Control Systems Design

課程用書

- **_Textbook:_**

  Richard C. Dorf and Robert H. Bishop, "MODERN CONTROL SYSTEMS" 10th Edition

- **_Reference:_**

  Benjamin C. Kuo, "AUTOMATIC CONTROL SYSTEMS" 7th Edition

  Katsuhiko Ogata, "Modern Control Engineering" 2th Edition

- **_MATLAB and SIMULINK:_**

  Mathworks Inc., "Matlab User's Guide" and "Simulink User's Guide"

授課方式

- 以投影片為主
- 模擬軟體 MATLAB、SIMULINK

課程內容

- ***Chapter 1 Introduction to Control Systems***
- ***Chapter 2 Mathematical Models of Systems***
- ***Chapter 3 State Variable Models***
- ***Chapter 4 Feedback Control System Characteristics***
- ***Chapter 5 The Performance of Feedback Control Systems***
- ***Chapter 6 The Stability of Linear Feedback Systems***
- ***Chapter 7 The Root Locus Method***
- ***Chapter 8 Frequency Response Methods***
- ***Chapter 9 Stability in The Frequency Domain***
- ***Chapter 10 The Design of Feedback Control Systems***

CHAPTER 1

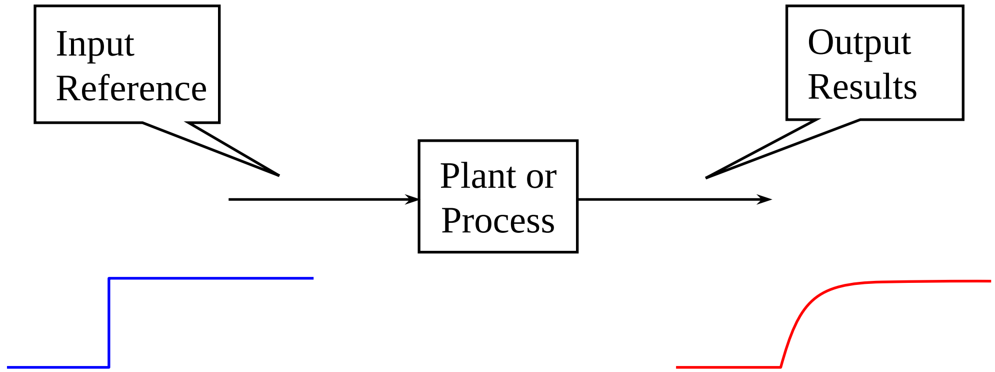
Introduction to Control Systems

FIGURE 1.2

Open-loop control system (without feedback).

An open-loop control system utilizes an actuating device to control the process directly without using feedback

Control :



- Manual Control
- Automatic Control
  - Automobile Cruise-Control
  - Airplane Flight Control
  - Robot Arm Control
  - Inkjet Printer Head Control
  - Disk Drive Read/Write Head Positioning Control

  ⋮

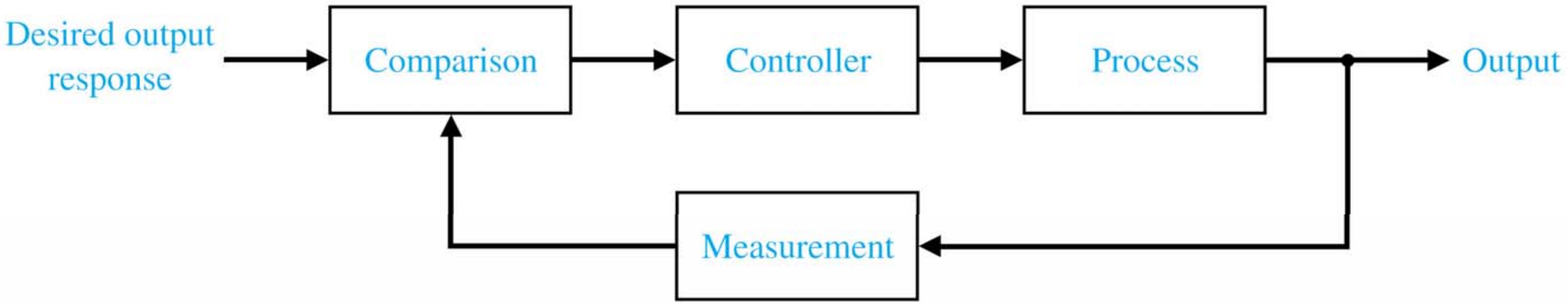


FIGURE 1.3 Closed-loop feedback control system (with feedback).

A closed-loop control system uses a measurement of the output and feedback of this signal to compare it with the desired output (negative feedback).

