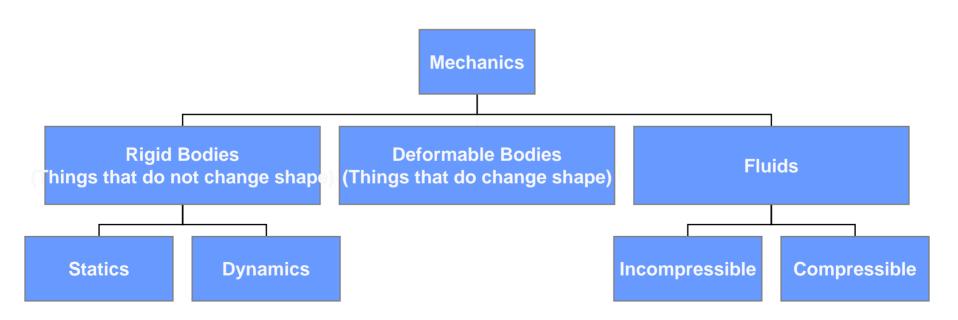
# MEKANIKA TEKNIK

OLEH:
PRAMONO
JURUSAN TEKNIK MESIN
FT – UNNES

# Apa itu Mekanika?

Cabang ilmu fisika yang berbicara tentang keadaan diam atau geraknya benda-benda yang mengalami kerja atau aksi gaya



# Review Sistem Satuan

- Four fundamental physical quantities. Length, Time, Mass, Force.
- We will work with two unit systems in static's: SI & US Customary.

Name	Length	Time	Mass	Force
International System of Units (SI)	meter (m)	second (s)	kilogram (kg)	$\frac{(N)}{\left(\frac{\text{kg} \cdot \text{m}}{\text{s}^2}\right)}$
U.S. Customary (FPS)	foot (ft)	second (s)	$\frac{\text{slug*}}{\left(\frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)}$	pound (lb)

Bagaimana konversi dari SI ke US atau sebaliknya?

# SISTEM GAYA

#### **GAYA**

Gaya adalah interaksi antara benda-benda yang berpengaruh terhadap bentuk atau gerak atau keduanya pada benda yang terlibat.

#### Gaya adalah besaran vektor.

- Besar (magnitude)
- Arah (direction and sense)
- Titik tangkap (point of application)

#### Satuan gaya:

#### SI units:

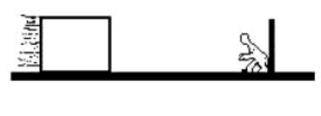
N (Newton)

kN (kilo Newton = 1000 Newton)

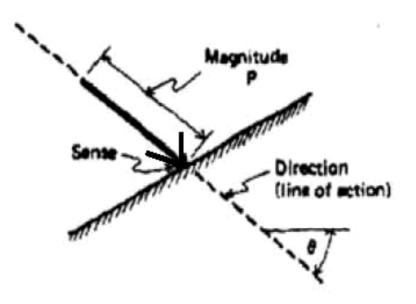
#### US units:

pound (lb, #)

kip (k) = 1000 pound



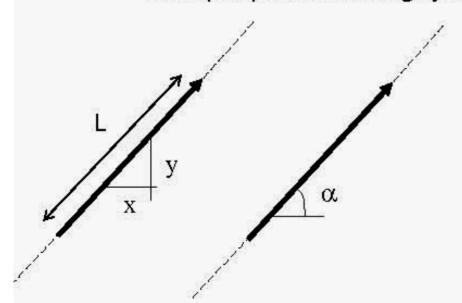




#### **GAYA**

- Presentasi gaya:
  - Secara matematis
  - Secara grafis
- Secara grafis:
  - sebagai garis:
    - panjang garis → besar gaya
    - arah garis → arah gaya
    - garis kerja 

      garis yang panjangnya tak tertentu yang terdapat pada vektor gaya tersebut



Besar gaya : L Newton, misal jika 1 Newton digambarkan dengan panjang garis 1 cm, maka 4 Newton ≈ 4 cm

Arah gaya dinyatakan dalam :  $tg \frac{y}{x}$ , atau besar sudut  $\alpha$  (°)

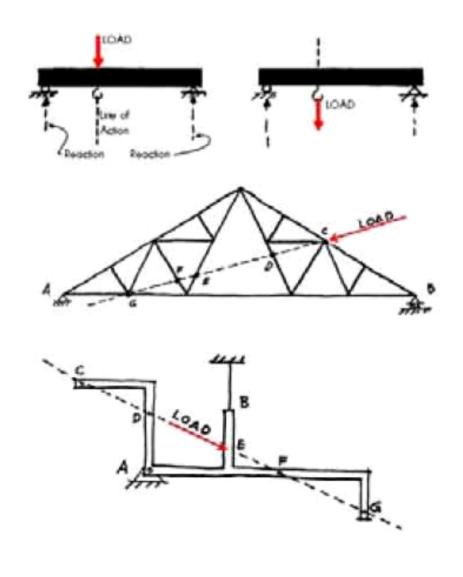
#### **GAYA**

#### Garis kerja gaya

 garis yang panjangnya tak tertentu yang terdapat pada vektor gaya tersebut

Titik tangkap suatu gaya dapat dipindahkan ke titik lain yang terletak pada garis kerjanya, tanpa mengubah efek translational dan rotasional dari gaya terhadap benda yang dibebani.

Gaya yang bekerja pada benda tegar dapat dipandang bekerja di MANA SAJA di sepanjang garis kerjanya.

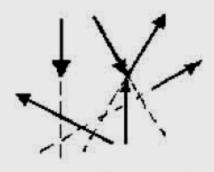


Garis kerja gaya

#### SISTEM GAYA

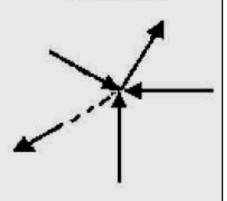
#### Macam sistem gaya:

- collinear
- coplanar
  - Concurrent
  - Parallel
  - Non-concurrent, non-paralel
- space
  - Noncoplanar, parallel
  - Noncoplanar, concurrent
  - Noncoplanar, nonconcurrent,

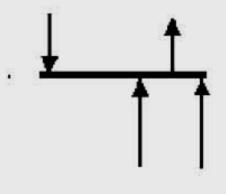


Non-Concurrent Non-Parallel

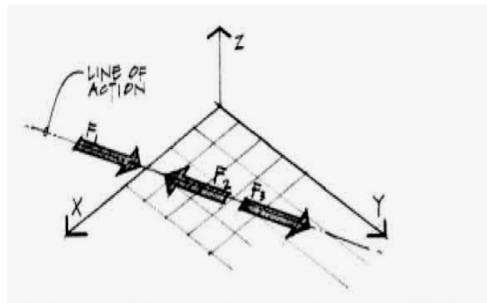




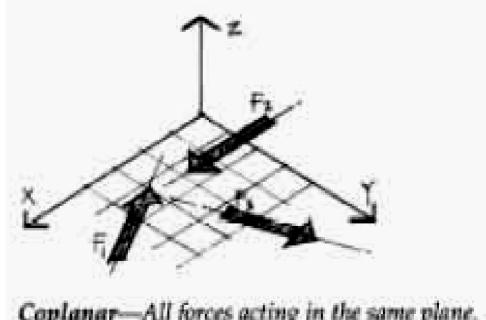
Concurrent



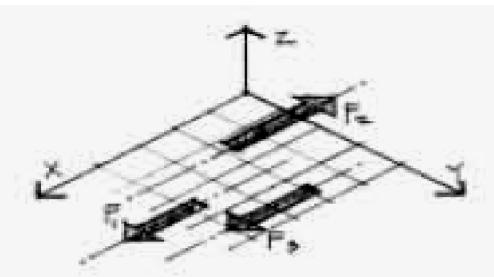
**Parallel** 



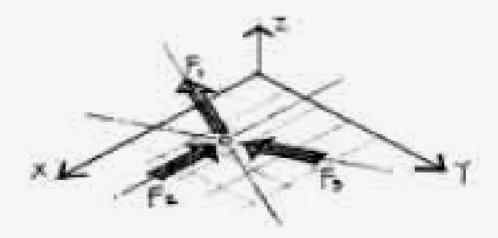
Collinear—All forces acting along the same straight line.



Coplanar-All forces acting in the same plane.

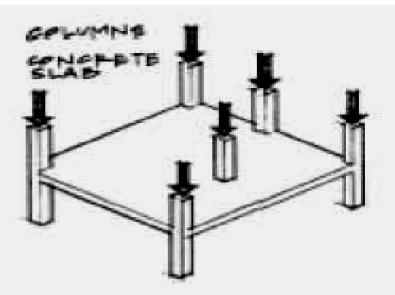


Coplanar, parallel—All forces are parallel and act in the same plane.

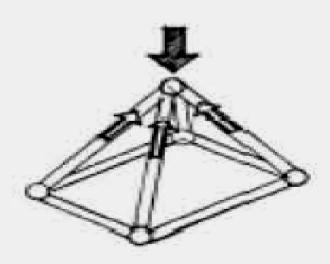


Coplanar, concurrent—All forces intersect at a common point and lie in the same plane.

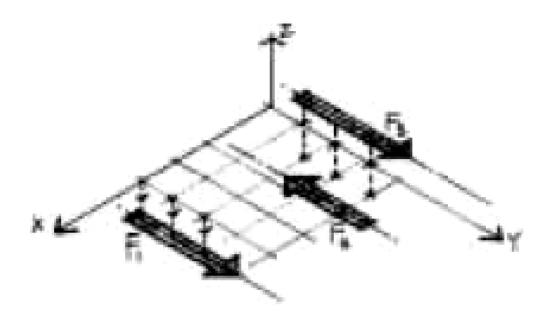
# SISTEM GAYA SPACE (3D)



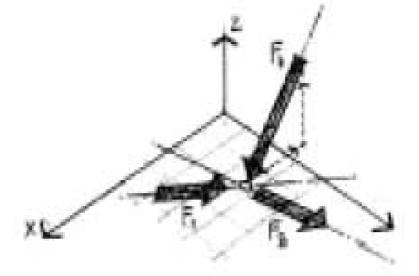
Column loads in a concrete building.



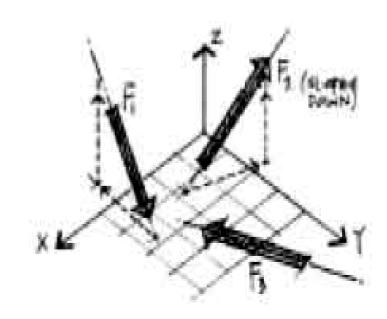
One component of a three-dimensional space frame.



Noncoplanar, parallel—All forces are parallel to each other, but not all lie in the same plane.



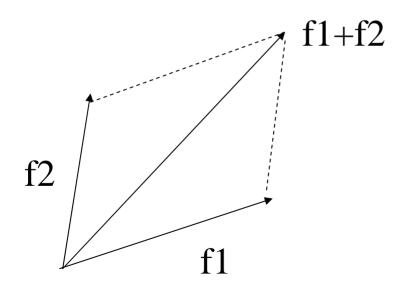
Noncoplanar, concurrent—All forces intersect point but do not all lie in the same plane.

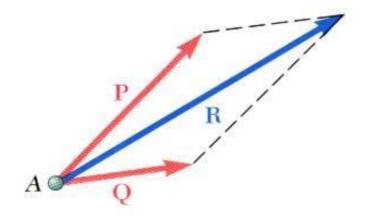


Noncoplanar, nonconcurrent-All forces are skewed.

# Fundamental Principles

 The parallelogram law for the addition of forces: Two forces acting on a particle can be replaced by a single force, called resultant, obtained by drawing the diagonal of the parallelogram which has sides equal to the given forces

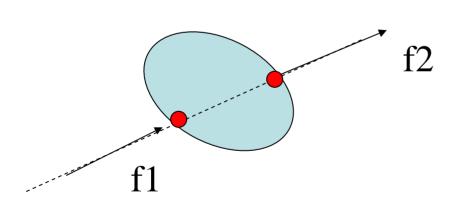




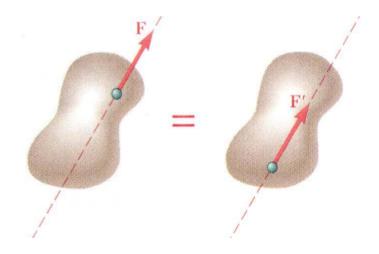
Parallelogram Law

# Fundamental Principles (cont')

• The principle of transmissibility: A force acting at a point of a rigid body can be replaced by a force of the the same magnitude and same direction, but acting on at a different point on the line of action



f1 and f2 are equivalent if their magnitudes are the same and the object is rigid.



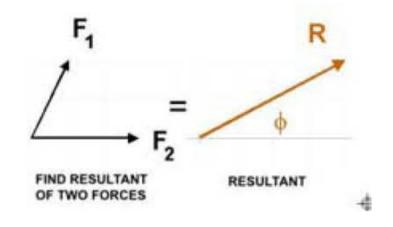
• Principle of Transmissibility

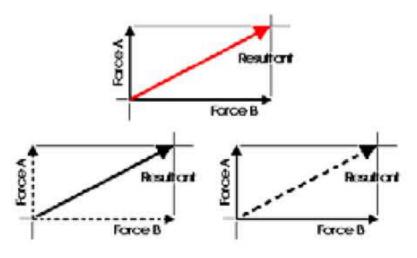
#### Resultan gaya

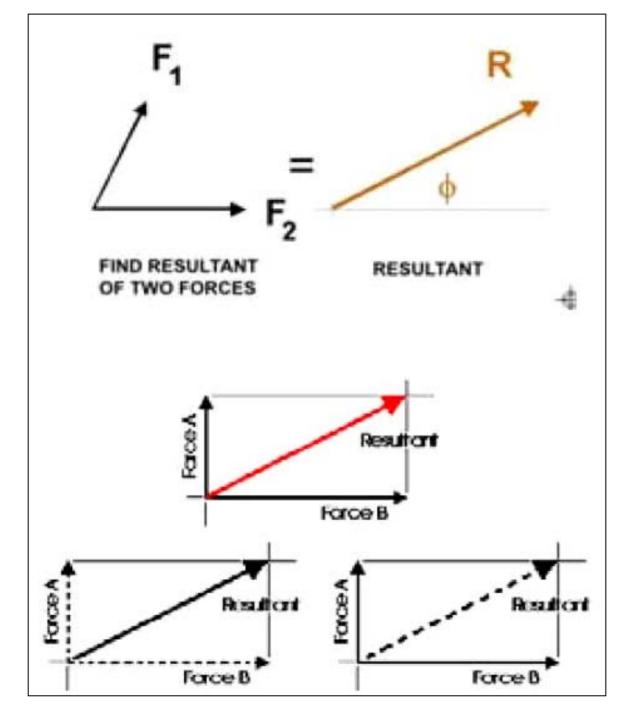
- Dua atau lebih gaya yang tak sejajar dapat dijadikan sederhana menjadi satu resultan gaya tanpa mengubah efek translasional maupun rotasional yg ditimbulkannya pada benda dimana gaya-gaya tsb. bekerja.
  - → pengaruh kombinasi gayagaya tsb setara dgn pengaruh satu gaya yang disebut sebagai resultan gaya-gaya tsb.

# Cara mencari besar dan arah resultan gaya:

- Paralellogram/jajaran genjang gaya atau segitiga gaya
- Polygon gaya
- aljabar

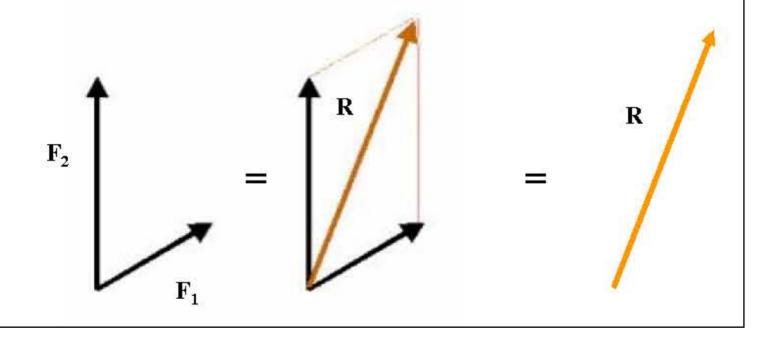




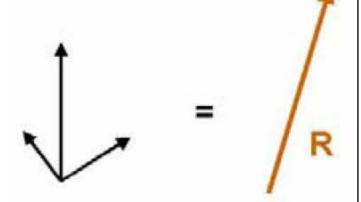


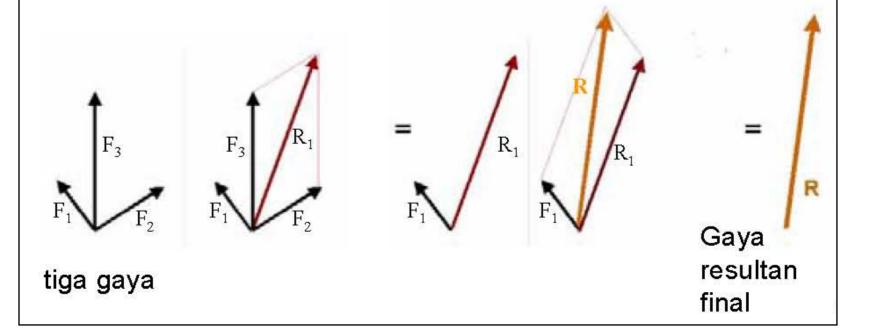
# Metoda paralellogram/jajaran genjang gaya :

 Resultan gaya dinyatakan dgn diagonal jajaran genjang yg dibentuk oleh kedua vektor gaya.



