 360° - 280° = 80°$ or $x = 80°$

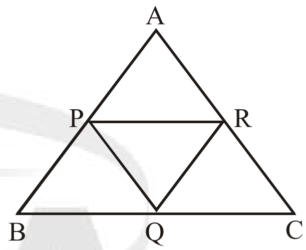
5. (b) In $\Delta ABC, \angle C = 180 - 90 - 30 = 60°$
$\therefore \Delta DCE = \frac{60}{2} = 30°$
Again in ΔDEC, $\angle CED = 180 - 90 - 30 = 60°$

6. (d)



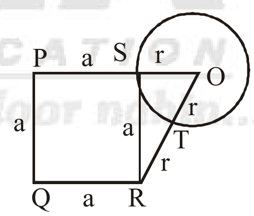
According to question,
$$(x + y) - \sqrt{x^2 + y^2} = \frac{x}{2}$$
$$(x + y) - \frac{x}{2} = \sqrt{x^2 + y^2}$$
$$\left(\frac{x}{2} + y\right)^2 = x^2 + y^2$$
$$\frac{x^2}{4} + y^2 + xy = x^2 + y^2$$
$$x^2 + 4xy = 4x^2$$
$$4xy = 3x^2 \Rightarrow 4y = 3x \Rightarrow \frac{y}{x} = \frac{3}{4}$$

7. (c) Consider for an equilateral triangle. Hence ΔABC consists of 4 such triangles with end points on mid points AB, BC and CA



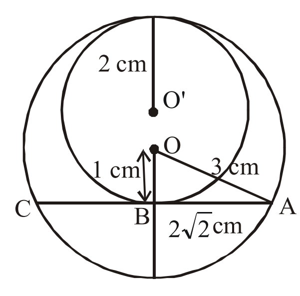
$\Rightarrow \frac{1}{4}$ ar (ΔABC) = ar (ΔPQR)
$\Rightarrow$ ar (ΔPQR) = 5 sq. units

8. (a)



In $\Delta SOR, a^2 + r^2 = (2r)^2 = 4r^2$
$\Rightarrow a^2 = 3r^2$ or $a = \sqrt{3}r$
$$\therefore \frac{\text{Area of circle}}{\text{Area of square}} = \frac{\frac{22}{7} \times r^2}{(\sqrt{3}r)^2} = \frac{22}{7 \times 3} = \frac{22}{21} = \frac{\pi}{3}$$

9. (d)



$AB = \sqrt{3^2 - 1^2} = 2\sqrt{2}$ cm
$\therefore$ AC = $4\sqrt{2}$ cm

10. (b) Let the common base be x m.
Now, area of the triangle = area of the parallelogram

$\frac{1}{2} \times x \times$ Altitude of the triangle $= x \times 100$

Altitude of the triangle = 200 m

11. (b) The sum of the interior angles of a polygon of n sides is given by the expression $(2n-4)\ \frac{\pi}{2}$

$\Rightarrow\ (2n-4) \times \frac{\pi}{2} = 1620 \times \frac{\pi}{180}$

$(2n-4) = \frac{1620 \times 2}{180} = 18$

or $2n = 22$
or $n = 11$

12. (c) $\frac{\text{Ratio of uncut portion}}{\text{Ratio of cut portion}} = \frac{(\pi \times 20 \times 20) - (100\pi)}{(4 \times \pi \times 5 \times 5)}$

$= \frac{300\pi}{100\pi} = \frac{3}{1}$

13. (a) AD = 24, BC = 12
In ΔBCE & ΔADE
since $\angle CBA = \angle CDA$ (Angles by same arc)
$\angle BCE = \angle DAE$ (Angles by same arc)
$\angle BEC = \angle DEA$ (Opp. angles)
$\therefore \angle BCE$ & $\angle DAE$ are similar Δs
with sides in the ratio 1 : 2
Ratio of area = 1:4 (i.e square of sides)

14. (c) Here $x_1 = 4,\ x_2 = -2,\ y_1 = -1,\ y_2 = 4$
and $m_1 = 3$ and $m_2 = 5$

$\therefore x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \frac{3(-2) + 5(4)}{3+5} = \frac{7}{4}$

and $y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{3(4) + 5(-1)}{3+5} = \frac{7}{8}$

$\therefore$ The required point is $\left(\frac{7}{4}, \frac{7}{8}\right)$

15. (a) In ΔABC, $DE \parallel BC$
By applying basic Proportionality theorem,

$\frac{AD}{DB} = \frac{AE}{EC}$

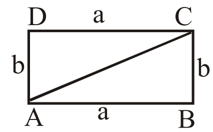
But $\frac{AD}{DB} = \frac{3}{5}$ (Given)

$\therefore\ \frac{AE}{EC} = \frac{3}{5}$ or $\frac{AE}{EC + AE} = \frac{3}{5+3}$ or $\frac{AE}{AC} = \frac{3}{8}$

or $\frac{AE}{5.6} = \frac{3}{8} \Rightarrow 8AE = 3 \times 5.6 \Rightarrow AE = 3 \times 5.6/8$

$\therefore$ AE = 2.1 cm.