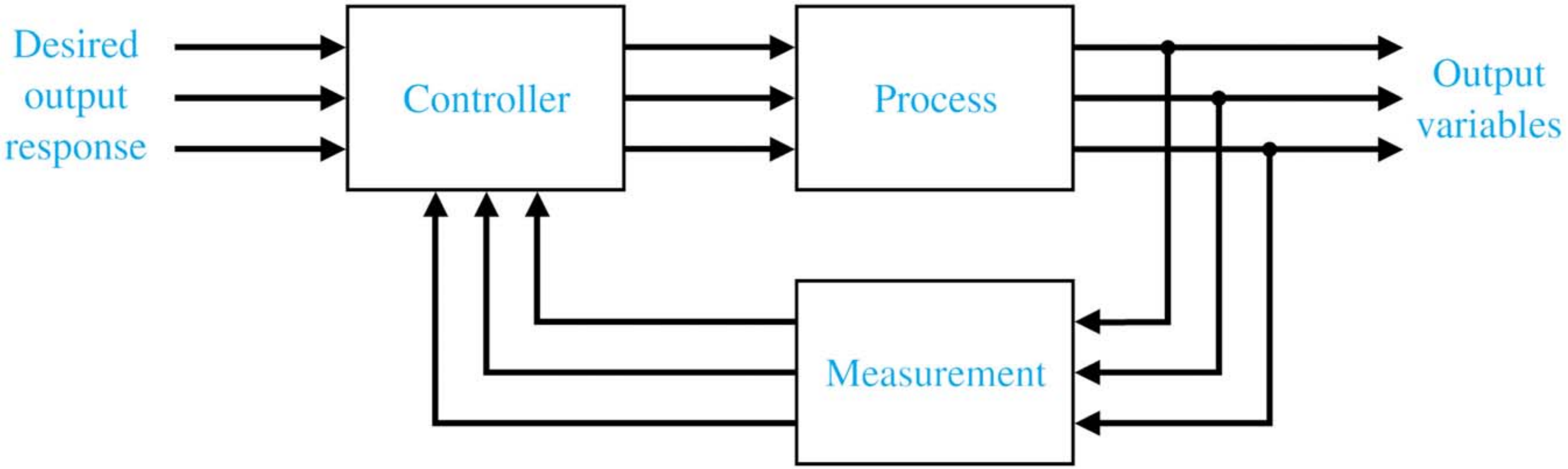


FIGURE 1.4 Multivariable control system.

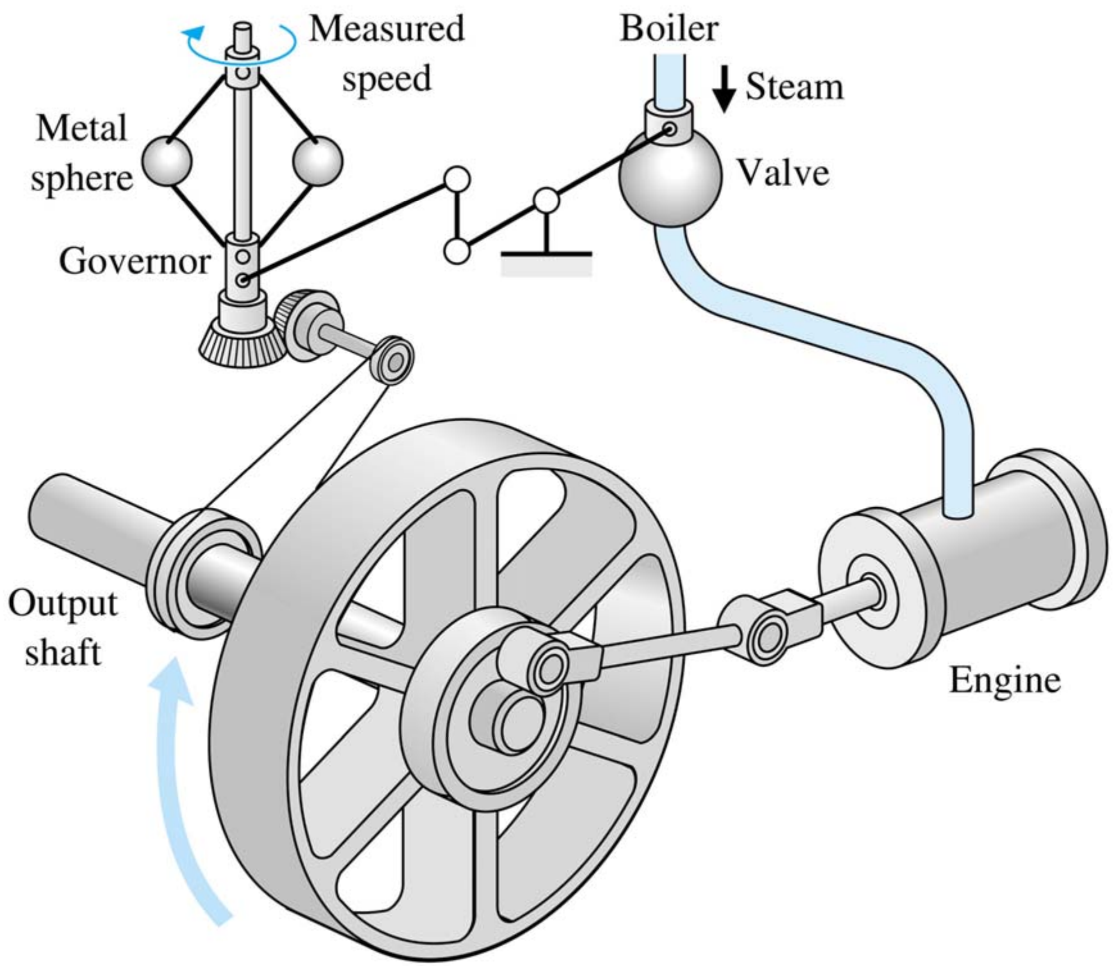


FIGURE 1.5 Watt’s flyball governor.