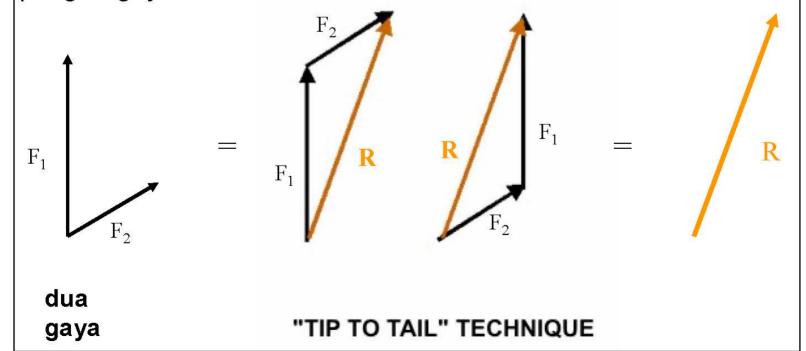
Mencari resultan gaya-gaya: metoda paralellogram/ jajaran genjang gaya





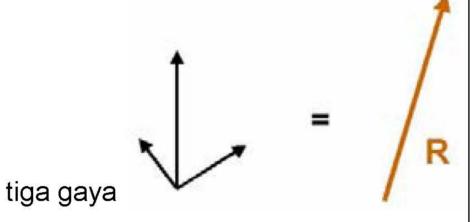
Mencari resultan gaya-gaya: Metoda polygon gaya

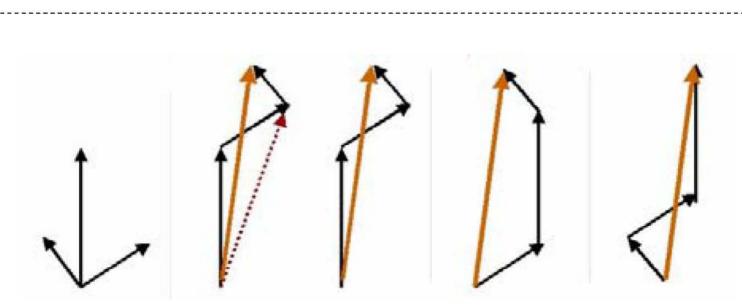
Masing-masing vektor gaya digambar berskala dan saling menyambung (ujung disambung dengan pangkal, urutan tidak penting). Garis penutup, yaitu garis yang berawal dari titik awal vektor pertama ke titik akhir vektor terakhir, merupakan gaya resultan dari semua vektor tsb. gaya resultan tersebut menutup poligon gaya.



Mencari resultan gaya-gaya:

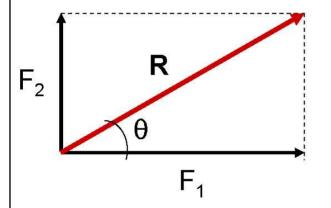
Metoda polygon gaya

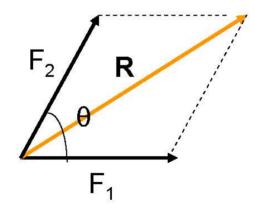


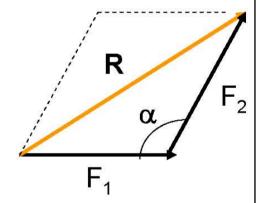


Mencari resultan gaya-gaya:

#### Cara aljabar







$$R = \sqrt{F_1^2 + F_2^2}$$

$$\tan \theta = F_2 I F_1$$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

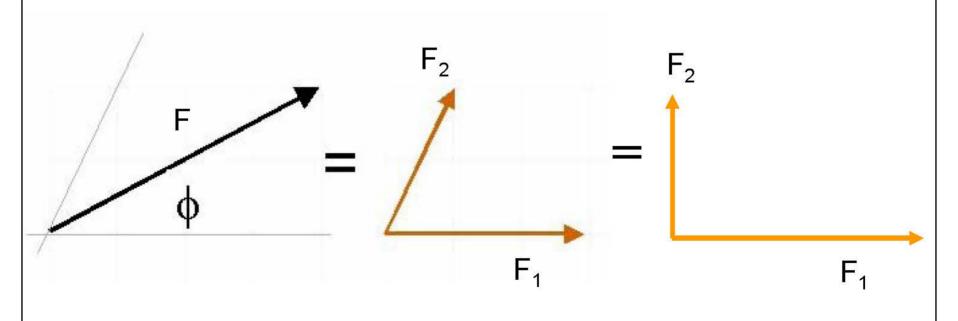
$$R = \sqrt{F_1^2 + F_2^2 - 2F_1F_2 \cos \alpha}$$

## **KOMPONEN GAYA**

### Komponen Gaya

Resolusi gaya:

Adalah penguraian gaya menjadi komponen-komponennya.



gaya F

komponen gaya :

F<sub>1</sub> dan F<sub>2</sub>

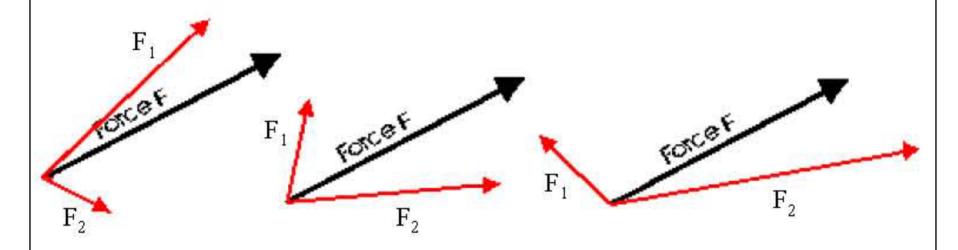
komponen gaya :

F<sub>1</sub> dan F<sub>2</sub>

### **KOMPONEN GAYA**

Mencari komponen gaya:

#### Cara grafis

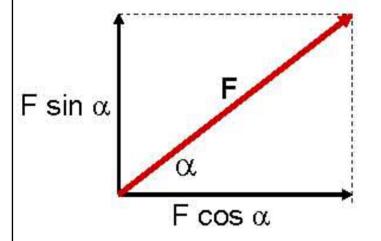


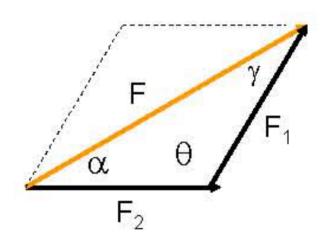
F<sub>1</sub> dan F<sub>2</sub> adalah komponen gaya dari gaya F

Sebuah gaya dapat diuraikan menjadi sistem komponen gaya yang berbeda-beda.

## **KOMPONEN GAYA**

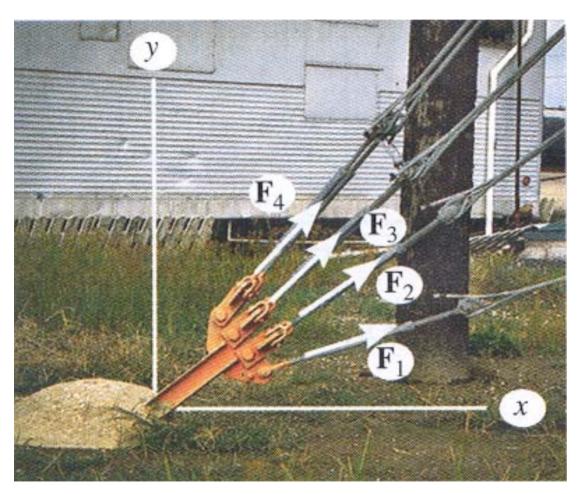
Mencari komponen gaya: Cara aljabar





$$\frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \gamma} = \frac{F}{\sin \theta}$$

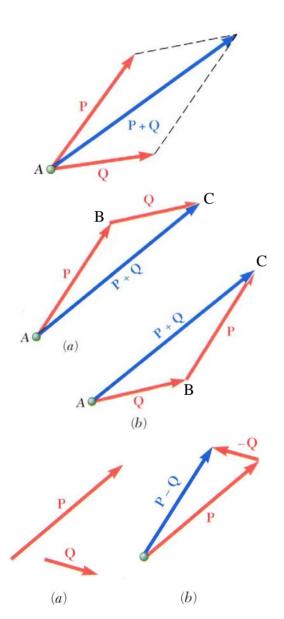
# APPLICATION OF VECTOR ADDITION



There are four concurrent cable forces acting on the bracket.

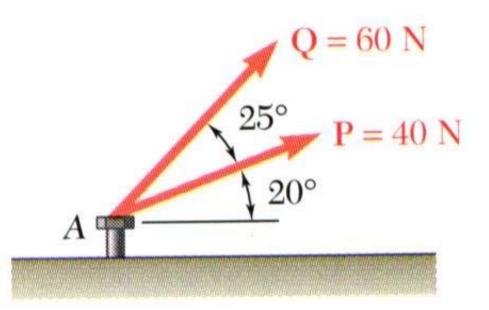
How do you determine the resultant force acting on the bracket?

# Addition of Vectors



- Trapezoid rule for vector addition
- Triangle rule for vector addition
- Law of cosines,  $R^{2} = P^{2} + Q^{2} - 2PQ \cos B$   $\vec{R} = \vec{P} + \vec{Q}$
- Law of sines,  $\frac{\sin A}{Q} = \frac{\sin B}{R} = \frac{\sin C}{A}$
- Vector addition is commutative,  $\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$
- Vector subtraction

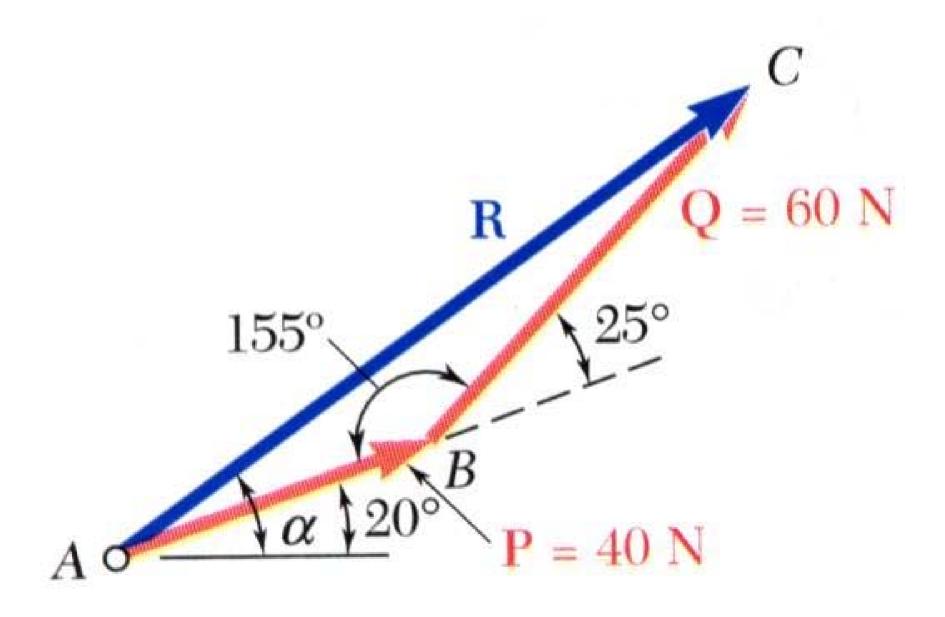
# Sample Problem



The two forces act on a bolt at *A*. Determine their resultant.

## **SOLUTION:**

• Trigonometric solution - use the triangle rule for vector addition in conjunction with the law of cosines and law of sines to find the resultant.



• Trigonometric solution - Apply the triangle rule. From the Law of Cosines,

$$R^{2} = P^{2} + Q^{2} - 2PQ\cos B$$
$$= (40N)^{2} + (60N)^{2} - 2(40N)(60N)\cos 155^{\circ}$$

$$R = 97.73$$
N

From the Law of Sines,

$$\frac{\sin A}{Q} = \frac{\sin B}{R}$$

$$\sin A = \sin B \frac{Q}{R}$$

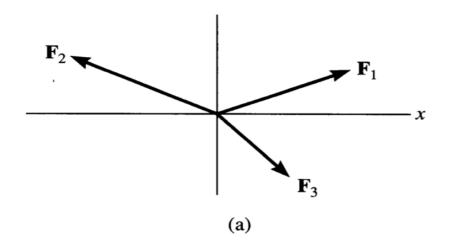
$$= \sin 155^{\circ} \frac{60 \text{N}}{97.73 \text{N}}$$

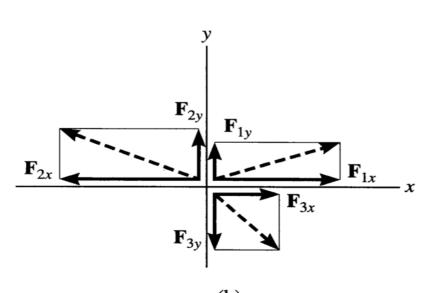
$$A = 15.04^{\circ}$$

$$\alpha = 20^{\circ} + A$$

$$\alpha = 35.04^{\circ}$$

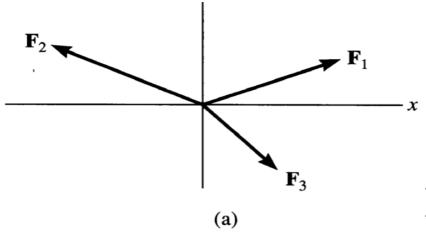
## ADDITION OF SEVERAL VECTORS

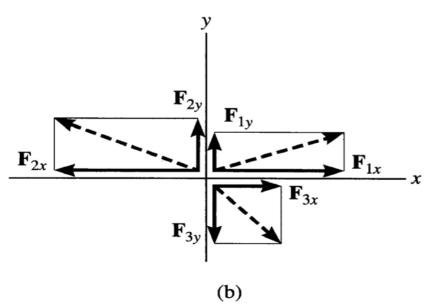




- Step 1 is to resolve each force into its components
- Step 2 is to add all the x components together and add all the y components together. These two totals become the resultant vector.
- Step 3 is to find the magnitude and angle of the resultant vector.

## Example of this process,





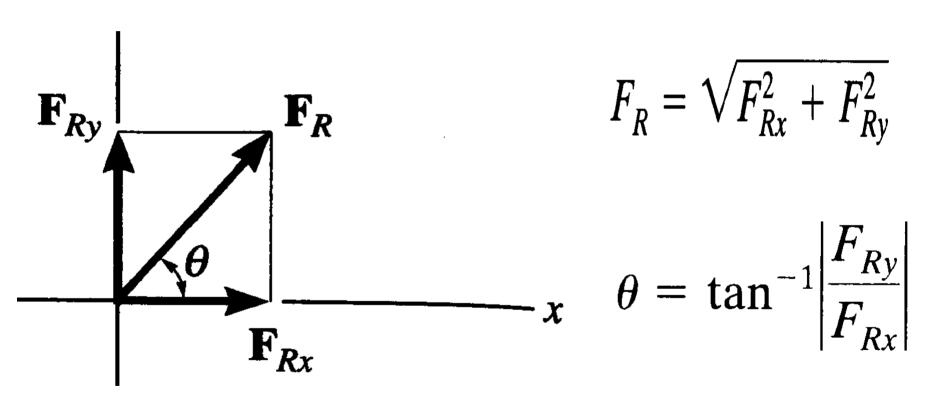
$$\mathbf{F}_{R} = \mathbf{F}_{1} + \mathbf{F}_{2} + \mathbf{F}_{3}$$

$$= F_{1x}\mathbf{i} + F_{1y}\mathbf{j} - F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{3x}\mathbf{i} - F_{3y}\mathbf{j}$$

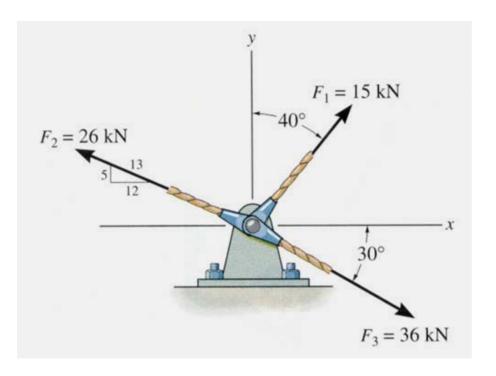
$$= (F_{1x} - F_{2x} + F_{3x})\mathbf{i} + (F_{1y} + F_{2y} - F_{3y})\mathbf{j}$$

$$= (F_{Rx})\mathbf{i} + (F_{Ry})\mathbf{j}$$

You can also represent a 2-D vector with a magnitude and angle.



#### **EXAMPLE**



**Given:** Three concurrent forces

acting on a bracket.