16. (c)



$AC + AB = 5AD$ or $AC + a = 5b$ ....(i)
$AC - AD = 8$ or $AC = b + 8$ ....(ii)
Using (i) and (ii) , $a + b + 8 = 5b$ or $a + 8 = 4b$ ...(iii)
Using Pythagorous theorem,

$a^2 + b^2 = (b+8)^2 = b^2 + 64 + 16b$

or $a^2 = 16b + 64 = (4b-8)^2 = 16b^2 + 64 - 64b$
[From (iii)]

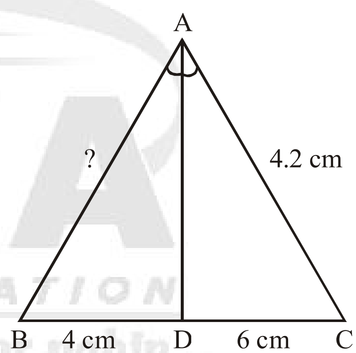
$\Rightarrow 16b^2 - 80b = 0$ or $b = 0$ or 5
Putting b = 5 in (iii), a = 4b – 8 = 20 – 8 = 12
Area of rectangle = $12 \times 5 = 60$

17. (b) $\Delta PAB \sim \Delta PQR$

$\frac{PB}{AB} = \frac{PR}{QR} \Rightarrow \frac{PB}{3} = \frac{6}{9}$

$\therefore$ PB = 2 cm

18. (a)

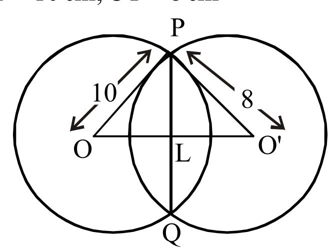


using angle bisector theorem

$\frac{AC}{AB} = \frac{DC}{BD} \Rightarrow \frac{4.2}{6} = \frac{AB}{4}$

$\therefore$ AB = 2.8 cm

19. (b) Here, OP = 10 cm; O'P = 8 cm



PQ = 12 cm

$\therefore$ PL = 1/2 PQ $\Rightarrow PL = \frac{1}{2} \times 12 \Rightarrow PL = 6$ cm

In rt. ΔOLP, $OP^2 = OL^2 + LP^2$
(using Pythagoras theorem)

$\Rightarrow (10)^2 = OL^2 + (6)^2 \Rightarrow OL^2 = 64$; OL = 8

In $\Delta O'LP$, $(O'L)^2 = O'P^2 - LP^2 = 64 - 36 = 28$

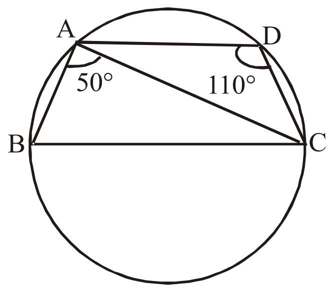
$O'L^2 = 28 \Rightarrow O'L = \sqrt{28}$

$O'L = 5.29$ cm

$\therefore OO' = OL + O'L = 8 + 5.29$

$OO' = 13.29$ cm

20. (a) $\angle ABC + \angle ADC = 180°$ (sum of opposites angles of cyclic quadrilateral is 180°)



$\Rightarrow \angle ABC + 110° = 180°$

(ABCD is a cyclic quadrilateral)

$\Rightarrow \angle ABC = 180 - 110 \Rightarrow \angle ABC = 70°$ $(\because AD \parallel BC)$

$\therefore \angle ABC + \angle BAD = 180°$ (Sum of the interior angles on the same side of transversal is 180°)

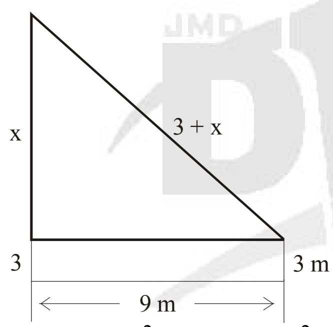
$70° + \angle BAD = 180°$

$\Rightarrow \angle BAD = 180° - 70° = 110°$

$\Rightarrow \angle BAC + \angle DAC = 110° \Rightarrow 50° + \angle DAC = 110°$

$\Rightarrow \angle DAC = 110° - 50° = 60°$

21. (b)

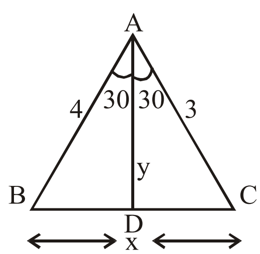


Using pythagoras, $x^2 + 81 = (3 + x)^2$

or $x^2 + 81 = 9 + x^2 + 6x \Rightarrow 6x = 72$ or $x = 12$m

Height of wall = 12 + 3 = 15 m

22. (b)



Using the theorem of angle of bisector,

$$\frac{BD}{DC} = \frac{AB}{AC} = \frac{4}{3} \Rightarrow BD = \frac{4}{7}x \ \& \ DC = \frac{3}{7}x$$

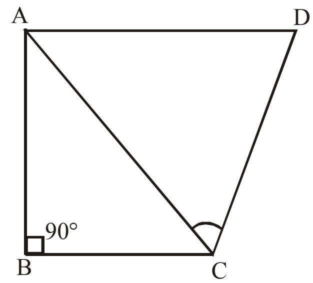
In ΔABD, by sine rule, $\frac{\sin 30}{4/7x} = \frac{\sin B}{y}$ ......(i)