Advantages of Feedback:

- *Increased accuracy (reduced the steady-state error)*
- *Reduced sensitivity to parameter variations*
- *Reduced effects of disturbances*
- *Increased speed of response and bandwidth*

Computer Aided Control System Design (CACSD)

- *Matlab & Simulink*
- *Matrixx*
- *Simnon*
- *Program CC*

⋮

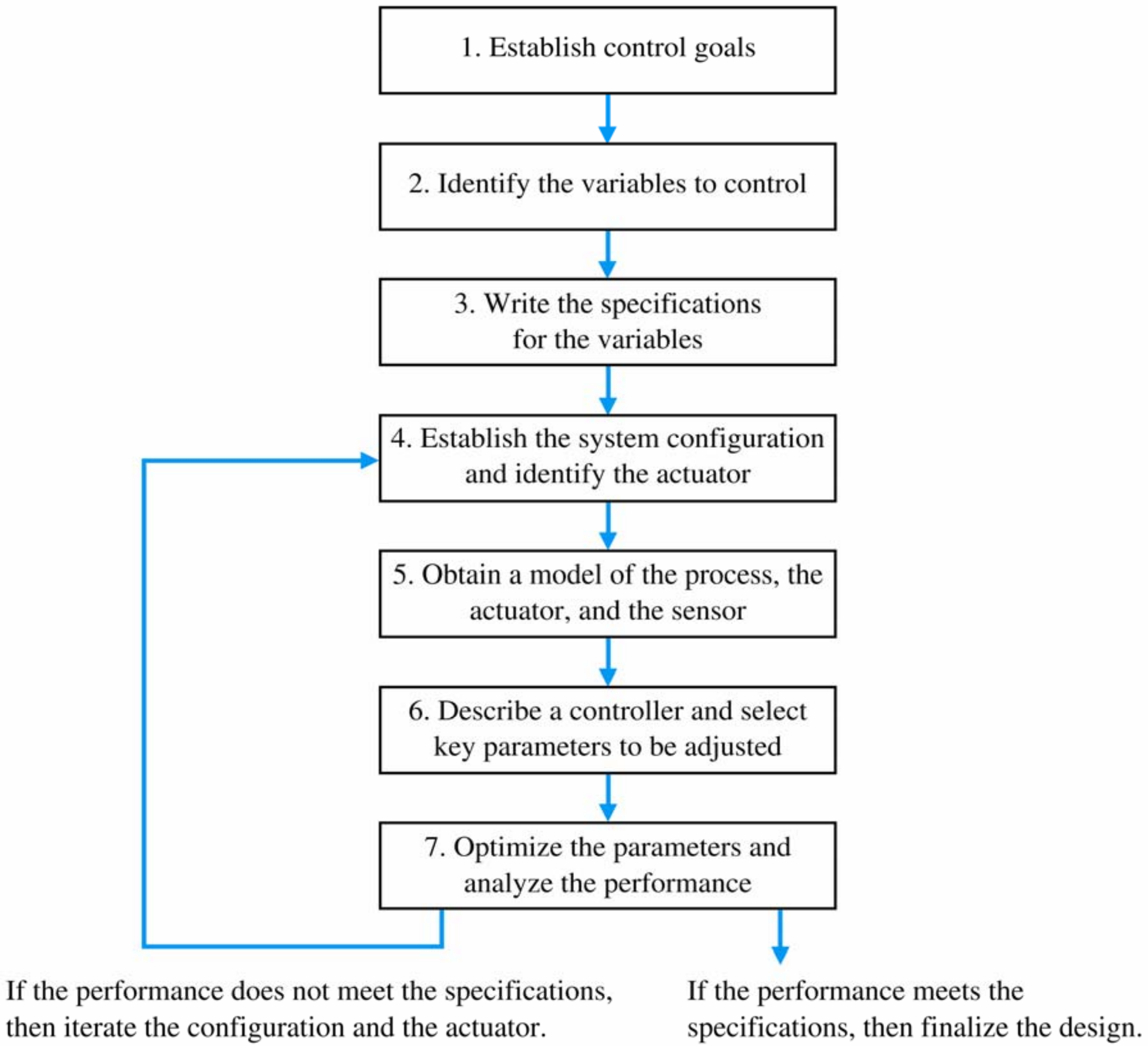


FIGURE 1.19 The control system design process.