Find: The magnitude and

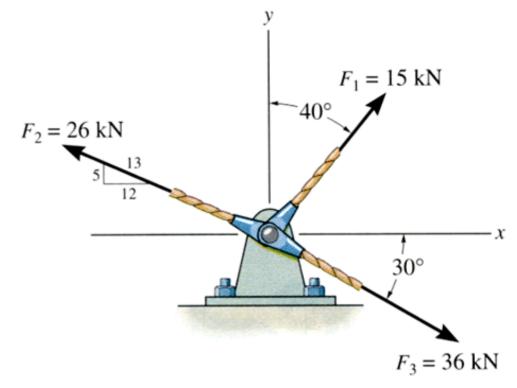
angle of the resultant

force.

## Plan:

- a) Resolve the forces in their x-y components.
- b) Add the respective components to get the resultant vector.
- c) Find magnitude and angle from the resultant components.

## **EXAMPLE** (continued)



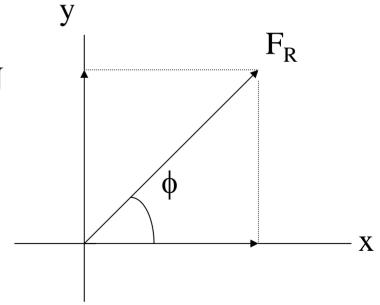
$$F_{1} = \{ 15 \sin 40^{\circ} i + 15 \cos 40^{\circ} j \} \text{ kN}$$
  
 $= \{ 9.642 i + 11.49 j \} \text{ kN}$   
 $F_{2} = \{ -(12/13)26 i + (5/13)26 j \} \text{ kN}$   
 $= \{ -24 i + 10 j \} \text{ kN}$   
 $F_{3} = \{ 36 \cos 30^{\circ} i - 36 \sin 30^{\circ} j \} \text{ kN}$   
 $= \{ 31.18 i - 18 j \} \text{ kN}$ 

#### **EXAMPLE** (continued)

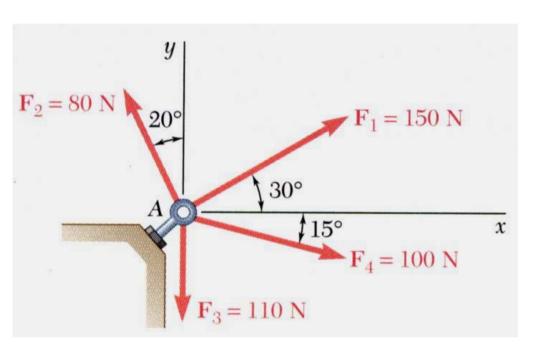
Summing up all the i and j components respectively, we get,

$$F_R = \{ (9.642 - 24 + 31.18) i + (11.49 + 10 - 18) j \} kN$$
  
=  $\{ 16.82 i + 3.49 j \} kN$ 

$$F_{R} = ((16.82)^{2} + (3.49)^{2})^{1/2} = 17.2 \text{ kN}$$
  
$$\phi = \tan^{-1}(3.49/16.82) = 11.7^{\circ}$$



## Sample Problem



Four forces act on bolt *A* as shown. Determine the resultant of the force on the bolt.

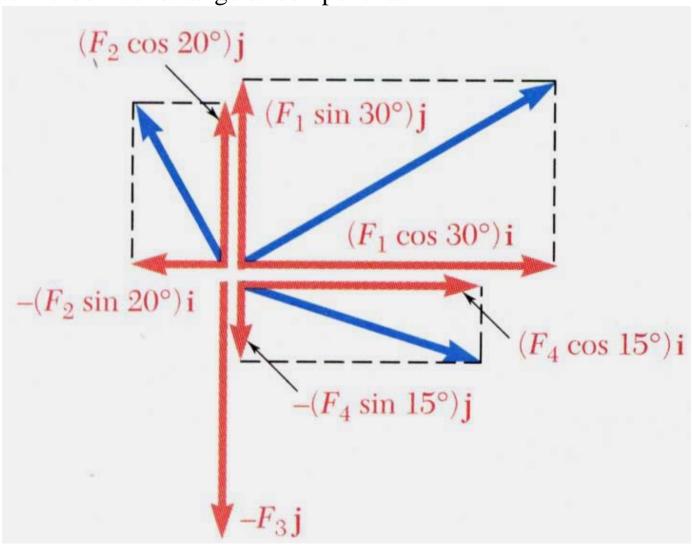
#### **SOLUTION:**

- Resolve each force into rectangular components.
- Determine the components of the resultant by adding the corresponding force components.
- Calculate the magnitude and direction of the resultant.

## Sample Problem (cont')

#### **SOLUTION:**

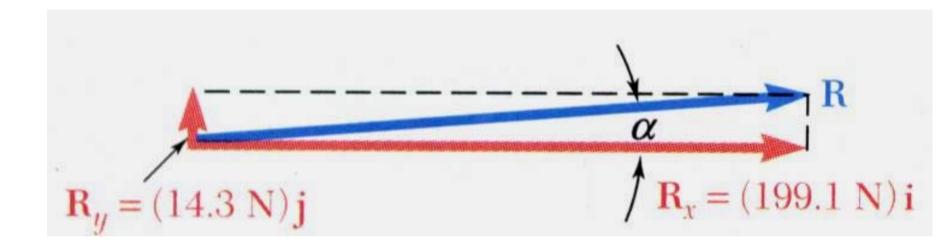
• Resolve each force into rectangular components.



## Sample Problem (cont')

force	mag	x - comp	y - comp
$\overrightarrow{F}_1$	150	+ 129 .9	+ 75 .0
$\vec{F}_2$	80	- 27 .4	+ 75 .2
$\vec{F}_3$	110	0	- 110 .0
$\vec{F}_4$	100	+ 96 .6	- 25 .9
		$R_{\chi} = +199.1$	$R_y = +14.3$

- Determine the components of the resultant by adding the corresponding force components.
- Calculate the magnitude and direction.



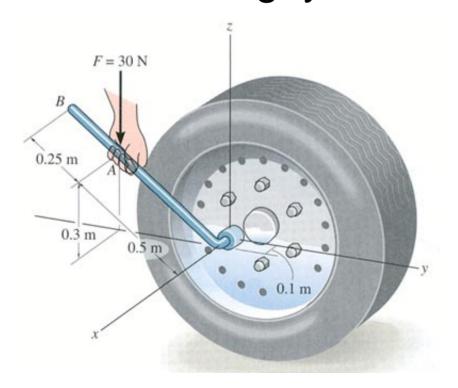
$$\tan \alpha = \frac{R_y}{R_x} = \frac{14.3 \text{ N}}{199.1 \text{ N}} \alpha = 4.1^{\circ}$$
  $\alpha = 4.1^{\circ}$ 

$$R = \frac{14.3 \text{ N}}{\sin \Box} = 199.6 \text{ N}$$
  $R = 199.6 \text{ N} \angle 4.1^{\circ}$ 

## MOMEN DAN KOPEL

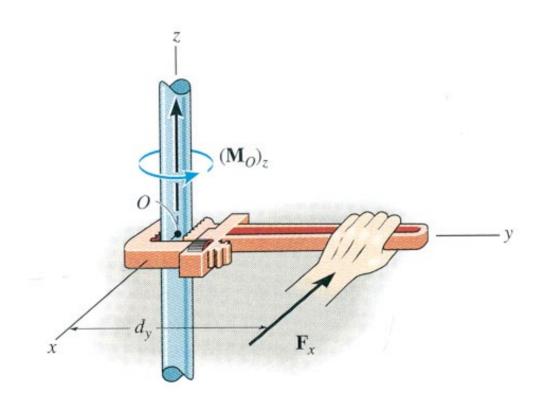
## Apa yang dipelajari sekarang?

Mengetahui dan memahami maksud dari momen gaya, momen kopel, dan cara memindah gaya





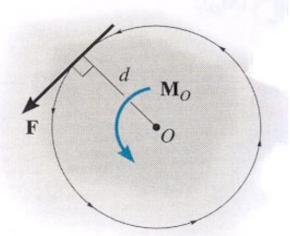
# Apa itu momen gaya?



The moment of a force about a point provides a measure of the tendency for rotation (sometimes called a torque).

#### MOMENT IN 2-D (continued)

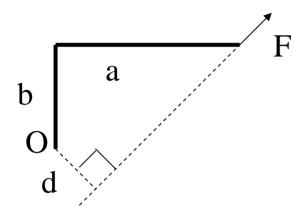
In the 2-D case, the <u>magnitude</u> of the moment is  $M_0 = F d$ 



As shown, d is the *perpendicular* distance from point O to the <u>line of action</u> of the force.

In 2-D, the <u>direction</u> of  $M_O$  is either clockwise or counter-clockwise depending on the tendency for rotation.

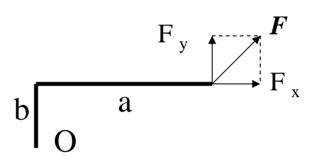
### Moment in 2-D



As shown, d is the *perpendicular* distance from point O to the <u>line</u> of action of the force.

$$M_0 = F d$$

and the direction is counter-clockwise.



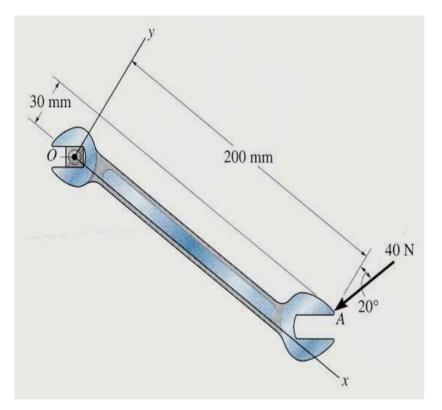
Often it is easier to determine  $M_O$  by using the components of F as shown.

$$\mathbf{M}_{\mathbf{O}} = (\mathbf{F}_{\mathbf{Y}} \ \mathbf{a}) - (\mathbf{F}_{\mathbf{X}} \ \mathbf{b})$$

$$CCW = (+)$$

$$CW = (-)$$

## Example 1



**Given:** A 40 N force is applied to the wrench.

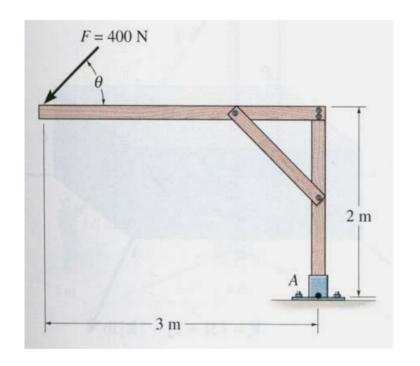
**Find:** The moment of the force at O.

Plan: 1) Resolve the force along x and y axes.

2) Determine M<sub>O</sub> using scalar analysis.

**Solution**: 
$$+ \uparrow F_y = -40 \cos 20^{\circ} \text{ N}$$
  
  $+ \rightarrow F_x = -40 \sin 20^{\circ} \text{ N}$   
  $+ \uparrow M_O = \{-(40 \cos 20^{\circ})(200) + (40 \sin 20^{\circ})(30)\} \text{N·mm}$   
  $= -7107 \text{ N·mm} = -7.11 \text{ N·m}$ 

#### **EXAMPLE 2**



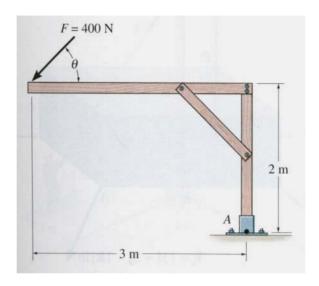
**Given:** A 400 N force is applied to the frame and  $\theta = 20^{\circ}$ .

**Find:** The moment of the force at A.

#### Plan:

- 1) Resolve the force along x and y axes.
- 2) Determine M<sub>A</sub> using scalar analysis.

#### **EXAMPLE 2** (continued)

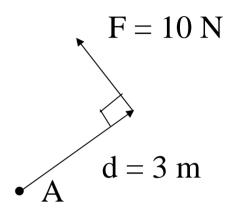


#### **Solution**

$$+ \uparrow F_y = -400 \cos 20^{\circ} \text{ N}$$
  
 $+ \rightarrow F_x = -400 \sin 20^{\circ} \text{ N}$   
 $+ M_A = \{(400 \cos 20^{\circ})(2) + (400 \sin 20^{\circ})(3)\} \text{ N} \cdot \text{m}$   
 $= 1160 \text{ N} \cdot \text{m}$ 

#### **CONCEPT QUESTION**

1. What is the moment of the 10 N force about point A  $(M_A)$ ?



- A) 10 N·m
- B) 30 N·m C) 13 N·m
- D)  $(10/3) \text{ N} \cdot \text{m}$  E)  $7 \text{ N} \cdot \text{m}$

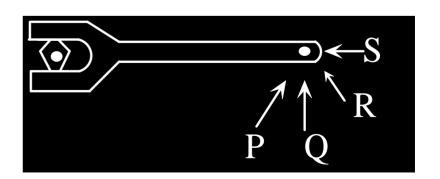
2. If a force of magnitude F can be applied in four different 2-D configurations (P,Q,R, & S), select the cases resulting in the maximum and minimum torque values on the nut. (Max, Min).

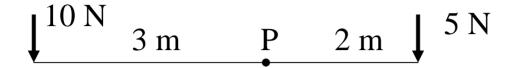
A) (Q, P)

B)(R, S)

C)(P,R)

D)(Q, S)



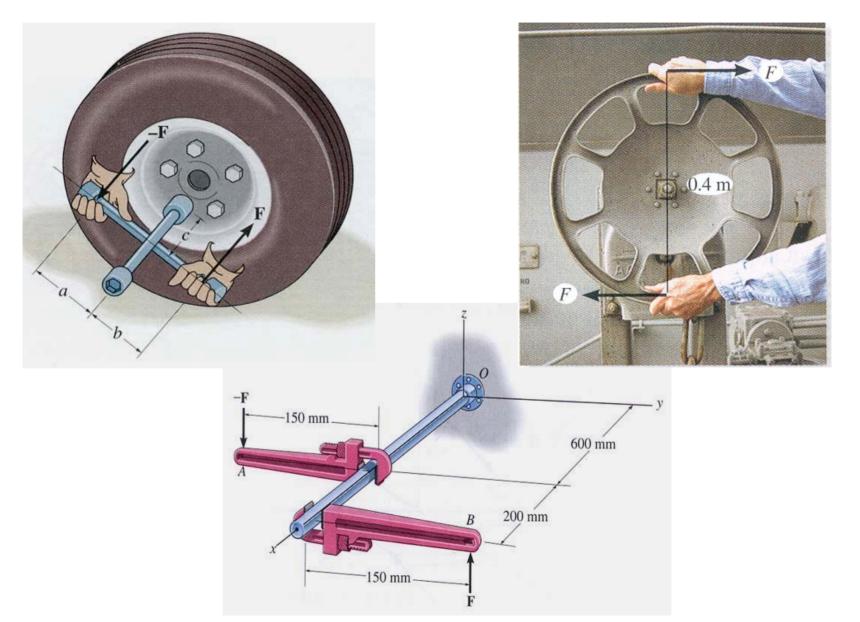


- 3. Using the CCW direction as positive, the net moment of the two forces about point P is

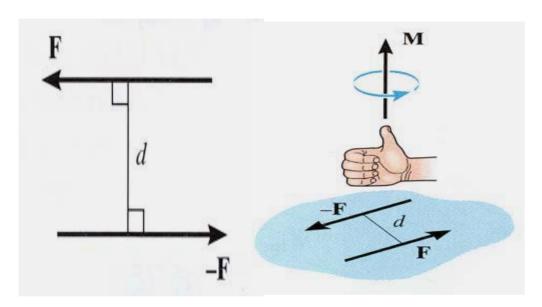
  - A) 10 N·m B) 20 N·m C) 20 N·m

- D) 40 N·m E) 40 N·m

# Apa itu momen kopel?



## Moment of a Couple



A couple is defined as two parallel forces with the same magnitude but opposite in direction separated by a perpendicular distance d.

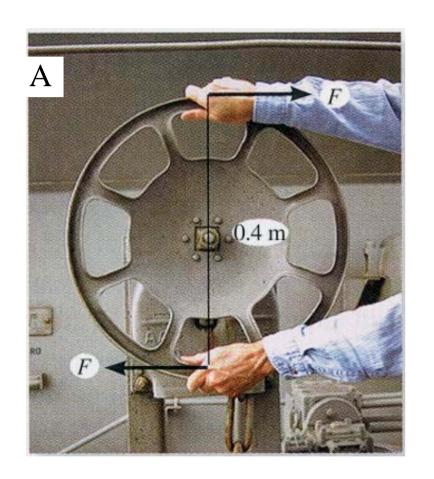
The moment of a couple is defined as

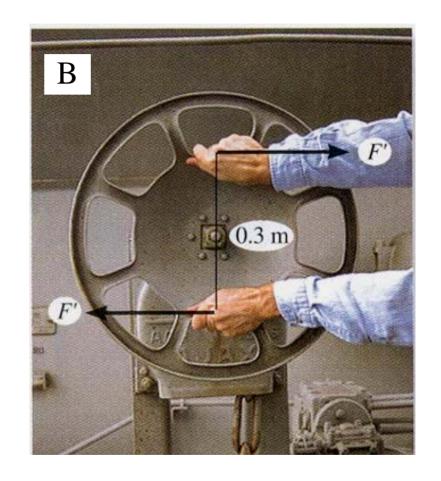
 $M_O = F d$  (using a scalar analysis) or as

 $M_0 = r \times F$  (using a vector analysis).

Here r is any position vector from the line of action of  $-\mathbf{F}$  to the line of action of  $\mathbf{F}$ .