In ΔABC, by sine rule; $\frac{\sin 60}{x} = \frac{\sin B}{3}$

or $\frac{\sqrt{3}}{2x} = \frac{\sin 30.y}{4/7x \times 3}$ [putting the value of sin B from (i)]

$$\Rightarrow y = \frac{\sqrt{3}}{2x} \times \frac{4}{7} x \times 3 \times \frac{2}{1} = \frac{12\sqrt{3}}{7}$$

23. (a) We have, $AD^2 = AB^2 + BC^2 + CD^2$



In ΔABC

$AC^2 = AB^2 + BC^2$

$\Rightarrow AD^2 = AC^2 + CD^2 \Rightarrow \angle ACD = 90°$

24. (c) Let n be the number of sides of the polygon

Now, sum of interior angles = 8 × sum of exterior angles

i.e. $(2n - 4) \times \frac{\pi}{2} = 8 \times 2\pi$

or $(2n - 4) = 32$

or $n = 18$

25. (a) Let the third vertex be (x, y)

$\therefore$ The centroid of the triangle is given (6, 1).

$$\Rightarrow \frac{x_1 + x_2 + x_3}{3} = 6 \Rightarrow \frac{3 + 11 + x}{3} = 6 \Rightarrow 14 + x = 18$$

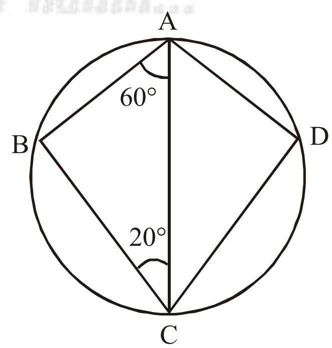
$\Rightarrow x = 4$

and $\frac{y_1 + y_2 + y_3}{3} = 1 \Rightarrow \frac{2 + 4 + y}{3} = 1 \Rightarrow 6 + y = 3$

$y = -3$

$\therefore$ Third vertex is (4, –3)

26. (c) In ΔABC, $\angle B = 180° - (60° + 20°)$ (By ASP)



$\Rightarrow \angle B = 100°$

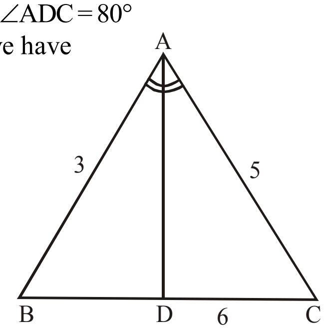
But $\angle B + \angle D = 180°$

($\because$ ABCD is a cyclic quadrilateral;

Sum of opposite is 180°)

$100° + \angle D = 180° \Rightarrow \angle ADC = 80°$

27. (b) As AD biseets CA, we have



$$\frac{BD}{AB} = \frac{DC}{AC}$$

or $\frac{DC}{BD} = \frac{5}{3}$

or $\frac{DC}{BD} + 1 = \frac{5}{3} + 1$

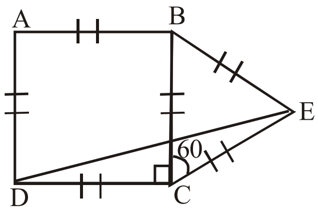
or $\dfrac{DC+BD}{BD}=\dfrac{5+3}{3}$

or $\dfrac{BC}{BD}=\dfrac{8}{3}$

or $BD=\dfrac{BC\times 3}{8}=\dfrac{6\times 3}{8}=\dfrac{9}{4}=2.25\text{ cm}$

28. (d) Let the radius of the semi- circle be R and that of the circle be r, then from the given data, it is not possible to express r in terms of R. Thus option (d) is the correct alternative.

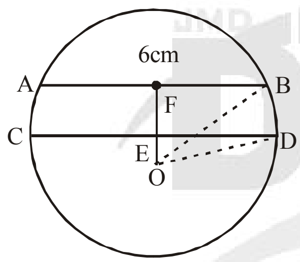
29. (a)



In $\Delta DEC, \angle DCE=90°+60°=150°$

$\angle CDE=\angle DEC=\dfrac{180-150}{2}=15°$

30. (a) Draw $OE\perp CD$ and $OF\perp AB$



$AB \parallel CD$ (Given)

Let 'r' be the radius of the circle

Now in rt. ΔOED,

$(OD)^2=(OE)^2+(ED)^2$

(using Pythagoras theorem)

$r^2=x^2+(6)^2 \quad \left(\because ED=\dfrac{1}{2}CD=\dfrac{1}{2}\times 12=6\text{cm}\right)$

$\Rightarrow r^2=x^2+36$ ....(i)

In rt. ΔOFB, $(OB)^2=(OF)^2+(FB)^2$

$\Rightarrow r^2=(x+3)^2+(3)^2 \Rightarrow r^2=x^2+6x+9+9$

$\Rightarrow r^2=x^2+6x+18$ ....(ii)

From (i) and (ii), we get $x^2+36=x^2+6x+18$

$\Rightarrow 36=6x+18 \Rightarrow 36-18=6x$

$18=6x \Rightarrow 3=x$

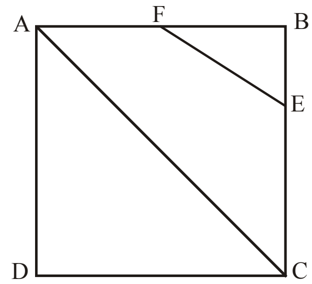
For (i), $r^2=(3)^2+(6)^2$

$r^2=9+36 \Rightarrow r^2=45$

$r=\sqrt{45}\Rightarrow r=3\sqrt{5}$ cm.