CHAPTER 2
Mathematical Models of Systems

- Differential Equations of Physical System
- Linear Approximations of Physical System
- The Laplace Transform
- The Transfer Function Of Linear System
- Block Diagram Models
- Signal-Flow Graph Model
- Computer Analysis of control system
- Design Examples
- The Simulation of System Using Matlab

Differential Equations of Physical System

The approach to dynamic system problem can be listed as follow:

1. Define the system and its components
2. Formulate the mathematical model and list the necessary assumptions.
3. Write the differential equations describing the model.
4. Solve the equations for the desired output variables.
5. Examine the solutions and the assumptions.
6. If necessary, reanalyze or redesign the system

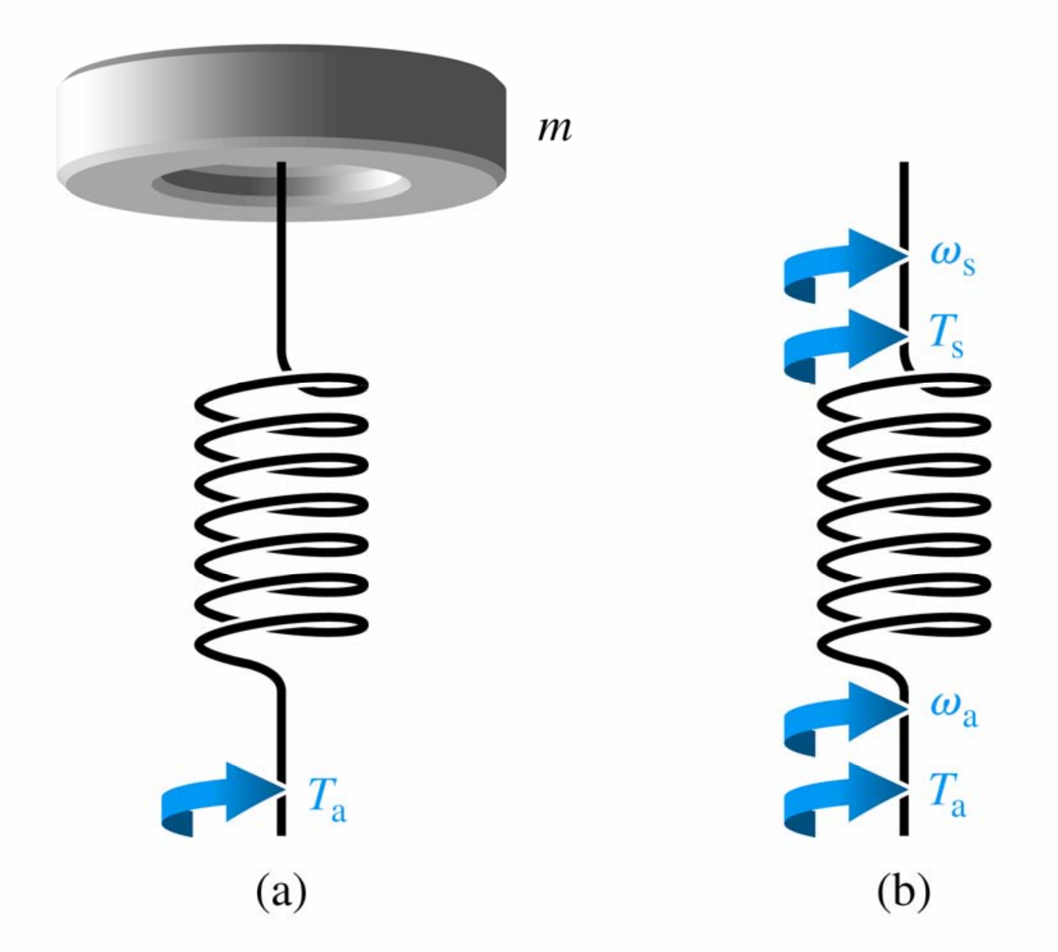
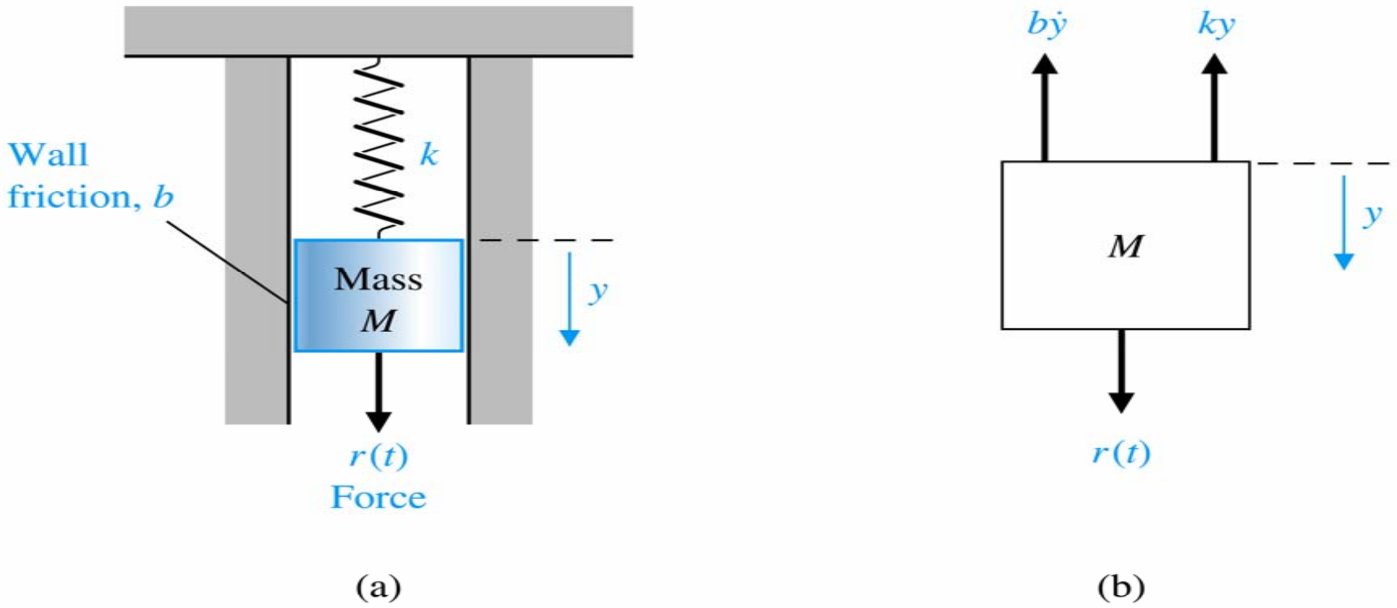


FIGURE 2.1 (a) Torsional spring-mass system.(b) Spring element.