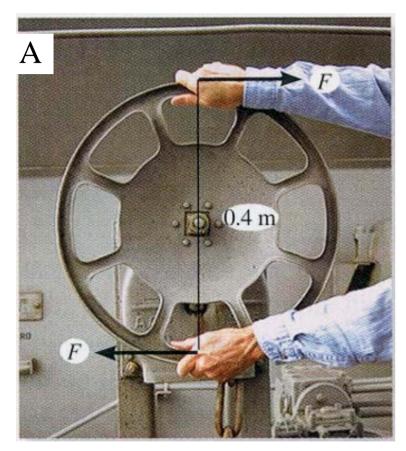
## **Problem Solving**





A torque or moment of  $12 \text{ N} \cdot \text{m}$  is required to rotate the wheel. Which one of the two grips of the wheel above will require less force to rotate the wheel?

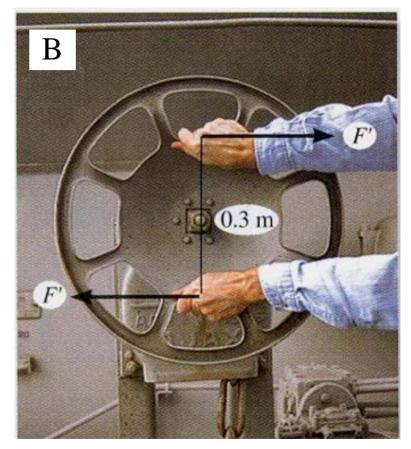
## Problem Solving (2-D)



M = F d

12 = F 0.4

F = 30 N

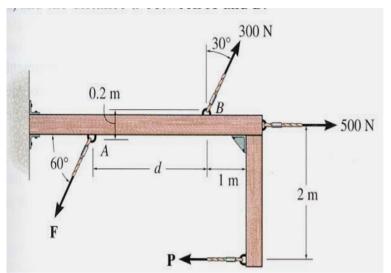


M = F d

12 = F 0.3

F = 40 N

#### PROBLEM SOLVING - SCALAR



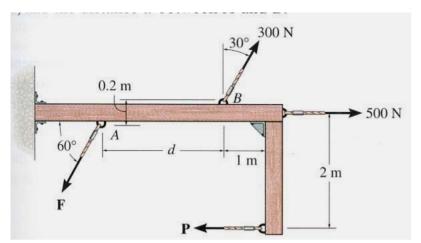
**Given**: Two couples act on the beam. The resultant couple is zero.

**Find**: The magnitudes of the forces P and F and the distance d.

#### PLAN:

- 1) Use definition of a couple to find P and F.
- 2) Resolve the 300 N force in x and y directions.
- 3) Determine the net moment.
- 4) Equate the net moment to zero to find d.

#### **Solution:**



From the definition of a couple

P = 500 N and

F = 300 N.

Resolve the 300 N force into vertical and horizontal components. The vertical component is (300 cos 30°) N and the horizontal component is (300 sin 30°) N.

It was given that the net moment equals zero. So

+ 
$$\Sigma M = -(500)(2) + (300 \cos 30^{\circ})(d) + (300 \sin 30^{\circ})(0.2) = 0$$

Now solve this equation for d.

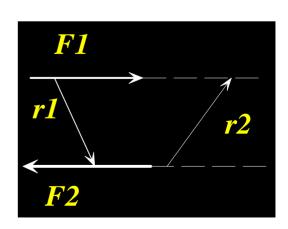
$$d = (1000 - 60 \sin 30^{\circ}) / (300 \cos 30^{\circ}) = 3.96 \text{ m}$$

#### **CONCEPT QUESTION**

- 1. In statics, a couple is defined as \_\_\_\_\_\_ separated by a perpendicular distance.
  - A) two forces in the same direction.
  - B) two forces of equal magnitude.
  - C) two forces of equal magnitude acting in the same direction.
  - D) two forces of equal magnitude acting in opposite directions.
- 2.  $F_1$  and  $F_2$  form a couple. The moment of the couple is given by \_\_\_\_\_.
  - A)  $r_1 \times F_1$

 $B) r_2 \times F_1$ 

- C)  $F_2 \times r_1$
- $D) r_2 \times F_2$



3. A <u>couple</u> is applied to the beam as shown. Its moment

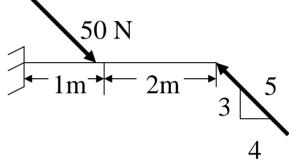
equals \_\_\_\_\_ N·m.

A) 50

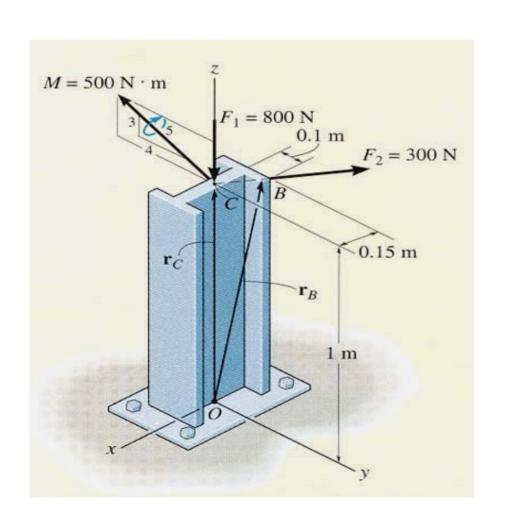
B) 60

C) 80

D) 100

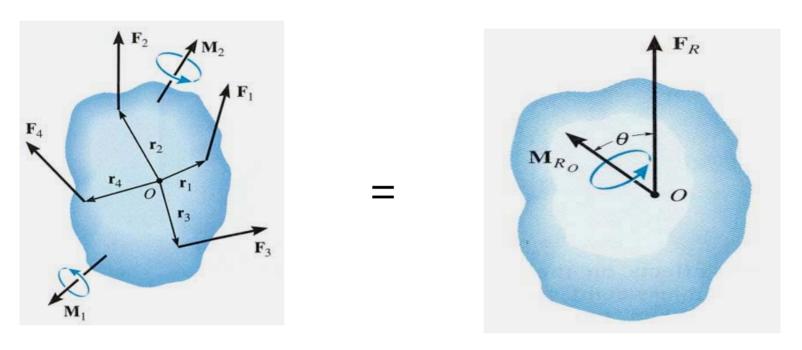


# Apa itu memindah gaya?



Several forces and a couple moment are acting on this vertical section of an I-beam. Can you replace them with just one force and one couple moment at point O that will have the same external effect? If yes, how will you do that?

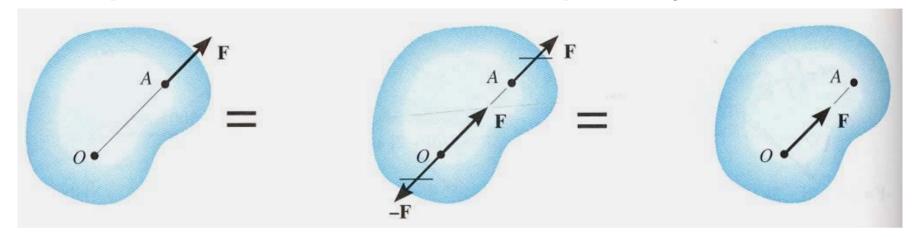
#### AN EQUIVALENT SYSTEM



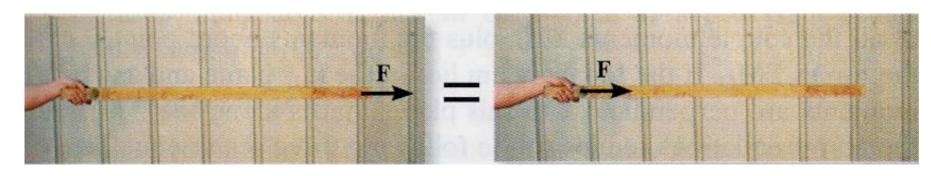
When a number of forces and couple moments are acting on a body, it is easier to understand their overall effect on the body if they are combined into a single force and couple moment having the same external effect

The two force and couple systems are called <u>equivalent systems</u> since they have the same external effect on the body.

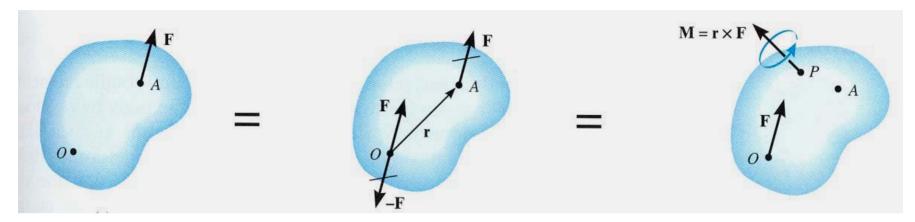
## Equivalent Force – Couple Systems



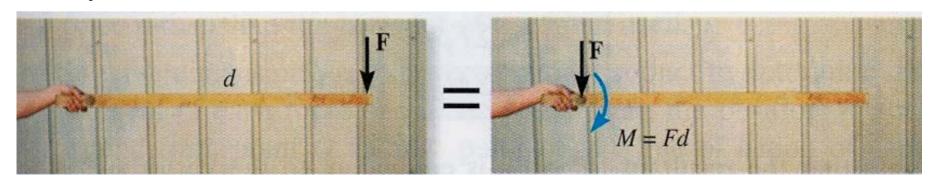
Moving a force from A to O, when both points are on the vectors' line of action, does not change the <u>external effect</u>. Hence, a force vector is called a <u>sliding vector</u>. (But the internal effect of the force on the body does depend on where the force is applied).



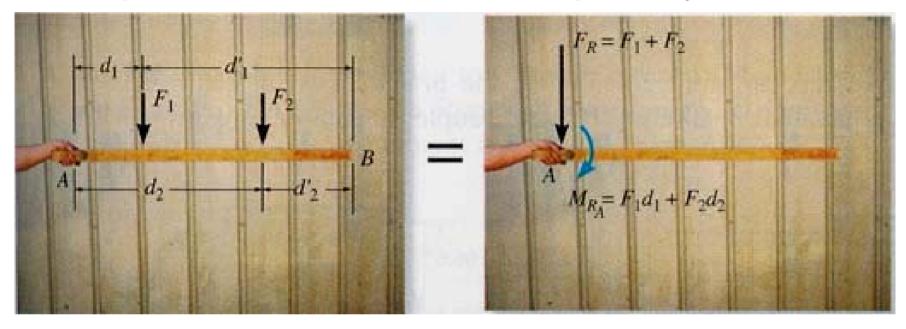
## Equivalent Force – Couple Systems



Moving a force from point A to O (as shown above) requires creating an additional couple moment. Since this new couple moment is a "free" vector, it can be applied at any point P on the body.



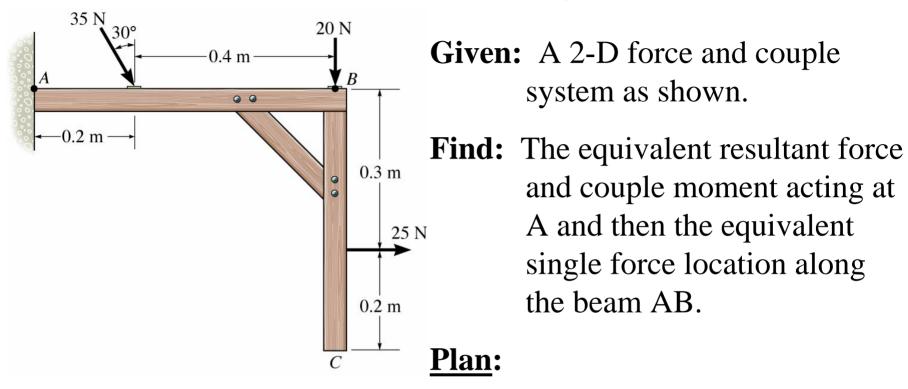
## Equivalent Force – Couple Systems



If the force system lies in the x-y plane (the 2-D case), then the reduced equivalent system can be obtained using the following three scalar equations.

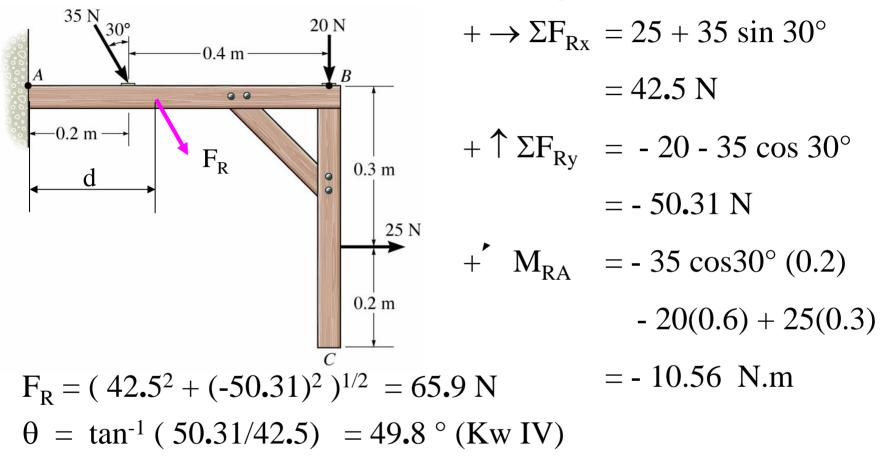
$$F_{R_x} = \Sigma F_x$$
 
$$F_{R_y} = \Sigma F_y$$
 
$$M_{R_O} = \Sigma M_c + \Sigma M_O$$

## Problem Solving (2-D)



- 1) Sum all the x and y components of the forces to find  $F_{RA}$ .
- 2) Find and sum all the moments resulting from moving each force to A.
- 3) Shift the  $F_{RA}$  to a distance d such that  $d = M_{RA}/F_{Ry}$

## Problem Solving (2-D)



The equivalent single force  $F_R$  can be located on the beam AB at a distance d measured from A.

$$d = M_{RA}/F_{Rv} = -10.56/(-50.31) = 0.21 \text{ m}.$$

# KESEIMBANGAN PARTIKEL (2D)

## Equilibrium of a Particle (2-D)

#### **Today's Objectives**:

Students will be able to:

- a) Draw a free body diagram (FBD), and,
- b) Apply equations of equilibrium to solve a 2-D problem.

For a given cable strength, what is the maximum weight that can be lifted?



# Apa pentingnya mekanika (statik) / keseimbangan ?



## Apa perbedaan partikel dan benda tegar?

- Particle: A very small amount of matter which may be assumed to occupy a single point in space.
- Rigid body: A combination of a large number of particles occupying fixed position with respect to each other.

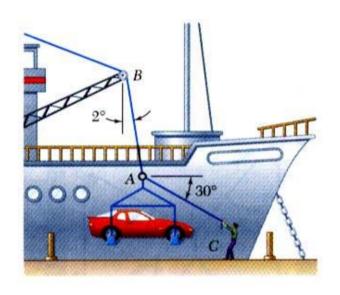
## Apa perbedaan Partikel dan Benda Tegar?

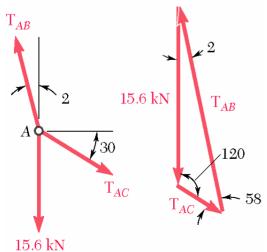
Partikel:

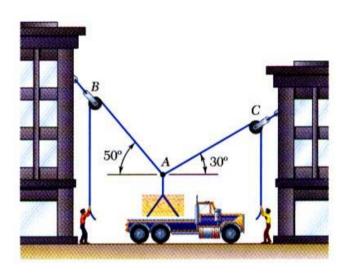
Mempunyai suatu massa namun ukurannya dapat diabaikan, sehingga geometri benda tidak akan terlibat dalam analisis masalah Benda Tegar:

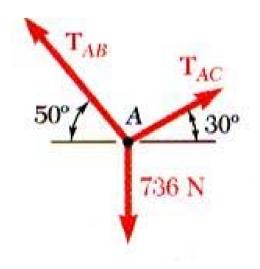
Kombinasi sejumlah partikel yang mana semua partikel berada pada suatu jarak tetap terhadap satu dengan yang lain

## Contoh Partikel

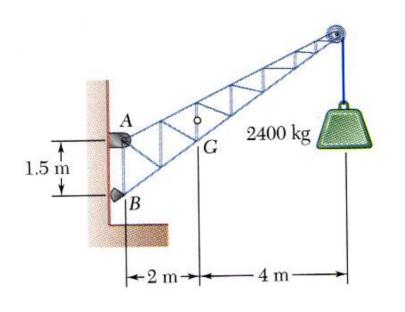


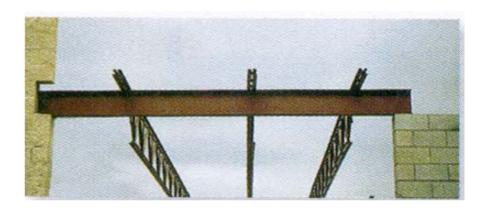


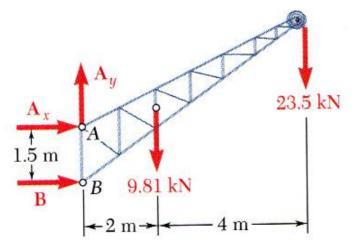


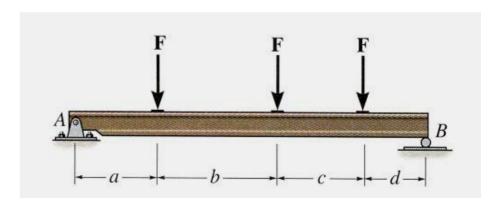


# Contoh Benda Tegar







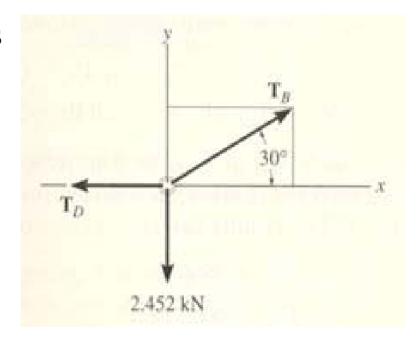


# THE WHAT, WHY AND HOW OF A FREE BODY DIAGRAM (FBD)

Free Body Diagrams are one of the most important things for you to know how to draw and use.