31. (b) Let the side of the square be x, then

$BE=\dfrac{x}{3}$ and $BF=\dfrac{x}{2}$



Area of $\Delta FEB=\dfrac{1}{2}\times\dfrac{x}{3}\times\dfrac{x}{2}=\dfrac{x^2}{12}$

Now, $\dfrac{x^2}{12}=108$

$\Rightarrow x^2=108\times 12=1296$

In ΔADC, we have

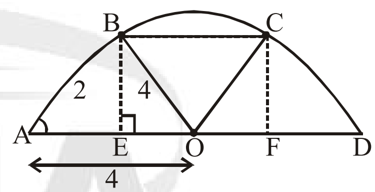
$AC^2=AD^2+DC^2$

$=x^2+x^2=2x^2$

$=2\times 1296=2592$

or $AC=\sqrt{2592}=36\sqrt{2}$

32. (b)



$BO=\text{radius}=4=AO$

$AE=2\cos A=2\times\left(\dfrac{2^2+4^2-4^2}{2\times 2\times 4}\right)=\dfrac{2}{4}=\dfrac{1}{2}$

$\therefore BC=AD-AE-FD=8-\dfrac{1}{2}-\dfrac{1}{2}=7\ (\because AE=FD)$

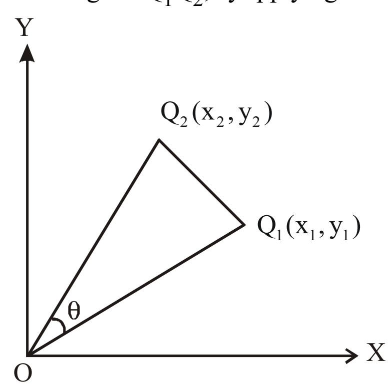
33. (b) Ratio $=-\left(\dfrac{-1+1-4}{5+7-4}\right)=\dfrac{1}{2}$

34. (c) Let fourth vertex be (x, y), then $\dfrac{x+8}{2}=\dfrac{2+5}{2}$

and $\dfrac{y+4}{2}=\dfrac{-2+7}{2}\Rightarrow x=-1, y=1$

35. (b) $x=\dfrac{2+5+3}{3}=\dfrac{10}{3}$ and $y=\dfrac{1+2+4}{3}=\dfrac{7}{3}$

36. (c) From triangle OQ_1Q_2, by applying cosine formula.



$$Q_1Q_2^2 = OQ_1^2 + OQ_2^2 - 2OQ_1.OQ_2 \cos Q_1OQ_2$$

or $(x_1 - x_2)^2 + (y_1 - y_2)^2$
$= x_1^2 + y_1^2 + x_2^2 + y_2^2 - 2OQ_1.OQ_2 \cos\theta$

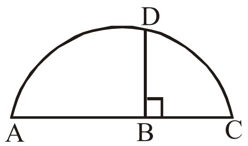
or $x_1x_2 + y_1y_2 = OQ_1.OQ_2 \cos Q_1OQ_2$

37. (a) $\angle EDC = \angle BAD = 45°$ (alternate angles)
$\therefore$ x = DEC = 180° – (50° + 45°) = 85°.

38. (d) $\frac{1}{2}[4-(2+16)+3(-16-4)+3(4+2)]$

$= \frac{1}{2}[56-60+18] = 7$

39. (a) m $\angle ADC = 90°$
(Angle subtended by the diameter on a circle is 90°)



$\therefore$ Δ ADC is a right angled triangle.
$\therefore$ $(DB)^2 = BA \times BC$ ..
(DB is the perpendicular to the hypotenuse)
$= 9 \times 4 = 36$
$\therefore$ DB = 6

40. (b) As F is the mid-point of AD, CF is the median of the triangle ACD to the side AD.
Hence area of the triangle FCD = area of the triangle ACF.
Similarly area of triangle BCE = area of triangle ACE.
$\therefore$ Area of ABCD = Area of (CDF + CFA + ACE + BCE)
= 2 Area (CFA + ACE) = 2 × 13 = 26 sq. units.

41. (a) a + 36° + 70° = 180° (sum of angles of triangle)
$\Rightarrow$ a = 180° – 36° – 70° = 74°
b = 36° + 70°(Ext. angle of triangle) = 106°
c = a – 50° (Ext. angle of triangle) = 74° – 50° = 24°.

42. (b) Since the sum of all the angle of a quadrilateral is 360°
We have $\angle ABC + \angle BQE + \angle DEF + \angle EPB = 360°$
$\therefore \angle ABC + \angle DEF = 180°$ [$\because$ BPE = EQB = 90°]

43. (b) m $\angle AHG = 180 - 108 = 72^0$
$\therefore$ $\angle AHG = \angle ABC$ .....(same angle with different names)
$\therefore$ $\Delta AHG - \Delta ABC$ .....(AA test for similarity)

$\frac{AH}{AB} = \frac{AG}{AC}$; $\frac{6}{12} = \frac{9}{AC}$

$\therefore AC = \frac{12 \times 9}{6} = 18$

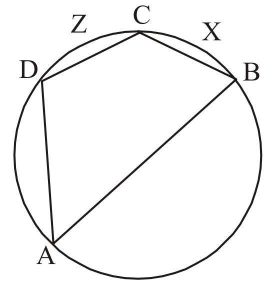
$\therefore$ HC = AC – AH = 18 – 6 = 12

44. (c) $b = \frac{1}{2}(48°)$

($\angle$at centre = 2 at circumference on same PQ) 24°
$\angle AQB = 90°$ ($\angle$In semi- circle)
$\angle QXB = 180° - 90° - 24°$ ($\angle$sum of Δ) = 66°

45. (a) $AO = \sqrt{OQ^2 - AQ^2} = \sqrt{5^2 - 4^2} = \sqrt{9} = 3$
Now, from similar Δs QAO and QOR
OR = 2OA = 2× 3 = 6 cm.