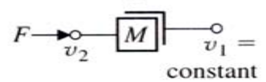
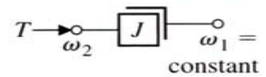
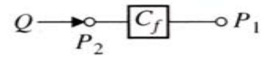
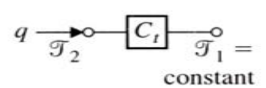
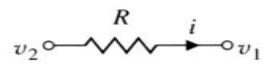
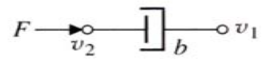
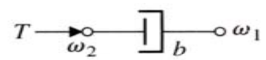
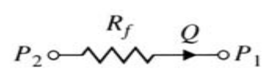
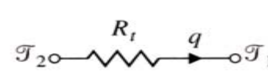


$$M\frac{d^2y(t)}{dt^2}+b\frac{dy(t)}{dt}+ky(t)=r(t)$$

FIGURE 2.2

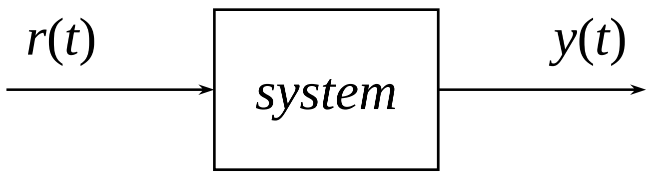
(a) Spring-mass-damper system.
(b) Free-body diagram.

Table 2.2 Summary of Describing Differential Equations for Ideal Elements

Type of Element	Physical Element	Describing Equation	Energy E or Power $\mathcal{P}$	Symbol
Inductive storage	Electrical inductance	$v_{21} = L\frac{di}{dt}$	$E = \frac{1}{2}Li^2$	
	Translational spring	$v_{21} = \frac{1}{k}\frac{dF}{dt}$	$E = \frac{1}{2}\frac{F^2}{k}$	
	Rotational spring	$\omega_{21} = \frac{1}{k}\frac{dT}{dt}$	$E = \frac{1}{2}\frac{T^2}{k}$	
	Fluid inertia	$P_{21} = I\frac{dQ}{dt}$	$E = \frac{1}{2}IQ^2$	
Capacitive storage	Electrical capacitance	$i = C\frac{dv_{21}}{dt}$	$E = \frac{1}{2}Cv_{21}^2$	
	Translational mass	$F = M\frac{dv_2}{dt}$	$E = \frac{1}{2}Mv_2^2$	
	Rotational mass	$T = J\frac{d\omega_2}{dt}$	$E = \frac{1}{2}J\omega_2^2$	
	Fluid capacitance	$Q = C_f\frac{dP_{21}}{dt}$	$E = \frac{1}{2}C_fP_{21}^2$	
	Thermal capacitance	$q = C_t\frac{d\mathcal{T}_2}{dt}$	$E = C_t\mathcal{T}_2$	
Energy dissipators	Electrical resistance	$i = \frac{1}{R}v_{21}$	$\mathcal{P} = \frac{1}{R}v_{21}^2$	
	Translational damper	$F = bv_{21}$	$\mathcal{P} = bv_{21}^2$	
	Rotational damper	$T = b\omega_{21}$	$\mathcal{P} = b\omega_{21}^2$	
	Fluid resistance	$Q = \frac{1}{R_f}P_{21}$	$\mathcal{P} = \frac{1}{R_f}P_{21}^2$	
	Thermal resistance	$q = \frac{1}{R_t}\mathcal{T}_{21}$	$\mathcal{P} = \frac{1}{R_t}\mathcal{T}_{21}$	

☑ Linear Approximations of Physical Systems

A linear system satisfied the properties of superposition and homogeneity.



$$r(t) \rightarrow y(t) \Leftrightarrow y(t) = g[r(t)]$$

$$\left.\begin{matrix} r_1(t) \rightarrow y_1(t) \\ r_2(t) \rightarrow y_2(t) \end{matrix}\right\} \Rightarrow r(t) = \alpha r_1(t) + \beta r_2(t) \rightarrow y(t) = \alpha y_1(t) + \beta y_2(t)$$