What? - It is a drawing that shows all external forces acting on the particle.

Why? - It helps you write the equations of equilibrium used to solve for the unknowns (usually forces or angles).

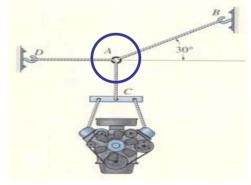


#### How?

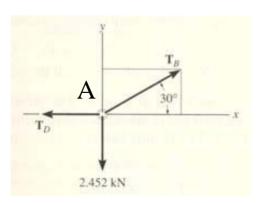
- 1. Imagine the particle to be isolated or cut free from its surroundings.
- 2. Show all the forces that act on the particle.

  Active forces: They want to move the particle.

  Reactive forces: They tend to resist the motion.
- 3. Identify each force and show all known magnitudes and directions. Show all unknown magnitudes and / or directions as variables .

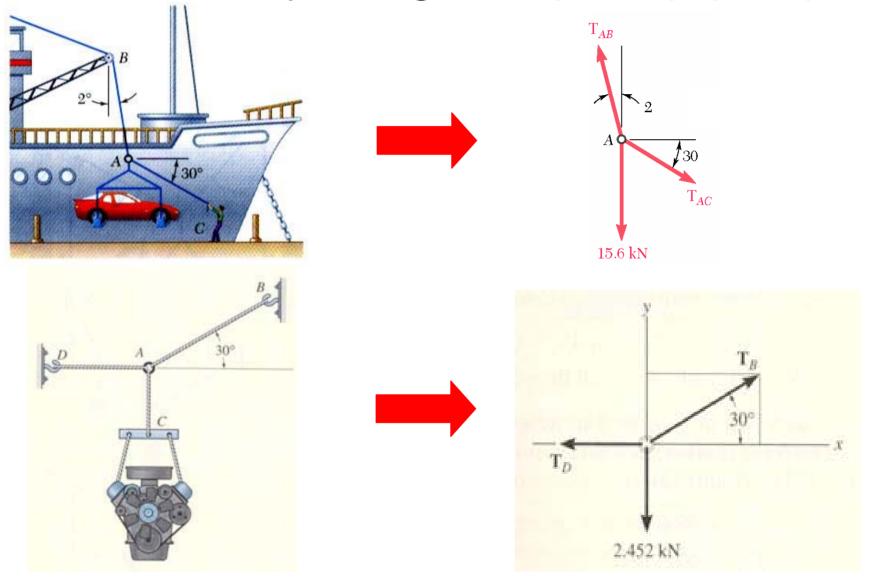


Note: Engine mass = 250 Kg

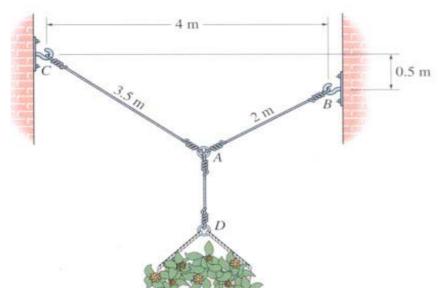


FBD at A

# Free Body Diagram (FBD) (2-D)



# Equations of Equilibrium (2-D)



Since particle A is in equilibrium, the net force at A is zero.

So 
$$F_{AB} + F_{AD} + F_{AC} = 0$$
  
or  $\Sigma F = 0$ 

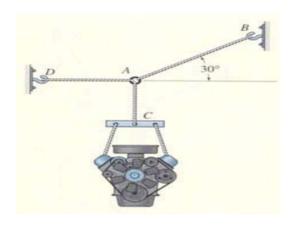
In general, for a particle in equilibrium,  $\Sigma \mathbf{F} = \mathbf{0}$  or  $\Sigma \mathbf{F}_{\mathbf{x}} \mathbf{i} + \Sigma \mathbf{F}_{\mathbf{y}} \mathbf{j} = \mathbf{0} = 0 \mathbf{i} + 0 \mathbf{j}$  (A vector equation)

#### Or, written in a scalar form,

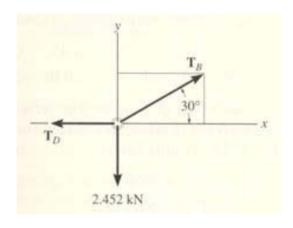
These are two scalar equations of equilibrium (EofE). They can be used to solve for up to **two** unknowns.

$$\Sigma F_x = 0$$
 and  $\Sigma F_y = 0$ 

#### **EXAMPLE**



Note: Engine mass = 250 Kg



FBD at A

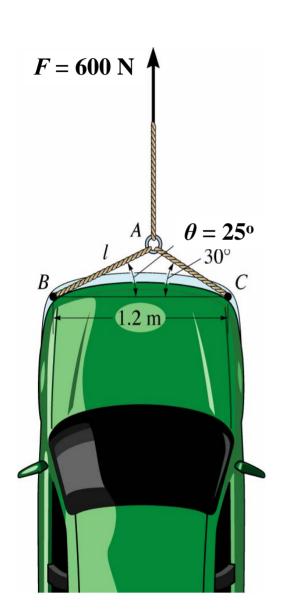
Write the scalar EofE:

$$+ \rightarrow \Sigma F_x = T_B \cos 30^\circ - T_D = 0$$
  
+ 
$$\uparrow \Sigma F_v = T_B \sin 30^\circ - 2.452 \text{ kN} = 0$$

Solving the second equation gives:  $T_B = 4.90 \text{ kN}$ 

From the first equation, we get:  $T_D = 4.25 \text{ kN}$ 

# Problem Solving (2-D)



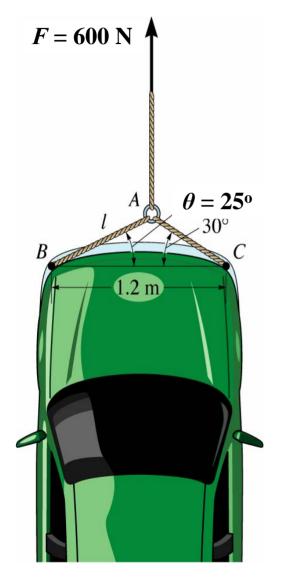
**Given:** The car is towed at constant speed by the 600 N force and the angle  $\theta$  is 25°.

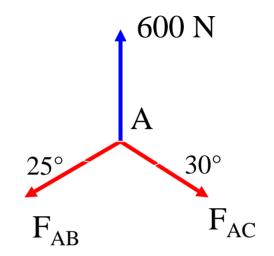
**Find:** The forces in the ropes AB and AC.

#### Plan:

- 1. Draw a FBD for point A.
- 2. Apply the EofE to solve for the forces in ropes AB and AC.

# Problem Solving (2-D)





FBD at point A

Applying the scalar EofE at A, we get;

$$+ \rightarrow \sum F_x = F_{AC} \cos 30^\circ - F_{AB} \cos 25^\circ = 0$$

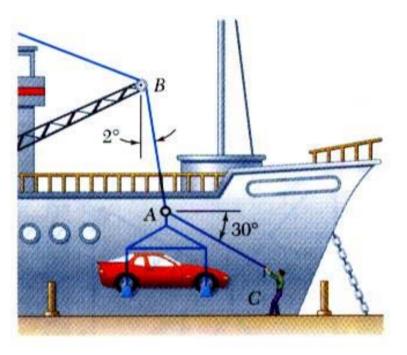
$$+ \rightarrow \sum F_{v} = -F_{AC} \sin 30^{\circ} - F_{AB} \sin 25^{\circ} + 600 = 0$$

Solving the above equations, we get;

$$F_{AB} = 634 \text{ N}$$

$$F_{AC} = 664 \text{ N}$$

# Example

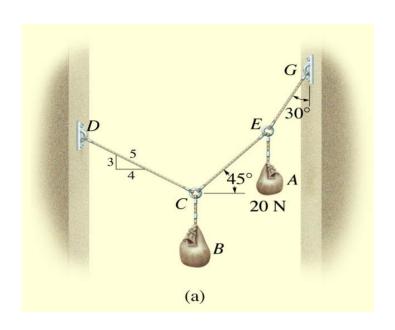


#### **SOLUTION:**

- Construct a free-body diagram for the particle at the junction of the rope and cable.
- Determine the unknown force magnitudes.

In a ship-unloading operation, a 15.6 kN automobile is supported by a cable. A rope is tied to the cable and pulled to center the automobile over its intended position. What is the tension in the rope?

#### **EXAMPLE**



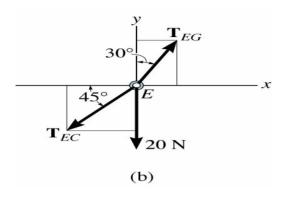
**Given:** Sack A weighs 20 N. and geometry is as shown.

**Find:** Forces in the cables and weight of sack B.

#### Plan:

- 1. Draw a FBD for Point E.
- 2. Apply EofE at Point E to solve for the unknowns ( $T_{EG} \& T_{EC}$ ).
- 3. Repeat this process at C.

#### **EXAMPLE** (continued)



A FBD at E should look like the one to the left. Note the assumed directions for the two cable tensions.

The scalar EofE are:

$$+ \rightarrow \Sigma F_x = T_{EG} \sin 30^\circ - T_{EC} \cos 45^\circ = 0$$

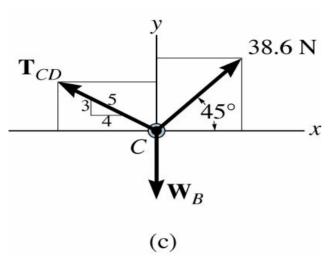
$$+ \uparrow \Sigma F_y = T_{EG} \cos 30^\circ - T_{EC} \sin 45^\circ - 20 N = 0$$

Solving these two simultaneous equations for the two unknowns yields:

$$T_{FC} = 38.6 \text{ N}$$

$$T_{EG} = 54.6 \text{ N}$$

#### **EXAMPLE** (continued)



Now move on to ring C. A FBD for C should look like the one to the left.

The scalar EofE are:

$$+ \rightarrow \Sigma F_{x} = 38.64 \cos 45^{\circ} - (4/5) T_{CD} = 0$$
  
 $+ \uparrow \Sigma F_{v} = (3/5) T_{CD} + 38.64 \sin 45^{\circ} - W_{B} = 0$ 

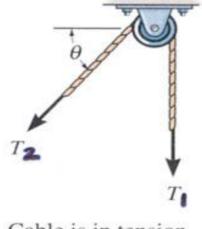
Solving the first equation and then the second yields

$$T_{CD} = 34.2 \text{ N}$$
 and  $W_B = 47.8 \text{ N}$ .

#### **READING QUIZ**

- 1) When a particle is in equilibrium, the sum of forces acting on it equals \_\_\_\_ . (Choose the most appropriate answer)
  - A) a constant

- B) a positive number C) zero
- D) a negative number E) an integer.
- 2) For a frictionless pulley and cable, tensions in the cable  $(T_1 \text{ and } T_2)$  are related as \_\_\_\_\_\_
  - A)  $T_1 > T_2$
  - B)  $T_1 = T_2$
  - C)  $T_1 < T_2$
  - D)  $T_1 = T_2 \sin \theta$



Cable is in tension

#### **ATTENTION QUIZ**

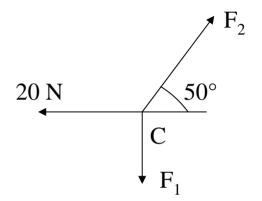
3. Using this FBD of Point C, the sum of forces in the x-direction  $(\Sigma F_X)$  is \_\_\_\_. Use a sign convention of  $+ \rightarrow$ .

A) 
$$F_2 \sin 50^\circ - 20 = 0$$

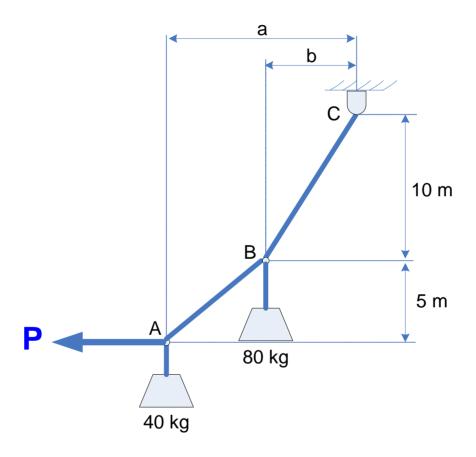
B) 
$$F_2 \cos 50^\circ - 20 = 0$$

C) 
$$F_2 \sin 50^\circ - F_1 = 0$$

D) 
$$F_2 \cos 50^\circ + 20 = 0$$



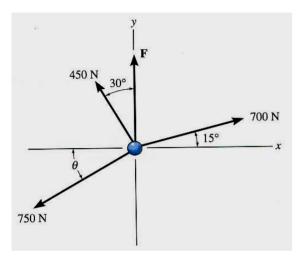
#### **SOAL TANTANGAN**



Jika b = 4 m, tentukan harga P dan jarak a

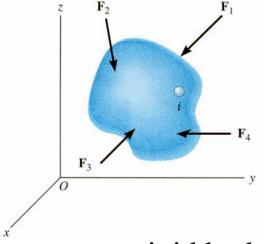
# KESEIMBANGAN BENDA TEGAR

### Apa Beda Partikel dengan Benda Tegar?



Forces on a particle

In contrast to the forces on a particle, the forces on a rigid-body are not usually concurrent and may cause rotation of the body (due to the moments created by the forces).

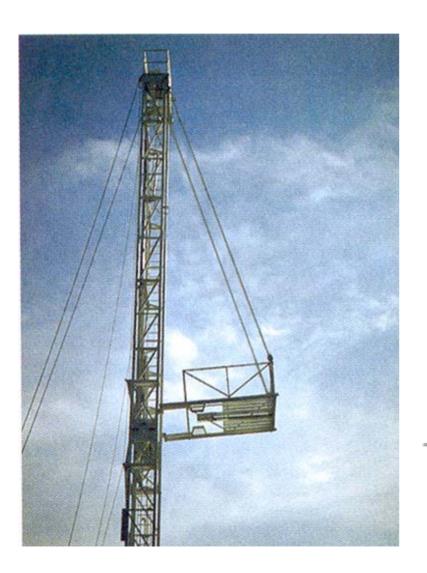


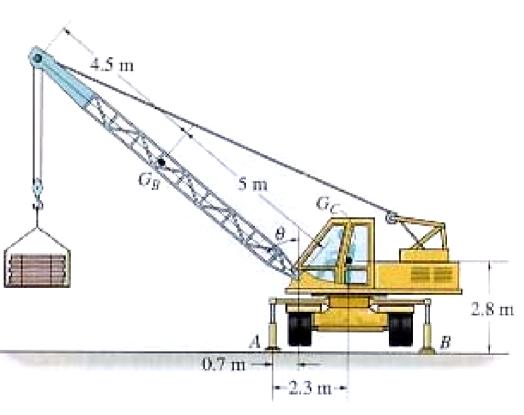
Forces on a rigid body

For a rigid body to be in equilibrium, the net force as well as the net moment about any arbitrary point O must be equal to zero.