46. (c) m $\angle DAB + 180° - 120° = 60°$
(Opposite angles of a cyclic quadrilateral)
m (arc BCD) = 2m $\angle DAB = 120°$.



$\therefore$ m (arc CXB) = m (BCD) – m (arc DZC)
= 120° – 70° = 50° .

47. (d) $\angle MBA = 180° - 95° = 85°$
$\angle AMB = \angle TMN$ ...(Same angles with different names)
$\therefore$ $\Delta MBA - \Delta MNT$ ......(AA test for similarity)

$\frac{MB}{MN} = \frac{AB}{NT}$ ....(proportional sides)

$\frac{10}{MN} = \frac{5}{9}$ $\therefore MN = \frac{90}{5} = 18.$

48. (d) The three length AB, BC, AC will be

$AB = \sqrt{[(-1+4)^2 + (4+2)^2]} = \sqrt{45}$

$BC = \sqrt{[(-4-3)^2 + (-2+2)^2]} = \sqrt{7^2 + 2^2} = \sqrt{53}$

$AC = \sqrt{4^2 + 8^2} = \sqrt{80}$
Perimeter = AB + BC + AC

49. (c) m $\angle ABM = 180° - 120° = 60°$
$\therefore$ Δ AMB is a 30° – 60° – 90° triangle.

$\therefore$ AM $\frac{\sqrt{3}}{2}$ AB = $\frac{\sqrt{3}}{2} \times 8 = 4\sqrt{3}$

MB = $\frac{1}{2}$ AB = $\frac{1}{2}$ x 8 = 4

$(AC)^2 = (AM)^2 + (MC)^2 = (4\sqrt{3})^2 + (4+7)^2$

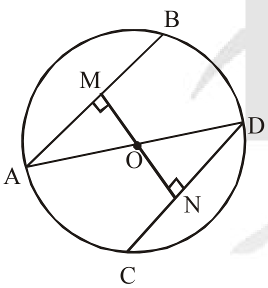
= 48 + 121 = 169 ; AC = $\sqrt{169}$ = 13.

50. (c) Let the line m cut *AB* and *CD* at point *P* and *Q* respectively
$\angle$ *DOQ* = *x* (exterior angle)
Hence, *Y* + 2*x* (corresponding angle)
$\therefore y = x$
...(1)
Also . $\angle$ *DOQ* = *x* (vertically opposite angles)
In Δ *OCD*, sum of the angles = 180°
$\therefore y + 2y + 2x + x = 180°$
$3x + 3y = 180°$
$x + y = 60$
...(2)
From (1) and (2) $x = y = 30 = 2y = 60$
$\therefore \angle ODS = 180 - 60 = 120°$
$\therefore \theta = 180 - 3x = 180 - 3(30) = 180 - 90 = 90°.$
$\therefore$ The required ratio = 90 : 120 = 3 : 4.

51. (c) Given AB is a circle and BT is a tangent, $\angle BAO = 32^\circ$
Here, $\angle OBT = 90^\circ$
[$\because$ Tangent is $\perp$ to the radius at the point of contact]
$OA = OB$ [Radii of the same circle]
$\therefore \angle OBA = \angle OAB = 32^\circ$
[Angles opposite to equal side are equal]
$\therefore \angle OBT = \angle OBA + \angle ABT = 90^\circ$ or $32^\circ + x = 90^\circ$.
$\angle x = 90^\circ - 32^\circ = 58^\circ$.
Also, $\angle AOB = 180^\circ - \angle OAB - \angle OBA$
$= 180^\circ - 32^\circ - 32^\circ = 116^\circ$
Now $Y = \frac{1}{2}$ AOB
[Angle formed at the center of a circle is double the angle formed in the remaining part of the circle]
$= \frac{1}{2} \times 116^\circ = 58^\circ$.

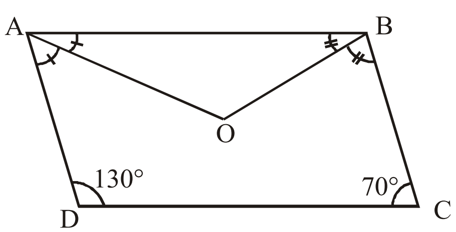
52. (b) $m\angle ACD = \frac{1}{2}$ M(are CXD) $= m\angle DEC$
$\therefore m\angle DEC = x = 40^\circ$
$\therefore m\angle ECB = \frac{1}{2}$ m (are EYC) $= m\angle EDC$
$\therefore m\angle ECB = y = 54^\circ$
$54 + x + z = 180^\circ$ (Sum of all the angles of a triangle)
$54 + 40 + z = 180^\circ$
$\therefore z = 86^\circ$.