$$g[\alpha r_1(t) + \beta r_2(t)] = \alpha g[r_1(t)] + \beta g[r_2(t)]$$

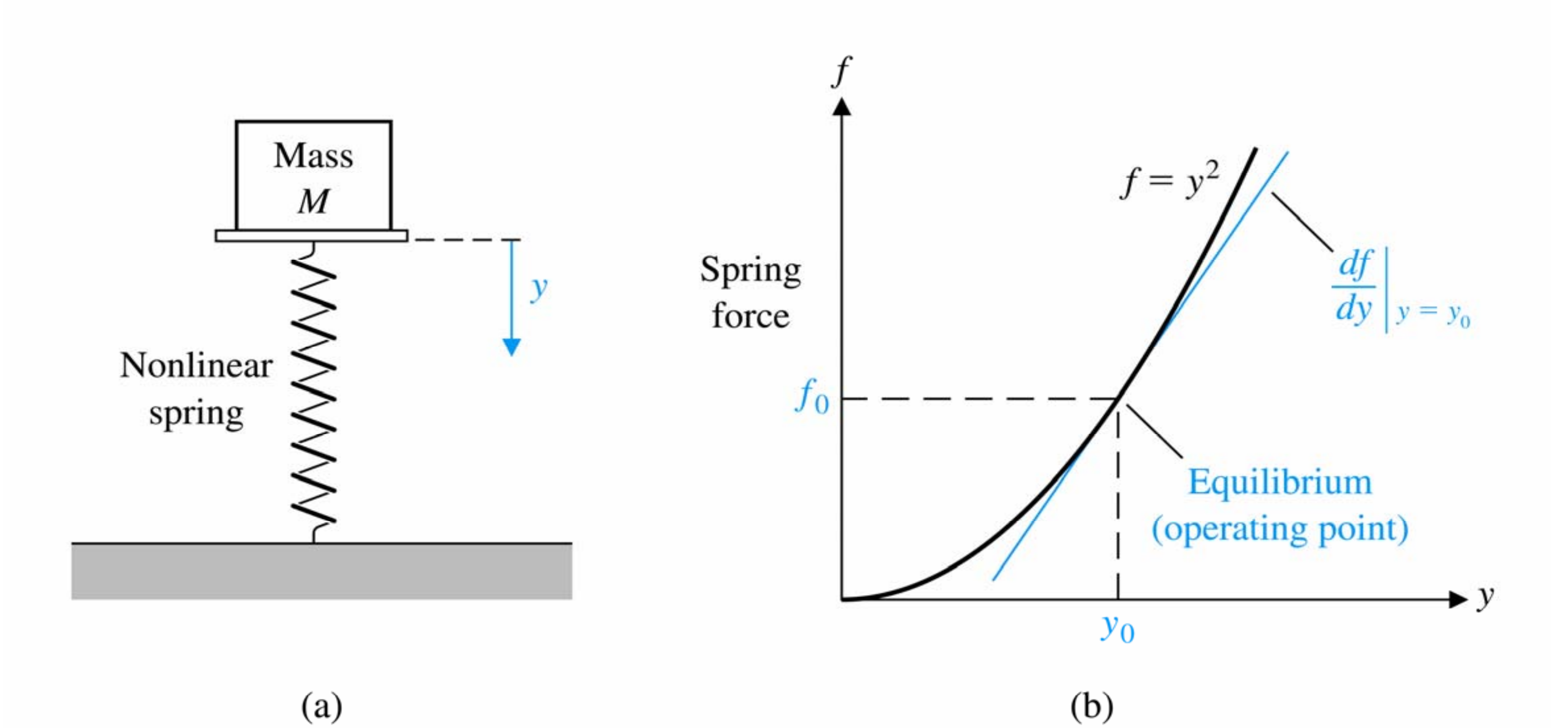


FIGURE 2.5

(a) A mass sitting on a nonlinear spring.
(b) The spring force versus y.

Linearization of a nonlinear systems:

$$y(t) = g[x(t)]$$

Expanding the nonlinear equation into a *Taylor series* about the operation point, then we have

$$y = g(x) = g(x_o) + \frac{d}{dx} g(x)\bigg|_{x=x_o} \frac{(x-x_o)}{1!} + \frac{d^2}{dx^2} g(x)\bigg|_{x=x_o} \frac{(x-x_o)^2}{2!} + \cdots$$

Neglecting all the high order terms, to yield

$$y = g(x_o) + \frac{d}{dx} g(x)\bigg|_{x=x_o} \frac{(x-x_o)}{1!} = y_0 + m \cdot (x - x_o)$$

$$\Rightarrow y - y_0 = m \cdot (x - x_o)$$

$$or \quad \Delta y = m \cdot \Delta x$$

$$\boxed{\begin{aligned} y &= g(x_1, \cdots, x_n) \\ &= g(x_{1o}, \cdots, x_{no}) + \frac{\partial g}{\partial x_1}\bigg|_{x=x_o} (x_1 - x_{1o}) + \frac{\partial g}{\partial x_2}\bigg|_{x=x_o} (x_2 - x_{2o}) + \cdots + \frac{\partial g}{\partial x_n}\bigg|_{x=x_o} (x_n - x_{no}) \end{aligned}}$$