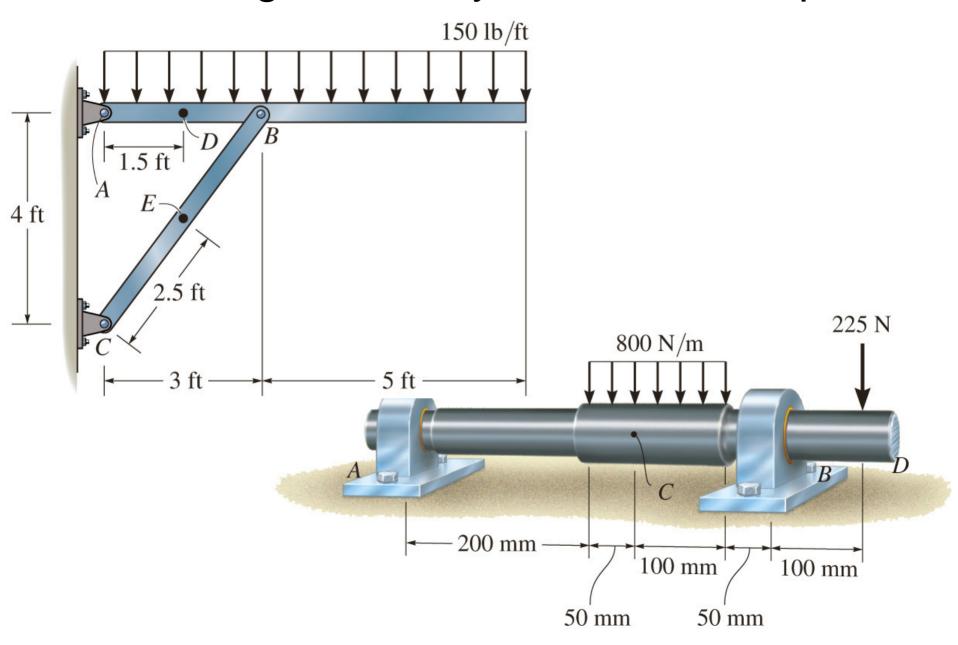
$$\sum \mathbf{F} = 0$$
 and  $\sum \mathbf{M_0} = 0$ 

### Benda Tegar Biasanya Memiliki Tumpuan

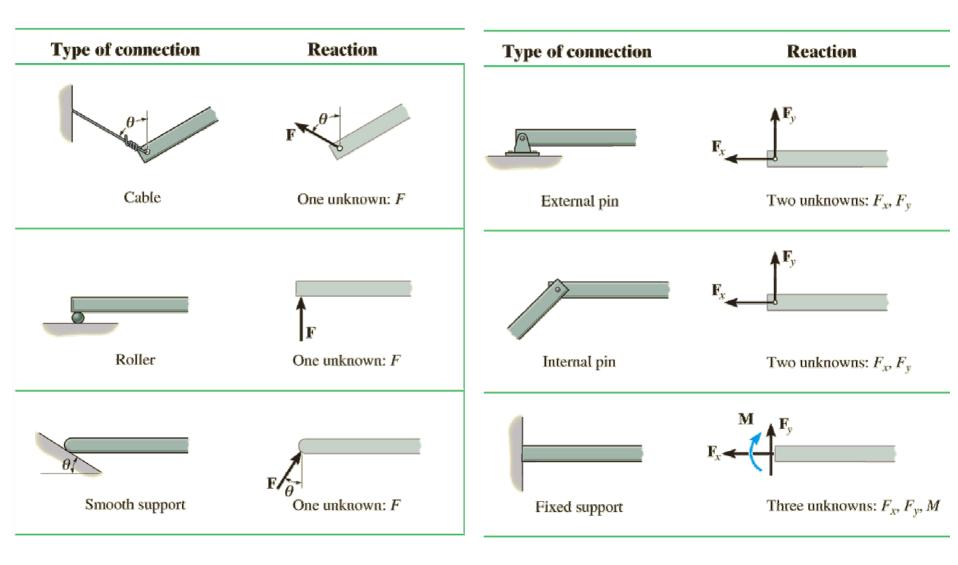




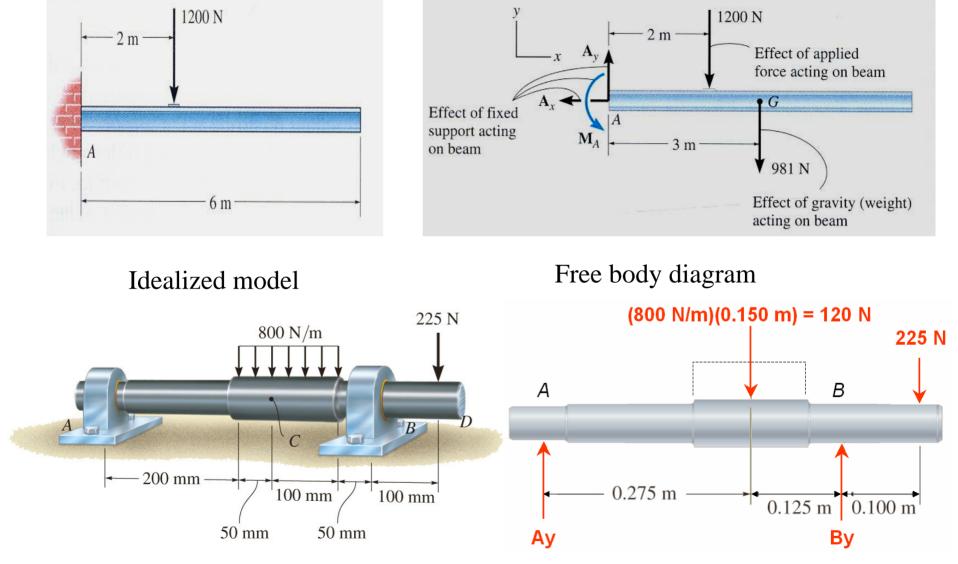
### Benda Tegar Biasanya Memiliki Tumpuan



## Macam-macam Tumpuan dan Reaksinya

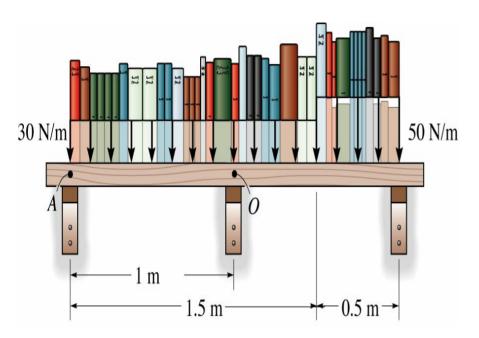


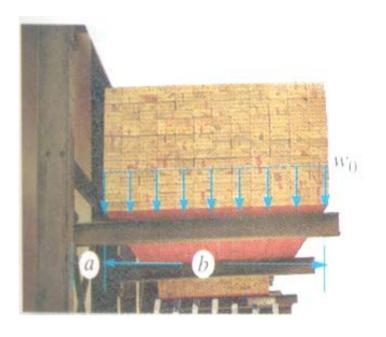
### Contoh Menggambar FBD nya

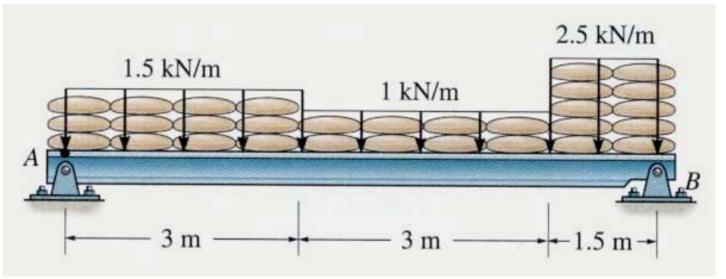


Lho kok ada beban yang segiempat, apa itu?

### Beban Terdistribusi

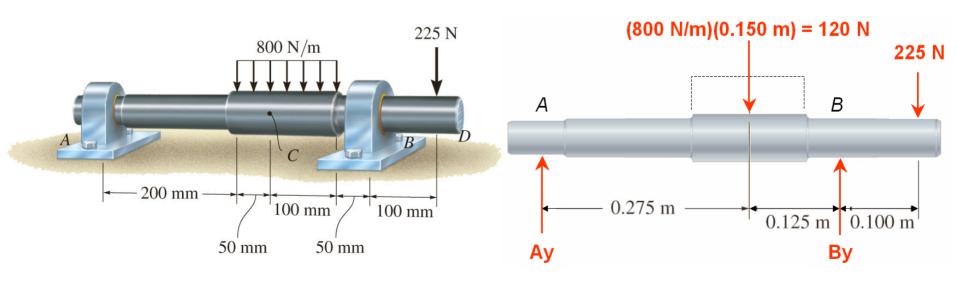




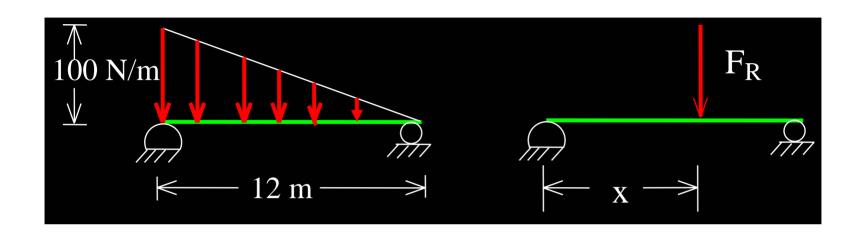


### Mencari Gaya Resultan pada Beban Terdistribusi

- Mencari titik berat dari beban terdistribusi
- Gaya resultan sama dengan luasan dari beban terdistribusi
- Gaya resultan terletak pada titik berat beban terdisribusi



### Kalo beban terdistribusinya berbentuk segitiga?



1. 
$$F_R =$$
\_\_\_\_\_

- A) 12 N
- B) 100 N
- C) 600 N
- D) 1200 N

$$2. x =$$
\_\_\_\_\_\_

A) 3 m

B) 4 m

C) 6 m

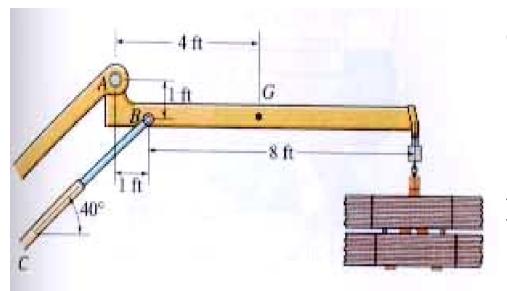
D) 8 m

### Prosedur Menyelesaikan Soal

- Gambar FBD dari soal
- Jangan lupa kasih perjanjian tandanya
- Gambar gaya reaksi yang ada
- Kalo ada beban terdistribusi, cari dulu besar gaya resultan, dan posisinya
- Hitung besar gaya reaksi di tumpuan, menggunakan

$$\sum Fx = 0$$
  $\sum Fy = 0$   $\sum Mo = 0$  titik O itu titik apa? Yang mana?

#### Contoh Soal 1



**Given**: Weight of the boom =

125 lb, the center of

mass is at G, and the

load = 600 lb.

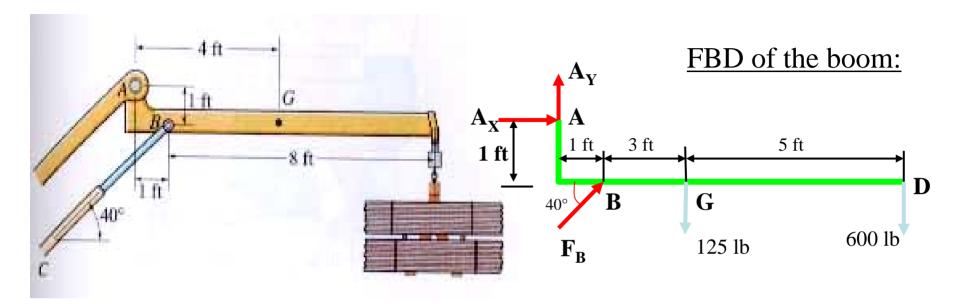
**Find:** Support reactions at A

and B.

#### Plan:

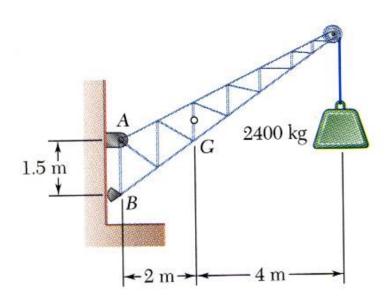
- 1. Put the x and y axes in the horizontal and vertical directions, respectively.
- 2. Draw a complete FBD of the boom.
- 3. Apply the EofE to solve for the unknowns.

### Contoh Soal 1 (Jawaban)



$$+ \sum M_A = -125 * 4 - 600 * 9 + F_B \sin 40^\circ * 1 + F_B \cos 40^\circ * 1 = 0$$
  
 $F_B = 4188 \text{ lb or } \underline{4190 \text{ lb}}$ 

#### Contoh Soal 2



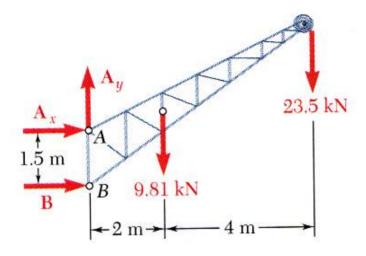
A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at *A* and a rocker at *B*. The center of gravity of the crane is located at *G*.

Determine the components of the reactions at *A* and *B*.

#### **SOLUTION:**

- Create a free-body diagram for the crane.
- Determine *B* by solving the equation for the sum of the moments of all forces about *A*. Note there will be no contribution from the unknown reactions at *A*.
- Determine the reactions at *A* by solving the equations for the sum of all horizontal force components and all vertical force components.
- Check the values obtained for the reactions by verifying that the sum of the moments about *B* of all forces is zero.

### Contoh Soal 2 (jawaban)



• Create the free-body diagram.

• Determine *B* by solving the equation for the sum of the moments of all forces about *A*.

$$\sum M_A = 0$$
:  $+B(1.5\text{m}) - 9.81 \text{kN}(2\text{m})$   
 $-23.5 \text{kN}(6\text{m}) = 0$   
 $B = +107.1 \text{kN}$ 

• Determine the reactions at A by solving the equations for the sum of all horizontal forces and all vertical forces.

$$\sum F_x = 0$$
:  $A_x + B = 0$ 

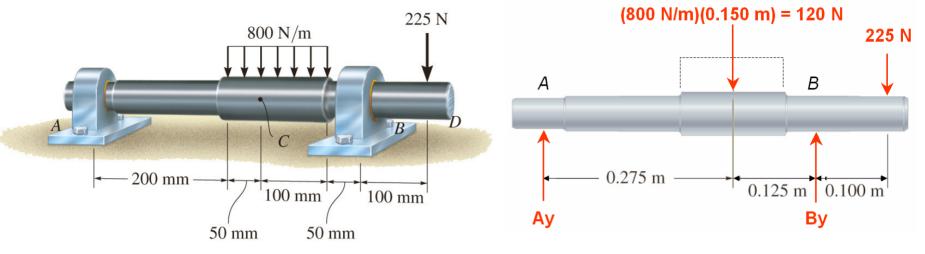
$$A_x = -107.1 \text{kN}$$

$$\sum F_y = 0$$
:  $A_y - 9.81 \text{kN} - 23.5 \text{kN} = 0$ 

$$A_y = +33.3 \text{kN}$$

Check the values obtained.

#### Contoh Soal 3



$$\Sigma M_A = 0 = .400 \, m(B_y) - 120 \, N(.275 \, m) - 225 \, N(.500 \, m)$$

$$B_{y} = \frac{-120 N (.275 m) - 225 N (.500 m)}{-.400 m}$$

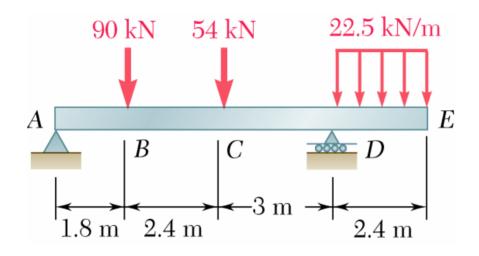
$$B_y = 363.75 \,\mathrm{N}$$

$$\sum_{+} Fy = 0 = Ay - 120 N + 363.75 N - 225 N$$

$$A_y = -18.75 N$$

$$A_{\rm y} = 18.75\,\mathrm{N}$$

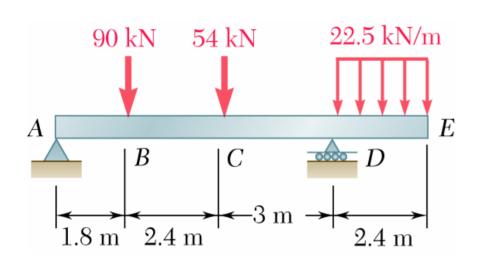
#### Contoh Soal 4

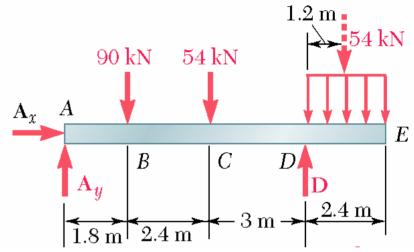


Given: The loading on the beam as shown.