53. (a)



Two parallel chords AB & CD & AB = CD = 8 cm
Diameter of circle = AD = 10 cm.
$\therefore$ radius $= AO = OD = \frac{10}{2} = 5$ cm
$AM = MB = \frac{AB}{2} = 4$ cm.
ΔAOM is Right angle Δ,
$AO^2 = AM^2 + OM^2$
$5^2 = 4^2 + OM^2$
$OM^2 = 25 - 16 = 9$
$\Rightarrow$ OM = 3 cm.
Similarly,
OM = ON = 3 cm
$\therefore$ Distance between parallel chords = MN
= OM + ON
= 3 + 3 = 6 cm

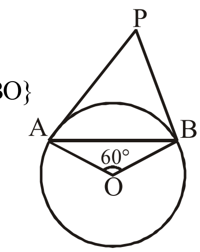
54. (d)



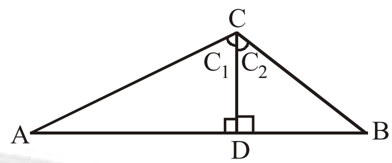
$A + B + C + D = 360$
$A + B = 360 - (130 + 70) = 160^\circ$
$\frac{A}{2} + \frac{B}{2} = 80^\circ$ ...(1)
In Δ AOB,
$\frac{A}{2} + \frac{B}{2} + 0 = 180^\circ$
$0 = 180^\circ - 80^\circ = 100^\circ$

55. (d)

56. (c) In Δ AOB, $\angle A + \angle B + \angle O = 180^\circ$
$\angle A + \angle B = 180 - 140^\circ = 40^\circ$
$\angle A = \angle B = 20^\circ$ {AO = BO}
$\angle PAO = 90^\circ$
$\angle PAB + \angle BAO = 90^\circ$
$\angle PAB = 90^\circ - 20^\circ = 70^\circ$



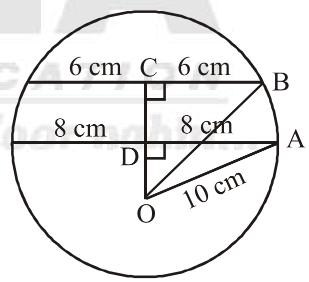
57. (c)



In Δ ADC,
$A + D + C_1 = 180^\circ$; $A + C_1 = 180^\circ - 90^\circ = 90^\circ$
In Δ BDC,
$B + D + C_2 = 180^\circ$; $B + C_2 = 180^\circ - 90^\circ = 90^\circ$
$A + C_1 = B + C_2$
$C_1 - C_2 = B - A$

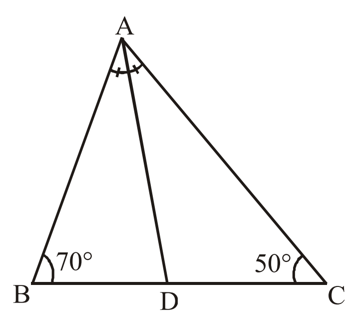
58. (b)

59. (b)



In ΔADO,
$OD = \sqrt{(AO)^2 - AD^2}$
$= \sqrt{100cm^2 - 64cm^2} = 6$ cm
In ΔBCO,
$OC = \sqrt{OB^2 - CB^2}$
$= \sqrt{100cm^2 - 36cm^2} = 8$ cm
distance between chords = OC – OD = 2cm

60. (c)



Given, $\frac{AB}{AC} = \frac{BD}{DC}$

According to angle bisector theorem which states that the angle bisector, like segment AO, divides the sides of the triangle proportionally. Therefore, $\angle A$ being the bisector of triangle.

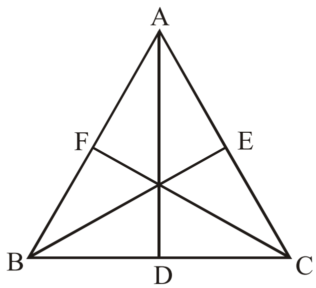
In ΔABC,

$\angle A + \angle B + \angle C = 180°$

$\angle A = 180° - 70° = 60°$

$\angle BAD = \frac{60°}{2} = 30°$

61. (b)



Let ABC be the triangle and D, E and F are midpoints of BC, CA and AB respectively.

Hence, in ΔABD, AD is median

$AB + AC > 2\,AD$ ...(1)

Similarly, we get

$BC + AC > 2\,CF$ ...(2)

$BC + AB > 2\,BE$ ...(3)

On adding the above in equations, we get

$(AB + AC + BC + AC + BC + AB) > 2\,(AD + BE + CF)$

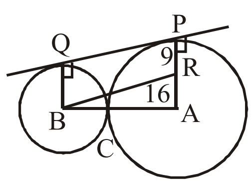
$2\,(AB + AC + BC) > 2\,(AD + BE + CF)$

$\therefore \quad AB + AC + BC > AD + BE + CF$

Thus, the perimeter of triangle is greater than the sum of the medians.

62. (c)

63. (b)



Let the two circles with centre A, B and radii 25 cm and 9 cm touch each other externally at point C.

Then $AB = AC + CB = 25 + 9 = 34$ cm.

Let PQ the direct common tangent. i.e., $BQ \perp PQ$ and $AP \perp PQ$. Draw $BR \perp AP$. Then BRQP is a rctangle.