Example 2.1 Pendulum oscillator model

$$T = Mgl\sin\theta$$

$$T - T_0 \cong Mgl\frac{\partial \sin\theta}{\partial\theta}\Big|_{\theta=\theta_0}(\theta-\theta_0)$$

$$where\ \ T_0 = 0.$$

$$T = Mgl(\cos 0^0)(\theta - 0)$$

$$= Mgl\theta$$

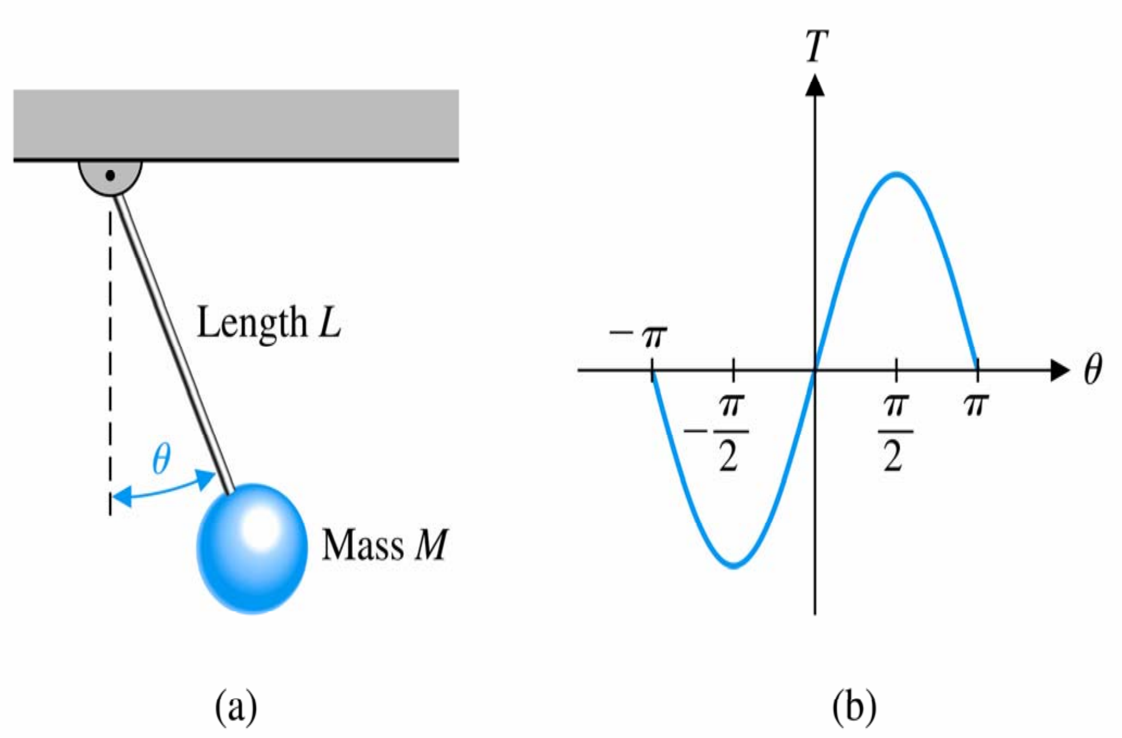


FIGURE 2.6

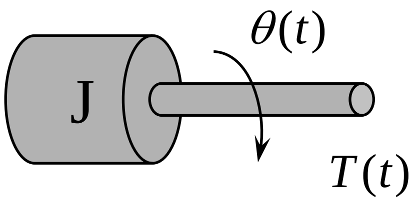
Pendulum oscillator.

☑ Rotational Motion

1. *Inertia*

Newton's Law for rotation motion :

$$\sum T(t) = J\alpha(t) = J\frac{d}{dt}\omega(t) = J\frac{d^2}{dt^2}\theta(t)$$

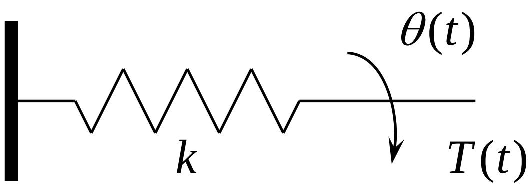


T : Torque J : Inertia α : Angular acceleration
ω : Angular velocity θ : Angular displacement