**Find**: Support reactions at A and B.

### Contoh Soal 4 (jawaban)





#### **SOLUTION:**

• Taking entire beam as a free-body, determine reactions at supports.

$$\sum M_A = 0:$$

$$D(7.2 \text{ m}) - (90 \text{ kN})(1.8 \text{ m}) - (54 \text{ kN})(4.2 \text{ m})$$
  
 $-(54 \text{ kN})(8.4 \text{ m}) = 0$ 

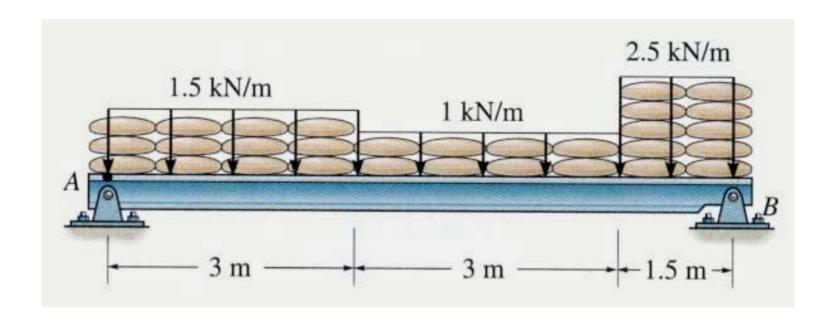
$$\sum F_y = 0:$$

$$A_y - 90 \text{ kN} - 54 \text{ kN} + 117 \text{ kN} - 54 \text{ kN} = 0$$

$$A_y = 81 \text{ kN}$$

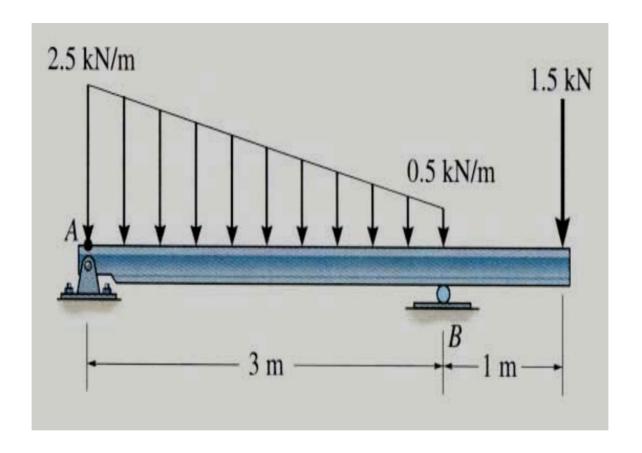
$$D = 117 \text{ kN}$$

#### Contoh Soal 5



Tentukan Reaksi di A dan B

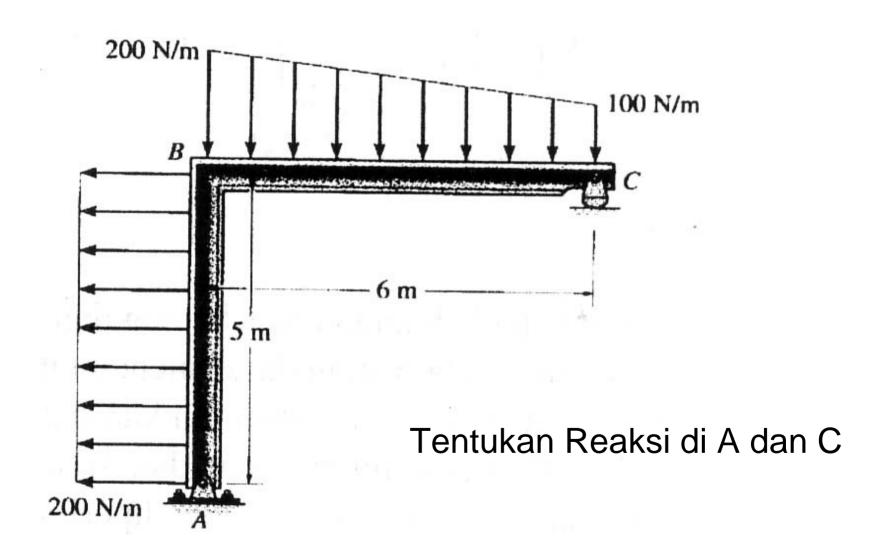
### Soal Tantangan



Given: The loading on the beam as shown.

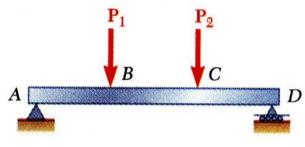
**Find**: Reaction at B and A

### Soal Tantangan (2)



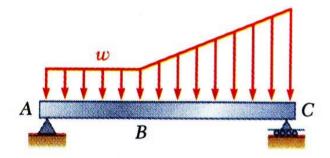
### BEAM GAYA INTERNAL, DIAGRAM GAYA GESER DAN MOMEN

### Definisi Beam



(a) Concentrated loads

- *Beam* structural member designed to support loads applied at various points along its length.
- Beam can be subjected to *concentrated* loads or *distributed* loads or combination of both.

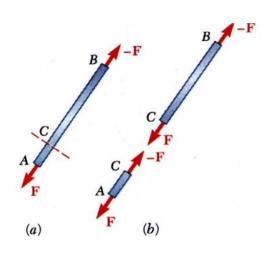


(b) Distributed load

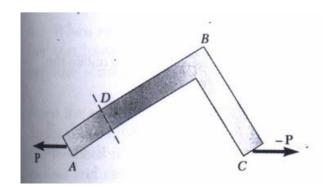
- Beam design is two-step process:
  - 1) determine shearing forces and bending moments produced by applied loads
  - 2) select cross-section best suited to resist shearing forces and bending moments

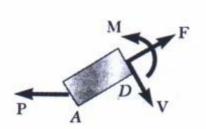
#### Apa itu Gaya Internal?

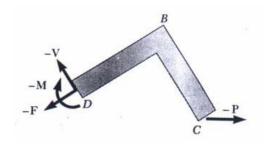
Gaya Internal: gaya yang mengikat bersama berbagai bagian struktur sehingga struktur tersebut menjadi kokoh



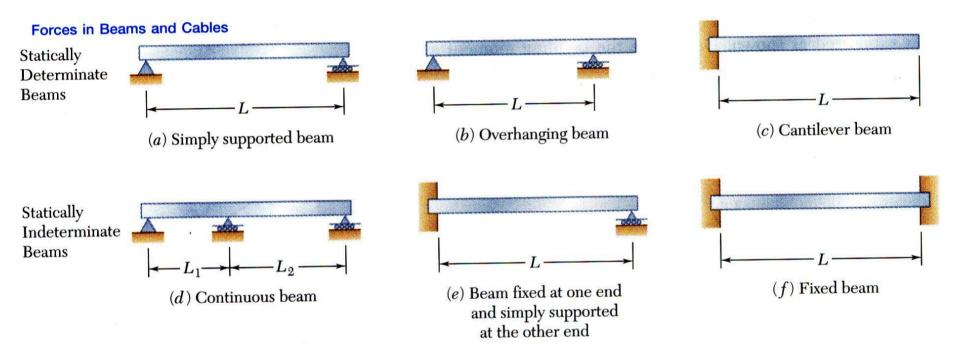
- Straight two-force member *AB* is in equilibrium under application of *F* and *-F*.
- *Internal forces* equivalent to *F* and *-F* are required for equilibrium of free-bodies *AC* and *CB*.





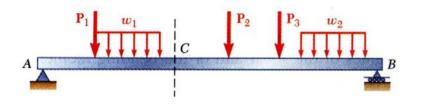


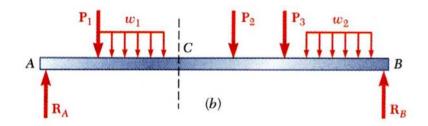
#### Reaksi pada Beam

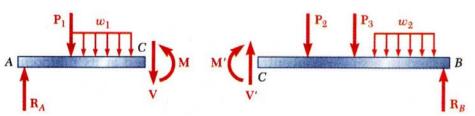


- Beams are classified according to way in which they are supported.
- Reactions at beam supports are determinate if they involve only three unknowns. Otherwise, they are statically indeterminate.

#### Gaya Geser dan Momen pada Beam

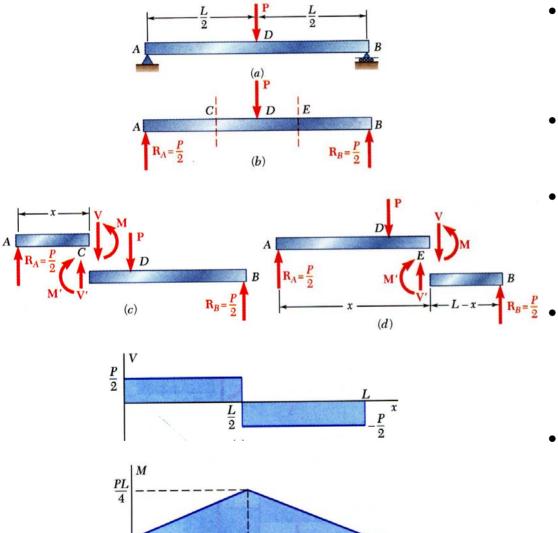






- Wish to determine bending moment and shearing force at any point in a beam subjected to concentrated and distributed loads.
- Determine reactions at supports by treating whole beam as free-body.
- Cut beam at *C* and draw free-body diagrams for *AC* and *CB*. By definition, positive sense for internal force-couple systems are as shown.
- From equilibrium considerations, determine *M* and *V* or *M*' and *V*'.

#### Diagram Gaya Geser dan Momen pada Beam



- Variation of shear and bending moment along beam may be plotted.
- Determine reactions at supports.
- Cut beam at *C* and consider member *AC*,

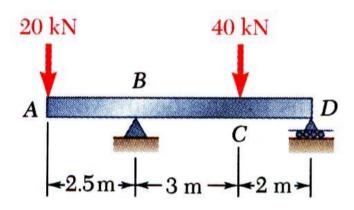
$$V = +P/2$$
  $M = +Px/2$ 

• Cut beam at *E* and consider member *EB*,

$$V = -P/2$$
  $M = +P(L-x)/2$ 

• For a beam subjected to concentrated loads, shear is constant between loading points and moment varies linearly.

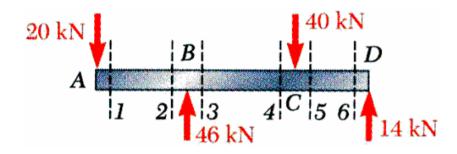
#### Contoh Soal 1

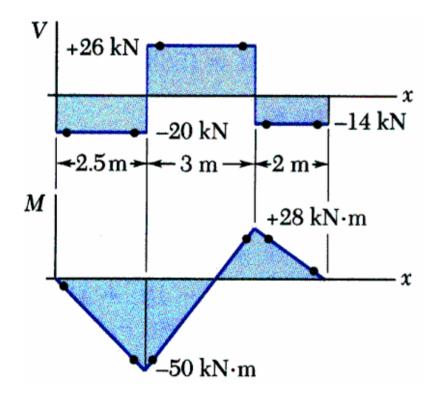


Draw the shear and bending moment diagrams for the beam and loading shown.

#### **SOLUTION:**

- Taking entire beam as a free-body, calculate reactions at *B* and *D*.
- Find equivalent internal force-couple systems for free-bodies formed by cutting beam on either side of load application points.
- Plot results.

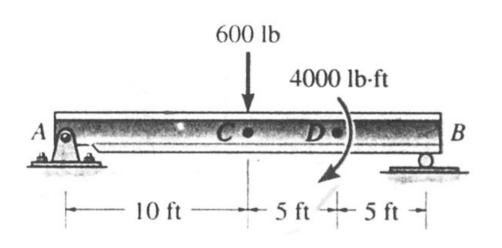




• Plot results.

Note that shear is of constant value between concentrated loads and bending moment varies linearly.

#### Contoh Soal 2

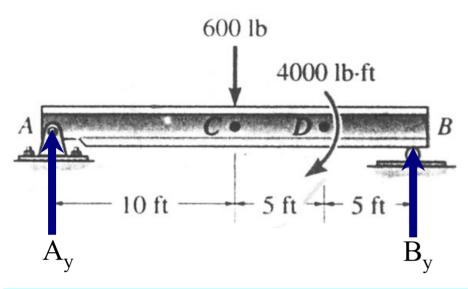


Given: A beam is supported by a hinge at A, a roller at B. Force applied at C. Moment applied at D.

Find: Draw the shear and bending moment diagrams

Plan:

- a) Draw a FBD of the beam.
- b) Calculate support reactions.
- c) Find equivalent internal force-couple systems for free-bodies formed by cutting beam on either side of load application points.



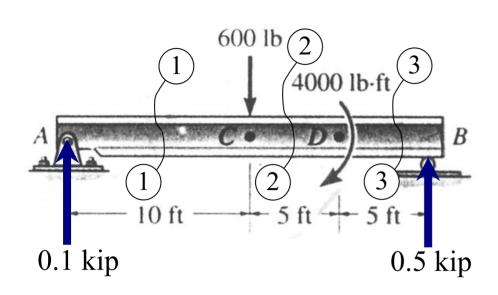
$$\Sigma M_B = 0$$
-  $A_y(20) + 0.6(10) - 4 = 0$ 
 $20A_y = 6 - 4$ 
 $A_y = 0.1 \text{ kip}$ 

$$\Sigma F_y = 0$$

$$A_y + B_y - 600 = 0$$

$$0.1 + B_y - 0.6 = 0$$

$$B_v = 0.5 \text{ kip}$$



$$\Sigma M_{1-1} = 0$$

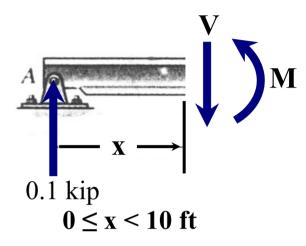
$$M - 0.1(x) = 0$$

$$M = 0.1(x)$$

$$x = 0 \text{ ft}$$
  $\rightarrow M_A = 0.1(0) = 0 \text{ kip-ft}$ 

$$x = 10 \text{ ft} \rightarrow M_C = 0.1(10) = 1 \text{ kip-ft}$$

#### Potongan 1-1



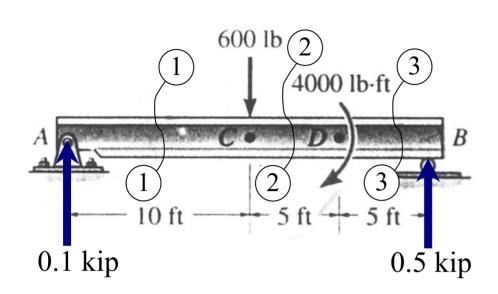
$$\Sigma F_{1-1} = 0$$

$$0.1 - V = 0$$

$$V = 0.1 \text{ kip}$$

$$x = 0 \text{ ft}$$
  $\rightarrow V_A = 0.1 \text{ kip}$ 

$$x = 10 \text{ ft } \to V_C = 0.1 \text{ kip}$$



$$\Sigma M_{2-2} = 0$$

$$M - 0.1(x) + 0.6(x - 10) = 0$$

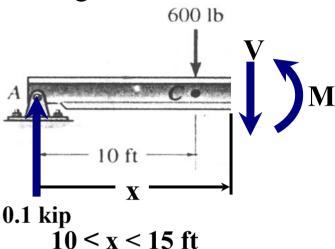
$$M - 0.1(x) + 0.6(x) - 6 = 0$$

$$M = -0.5(x) + 6$$

$$x = 10 \text{ ft} \rightarrow M_C = -0.5(10) + 6 = 1 \text{ kip-ft}$$

$$x = 15 \text{ ft} \rightarrow M_D = -0.5(15) + 6 = -1.5 \text{ kip-ft} \quad x = 15 \text{ ft} \rightarrow V_D = -0.5 \text{ kip}$$

Potongan 2-2



$$\Sigma F_{2-2} = 0$$

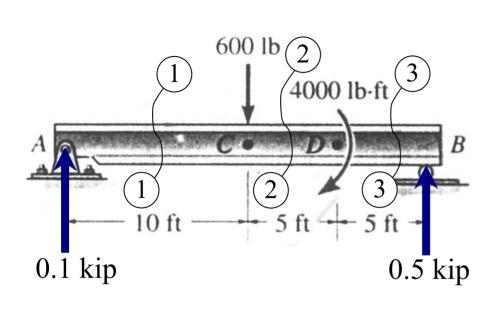
$$0.1 - 0.6 - V = 0$$

$$V = 0.1 - 0.6 \text{ kip}$$

$$V = -0.5 \text{ kip}$$

$$x = 10 \text{ ft } \rightarrow V_C = -0.5 \text{ kip}$$

$$x = 15 \text{ ft } \rightarrow V_D = -0.5 \text{ kip}$$



$$\Sigma M_{3-3} = 0$$

$$M - 0.1(x) + 0.6(x - 10) - 4 = 0$$

$$M - 0.1(x) + 0.6(x) - 6 - 4 = 0$$

$$M = -0.5(x) + 10$$

$$x = 15 \text{ ft} \rightarrow M_D = -0.5(15) + 10 = 2.5 \text{ kip-ft} \quad x = 15 \text{ ft} \rightarrow V_D = -0.5 \text{ kip}$$

$$x = 20 \text{ ft}$$
  $\rightarrow M_B = -0.5(20) + 10 = 0 \text{ kip-ft}$   $x = 20 \text{ ft}$   $\rightarrow V_D = -0.5 \text{ kip}$ 

$$\Sigma F_{3-3} = 0$$

$$0.1 - 0.6 - V = 0$$

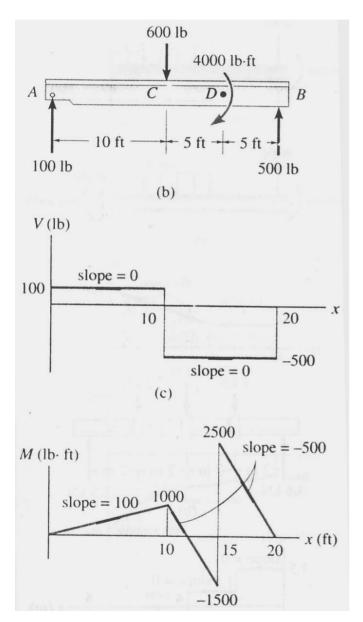
15 < x < 20 ft

$$V = 0.1 - 0.6 \text{ kip}$$

$$V = -0.5 \text{ kip}$$

$$x = 15 \text{ ft } \rightarrow V_D = -0.5 \text{ kip}$$

$$x = 20 \text{ ft } \rightarrow V_D = -0.5 \text{ kip}$$



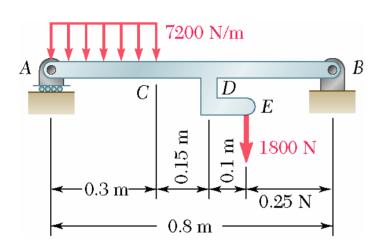
#### Diagram Geser dan Momen

$$\Sigma M_{1-1} = 0$$
  $\Sigma F_{1-1} = 0$   $V = 0.1 \text{ kip}$   $M_A = 0 \text{ kip-ft}$   $V_A = 0.1 \text{ kip}$   $M_C = 1 \text{ kip-ft}$   $V_C = 0.1 \text{ kip}$ 

$$\Sigma M_{2-2} = 0$$
  $\Sigma F_{2-2} = 0$   $M = -0.5(x) + 6$   $V = -0.5 \text{ kip}$   $M_C = 1 \text{ kip-ft}$   $V_C = -0.5 \text{ kip}$   $M_D = -1.5 \text{ kip-ft}$   $V_D = -0.5 \text{ kip}$ 

$$\Sigma M_{3-3} = 0$$
  $\Sigma F_{3-3} = 0$   $V = -0.5 \text{ kip}$   $M_D = 2.5 \text{ kip-ft}$   $V_D = -0.5 \text{ kip}$   $V_B = -0.5 \text{ kip}$ 

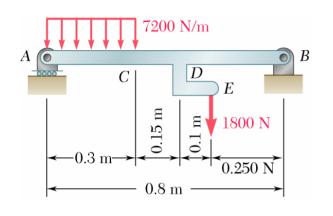
#### Contoh Soal 3

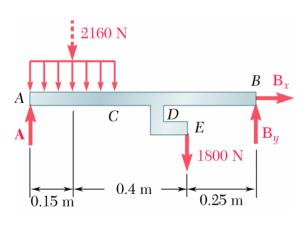


Draw the shear and bending moment diagrams for the beam AB. The distributed load of 7200 N/m. extends over 0.3 m of the beam, from A to C, and the 1800 N load is applied at E.