(Tangent $\perp$ radius at point of contact)

In ΔABR,

$AB^2 = AR^2 + BR^2$

$(34)^2 = (16)^2 + (BR)^2$

$BR^2 = 1156 - 256 = 900$

$BR = \sqrt{900} = 30$ cm

64. (a) In ΔABC,

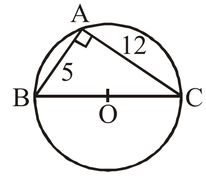
$BC^2 = AB^2 + AC^2$

$BC^2 = (5)^2 + (12)^2$

$BC^2 = 25 + 144$

$BC^2 = 169$

$BC = \sqrt{169} = 13$ cm



Radius of triangle $= \frac{BC}{2} = \frac{13}{2} = 6.5$ cm

65. (b) Putting $x = 0$ in $4x + 3y = 12$ we get $y = 4$

Putting $y = 0$ in $4x + 3y = 12$ we get $x = 3$

The triangle so formed is right angle triangle with points (0, 0) (4, 0) (0, 3)

So diameter is the hypotenus of triangle $= \sqrt{16+9}$
$= 5$ unit

radius = 2.5 unit

66. (b) Circum Radius (R) $= \frac{abc}{4 \times \text{Area of triangle}}$

[where a, b and c are sides of triangle]

Area of Triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$\left[\therefore s = \frac{a+b+c}{2} = 24\right]$

Area of Triangle $= \sqrt{24 \times 12 \times 8 \times 4} = 8 \times 3 \times 4\ \text{cm}^2$

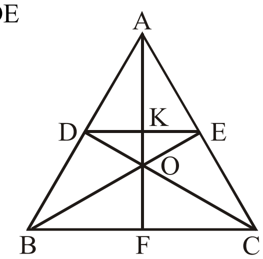
$R = \frac{12 \times 16 \times 20}{4 \times 8 \times 3 \times 4} = 10$ cm

67. (c) Area of $\Delta ABD = 16\ \text{cm}^2$

Area of $\Delta ABC = 2 \times$ Area of ΔABD [$\because$ In triangle, the midpoint of the opposite side, divides it into two congruent triangles. So their areas are equal and each is half the area of the original triangle]

$\Rightarrow 32\ \text{cm}^2$

68. (d) Area of $\Delta ODE = \frac{1}{2} OK \times DE$



$= \frac{1}{2}\left(\frac{1}{2} BC \times OK\right)$

$= \frac{1}{4}[BC \times (AO - AK)]$

$= \frac{1}{4}\left[BC \times \left(\frac{2}{3} AF - \frac{1}{2} AF\right)\right]$

$= \frac{1}{4} \times \frac{1}{3}\left[\frac{1}{2} AF \times BC\right] = \frac{1}{12}$ area of $\Delta ABC = 1 : 12$

69. (d) Parallelogram Area $= l \times b$

Rhombus Area $= l \times b$

Triangle Area $= \frac{l \times b}{2}$

Therefore R = P = 2T.

70. (a) Since AB is a diameter. Then $\angle APB = 90°$ (angle in the semicircle)

$\Delta BPN \sim \Delta APB$

So, $BN = BP^2 / AB$

$BN = \frac{6 \times 6}{10} = 3.6$ cm

71. (b) In ΔAOM

$r^2 = AM^2 + x^2$

$AM^2 = r^2 - x^2$ ...(1)

In $\Delta AMO'$

$r^2 = (r-x)^2 + AM^2$

$AM^2 = r^2 - (r-x)^2$ ...(2)

From eqns. (1) & (2)

$r^2 - x^2 = r^2 - (r-x)^2$

$\Rightarrow 2rx = r^2$

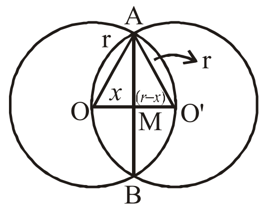
$\Rightarrow x = \frac{r}{2}$

From eq. (1)

$$AM^2 = r^2 - \left(\frac{r}{2}\right)^2 = \frac{3}{4}r^2$$

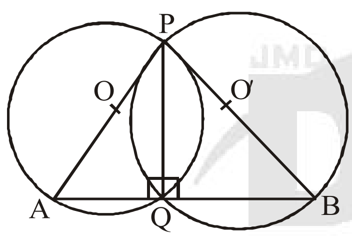
$$AM = \frac{\sqrt{3}}{2}r$$

Length of chord $AB = 2AM = 2 \times \frac{\sqrt{3}}{2}r = \sqrt{3}r$



72. (d)

73. (d)



$\angle AQP = \frac{\pi}{2}$ (Angle in the semicircle is 90°)

$\angle BQP = \frac{\pi}{2}$ (Angle in the semicircle is 90°)

$\angle AQB = \angle AQP + \angle BQP = \frac{\pi}{2} + \frac{\pi}{2} \Rightarrow \pi$ or 180°

74. (b) In ΔAOB

AO = BO (radii of circles)

$\therefore \angle ABO = \angle BAO = 30°$

In ΔBOC

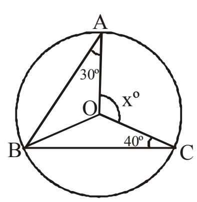
BO = CO (radii of circles)

$\therefore \angle BCO = \angle OBC = 40°$

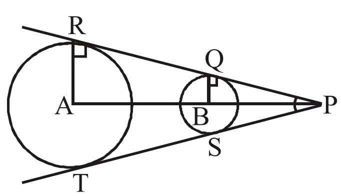
$\angle ABC = \angle ABO + \angle OBC$

$\angle ABC = 30° + 40° = 70°$

$2 \times \angle ABC = \angle AOC \Rightarrow x° = 140$



75. (a)



Δ PQB and Δ PRA are similar triangle by AAA criteria.

$$\therefore \frac{AP}{BP} = \frac{AR}{BQ} = \frac{5}{2}$$

P divides AB externally in the ratio of 5 : 2

76. (a) In ΔOBP.