#### **SOLUTION:**

- Taking entire beam as free-body, calculate reactions at *A* and *B*.
- Determine equivalent internal forcecouple systems at sections cut within segments *AC*, *CD*, and *DB*.
- Plot results.





#### **SOLUTION:**

 Taking entire beam as a free-body, calculate reactions at A and B.

$$\sum M_A = 0$$
:

$$B_v(0.8 \,\mathrm{m}) - (2160 \,\mathrm{N})(0.15 \,\mathrm{m}) - (1800 \,\mathrm{N})(0.55 \,\mathrm{m}) = 0$$

$$B_{y} = 1642 \text{ N}$$

$$\sum M_B = 0$$
:

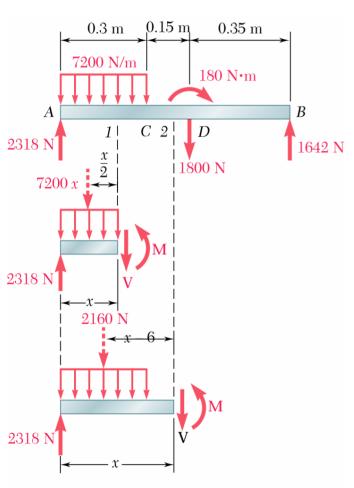
$$(2160 \text{ N})(0.65 \text{ m}) + (1800 \text{ N})(0.25 \text{ m}) - A(0.8 \text{ m}) = 0$$

$$A = 2318 \,\mathrm{N}$$

$$\sum F_{x} = 0$$
:

$$B_{\chi} = 0$$

• Note: The 1800 N load at E may be replaced by a 1800 N force and 180 Nm. couple at D.



• Evaluate equivalent internal force-couple systems at sections cut within segments *AC*, *CD*, and *DB*.

From A to C:

$$\sum F_y = 0$$
: 2318 – 7200 $x - V = 0$ 

$$V = 2318 - 7200x$$

$$\sum M_1 = 0 : -2318x + 7200x(\frac{1}{2}x) + M = 0$$

$$M = 2318x - 3600x^2$$

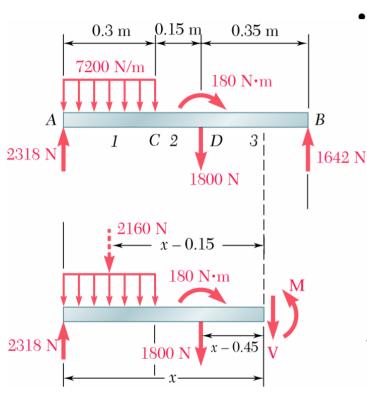
From *C* to *D*:

$$\sum F_{v} = 0$$
: 2318 – 2160 –  $V = 0$ 

$$V = 158 \text{ N}$$

$$\sum M_2 = 0 :-2318x + 2160(x - 0.15) + M = 0$$

$$M = (324 + 158x) \,\mathrm{N} \cdot \mathrm{m}$$



 Evaluate equivalent internal force-couple systems at sections cut within segments AC, CD, and DB.

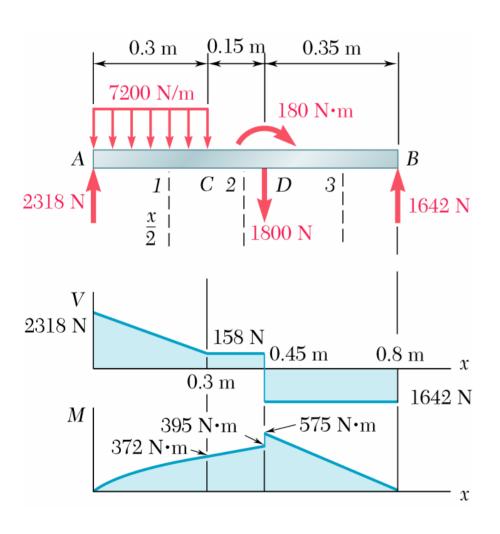
From *D* to *B*:

$$\sum F_y = 0$$
: 2318 - 2160 - 1800 -  $V = 0$ 

$$\sum M_2 = 0:$$

$$-2318x + 2160(x - 0.15) - 180 + 1800(x - 0.45) + M = 0$$

$$M = (1314 - 1642x) \text{ N} \cdot \text{m}$$



• Plot results.

From A to *C*:

$$V = 3218 - 7200x$$

$$M = 3218x - 20x^2$$

From *C* to *D*:

$$V = 158 \text{ N}$$

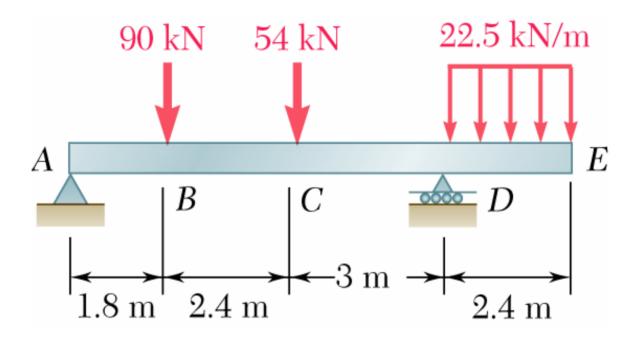
$$M = (324 + 158x) \,\mathrm{N} \cdot \mathrm{m}$$

From *D* to *B*:

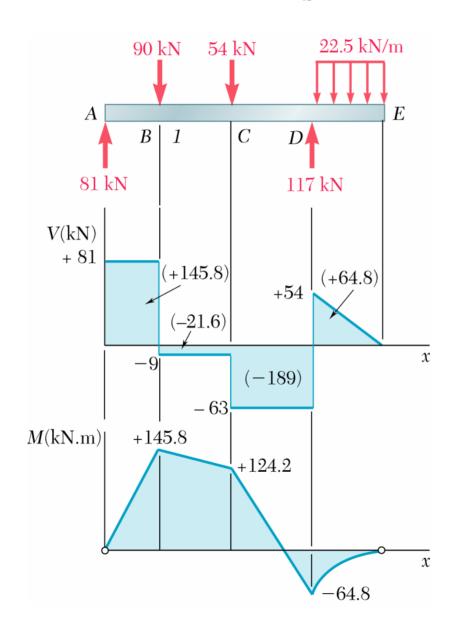
$$V = -1642 \text{ N}$$

$$M = (1314 - 1642x) \,\mathrm{N} \cdot \mathrm{m}$$

#### Contoh Soal 4



Draw the shear and bending-moment diagrams for the beam and loading shown.



### What's Stopping You?

Mu

## Friction

F<sub>n</sub>

Hard to Live With It, Can't Live Without It

Coefficient of Friction



### **Consider 2 Types of Friction**

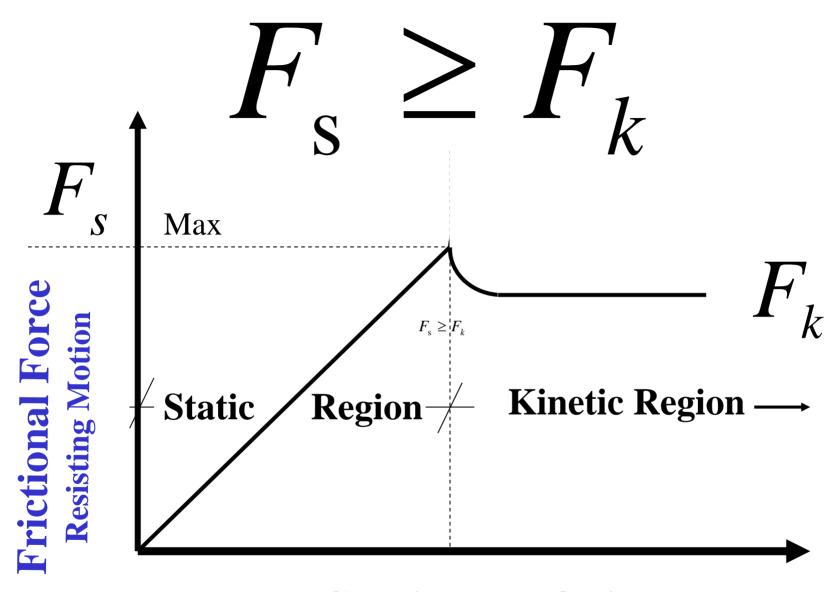
Force of Static Friction

This value represents the relative force necessary to make an object move

F<sub>k</sub> Force of Kinetic Friction

This value represents the relative force necessary to keep an object moving at a constant rate







Force Causing the Object to Move

### Frictional Forces Occur When Materials are in N **Contact Surfaces in Spring Scale Contact**



 $F_s$  = Force of Static Friction (Resists Motion)

**N** = Force Normal Holds Surfaces in Contact

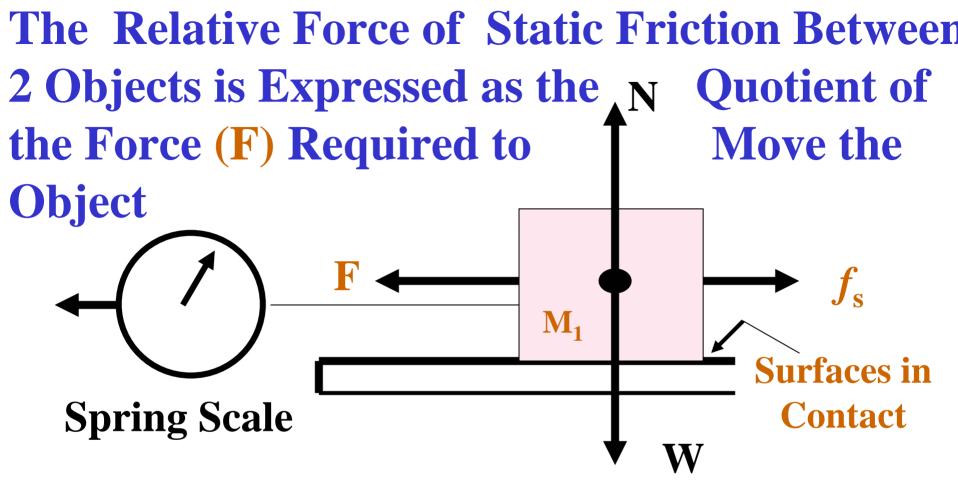
W = Weight of Object (Mass x Gravity)



## Friction is a Force That **Resists Motion Surfaces in Spring Scale Contact**



The Pink Block  $M_1$  Will not Move Until the Force F (Pull on the scale) Exceeds the Force of Static Friction  $f_s$ 



Divided by the Weight W of the Object



This is Called the Coefficient of Friction

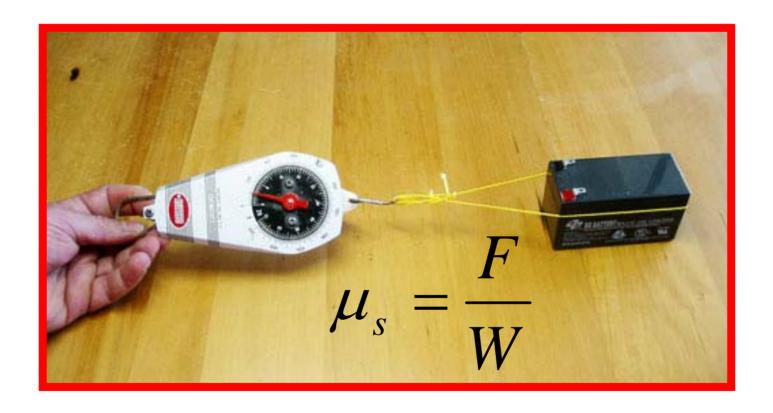
$$\mu_s = \frac{F}{W} \quad \begin{array}{c} \text{Coefficient of Static} \\ \text{Friction} \quad \text{N} \\ \\ \text{Spring Scale} \\ \end{array}$$

 $\mu_s$  = Coefficient of Friction

F = Force Required to Cause Motion

W =Weight of Object

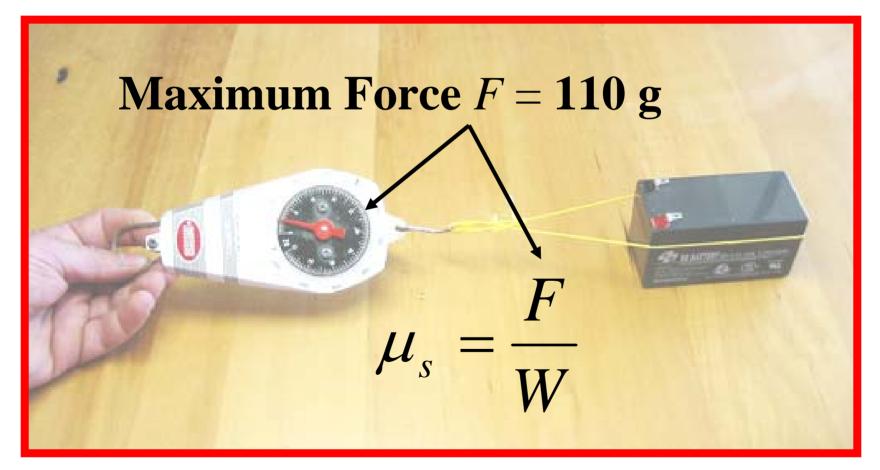
# Using the Gears-IDS Battery to Calculate The $\mu_s$ Static



**Coefficient of Friction** 



## Record the Maximum Force (F) (Before the Battery Begins to Move)





# Record the Weight (W) of the Battery

$$\mu_s = \frac{F}{W}$$

580 g







# The Coefficient of Static Friction Between the Wood Desktop and the Plastic Battery is Described Algebraically:

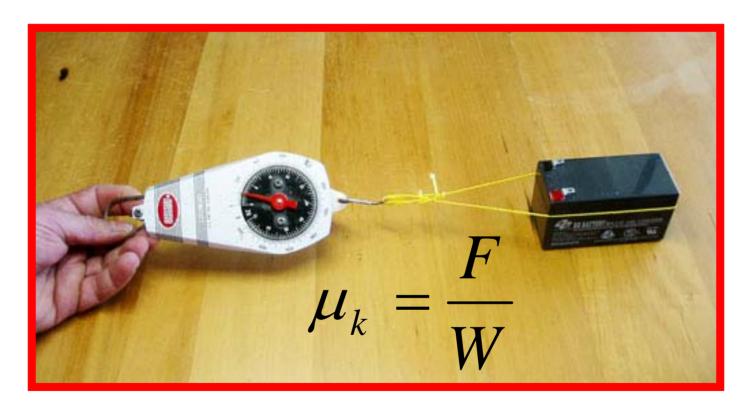
$$\mu_s = \frac{F}{W} = \frac{110 \text{ g}}{8}$$

$$= \frac{110 \text{ g}}{W} = 580 \text{ g}$$

$$\mu_s = .190$$



## The Coefficient of Kinetic Friction Can be Found Using the Same Technique



Record the Force Required to Move Georg the Battery at a Constant Rate

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Coefficients of Friction Between Various Surfaces Have Significant Impact on the Design and Construction of Mechanisms.

The Design of Competitive Mechanisms with Wheels Requires **Extensive Testing in Order to** Optimize the Torque, Speed and Traction (Friction) of the Drive Systems.

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## The End