OB = OP ($\because$ radius)

$\therefore \angle OBP = \angle OPB = 35°$

In ΔAOP

OA = OP ($\because$ radius)

$\therefore \angle OAP = \angle OPA = 25°$

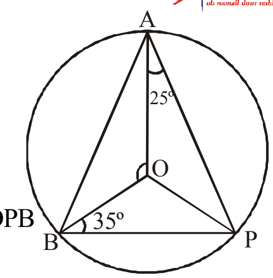
Now, $\angle APB = \angle OPA + \angle OPB$

$= 25° + 35° = 60°$

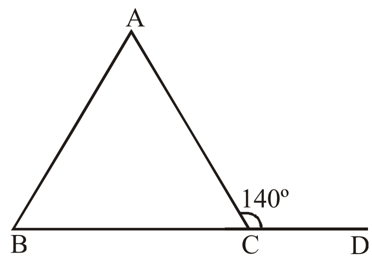
Hence, $\angle AOB = 2\angle APB$

(Angle be substended by are at centre is twice)

$= 2 \times 60° = 120°$



77. (d)



$\angle ACB + \angle ACD = 180°$ (linear pair)

$\therefore \angle ACB = 180° - 140° = 40°$

In ΔABC

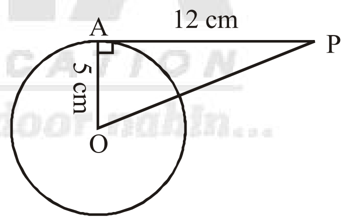
$\angle BAC + \angle ABC + \angle ACB = 180°$

$\angle BAC + 3\angle BAC + 40° = 180°$

$4\angle BAC = 180° - 40°$

$$\angle BAC = \frac{140}{4} = 35°$$

78. (c)



AP is a tangent and OA is a radius.

Therefore, OA is $\perp$ at AP.

So, In ΔOAP

$OP^2 = 5^2 + 12^2$

$OP^2 = 25 + 144 = 169$

OP = 13 cm

79. (b) In ΔABC, $\angle ACB = 90°$

$\therefore \angle ACB + \angle ACD$

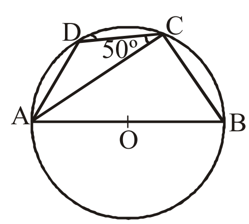
$\Rightarrow 90° + 50° = 140°$

As angle mode by triangle in semicircle is equal to 90°.

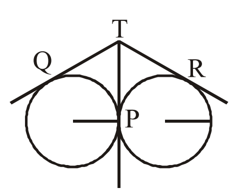
$\therefore$ In quad. ABCD $\angle BAD + \angle BCD = 180°$

angle of (opp. pair of quad is equal to 180°)

$\angle BAD = 180° - 140° = 40°$



80. (d)



TP = TQ [The length of tangents drawn from an external point to a circle are equal]

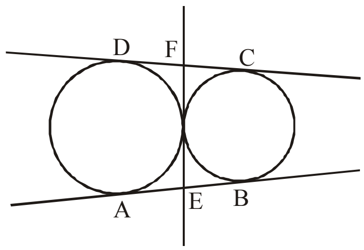
Similarly, TP = TR

Using both equation, we get

TQ = TR

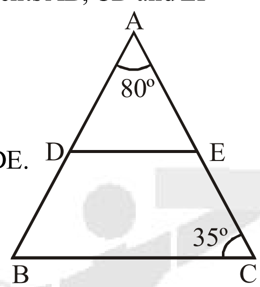
The relation of TQ and TR is TQ = TR.

81. (b)



There are three common tangents AB, CD and EF

82. (b) DE is parallel to BC



So $\angle AED = \angle C = 35°$

Since $\angle A = 80°$

Then $\angle ADE = 65°$

$\angle EDB$ is supplement to $\angle ADE$.

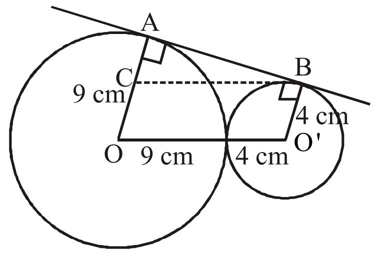
So, $\angle EDB = 180° - \angle ADE$

$= 180° - 65° = 115°$

83. (d) Area of triangle = Inradius × Semi-perimeter

$= 6 \times 16 = 96$ sq. cm

84. (d)



In figure, AC = AO – CO

= 9 cm – 4 cm = 5 cm {CO = BO'}

Also, CB = OO' = 13 cm

In Δ ABC

$AB = \sqrt{CB^2 - AC^2}$

$= \sqrt{(13cm)^2 - (5cm)^2}$

= 12 cm