2. *Torsional Spring*

$$T(t) = k\theta(t)$$

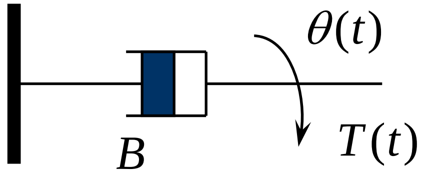
k : Torsional spring constant



3. *Friction for rotational motion : viscous, static, coulomb friction*

$$T(t) = B\frac{d}{dt}\theta(t)$$

B : Viscous friction coefficient



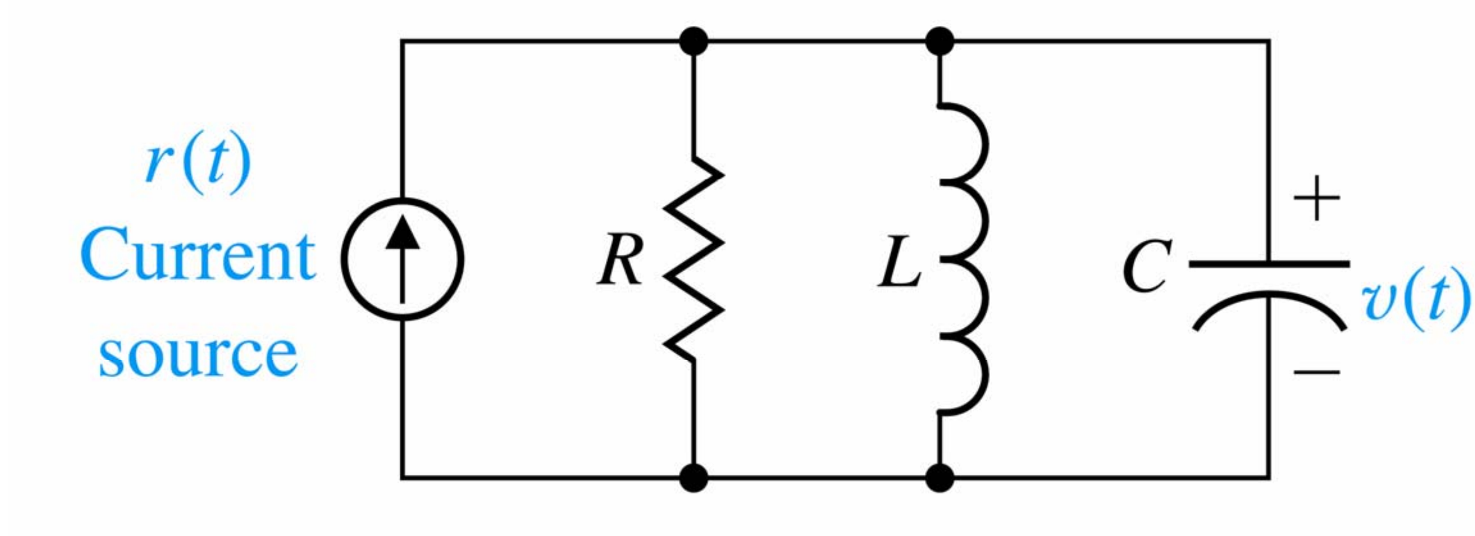


FIGURE 2.3

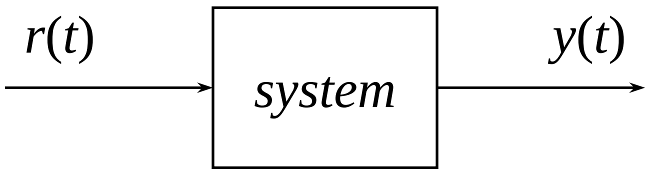
$$\frac{v(t)}{R} + C\frac{dv(t)}{dt} + \frac{1}{L}\int_0^t v(t)dt = r(t)$$

RLC circuit.

☑ The Transfer Function of Linear Systems

Definition:

- ***The ratio of the Laplace transform of the output variable to the Laplace transform of the input variable, with all initial conditions assumed to be zero.***
- ***The Laplace transform of the impulse response, with all the initial conditions set to zero.***

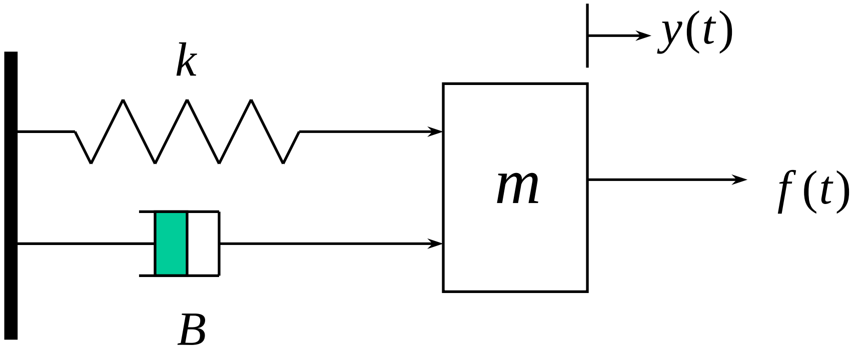


$$\frac{Y(s)}{R(s)} = G(s)$$

or

$$G(s) = L[y(t)]$$

Example : Mass-Spring-Friction System



$$f_1(t) = ma(t) = m\frac{d}{dt}v(t) = m\frac{d^2}{dt^2}y(t)$$

$$f_2(t) = ky(t)$$

$$f_3(t) = B\frac{d}{dt}y(t)$$

$$\sum f(t) = f_1 + f_2 + f_3$$

$$= m\frac{d^2}{dt^2}y(t) + B\frac{d}{dt}y(t) + ky(t)$$

$$\ddot{y}(t) + \frac{B}{m}\dot{y}(t) + \frac{k}{m}y(t) = \frac{1}{m}f(t)$$

Taking the Laplace transform with zero initial conditions, we have

$$s^2Y(s) + s\frac{B}{m}Y(s) + \frac{k}{m}Y(s) = \frac{1}{m}F(s)$$

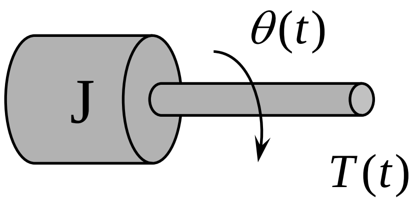
Then the transfer function between $Y(s)$ and $F(s)$ is obtained

$$\frac{Y(s)}{F(s)} = \frac{1}{s^2m + sB + k}$$

1. *Inertia*

Newton's Law for rotation motion :

$$\sum T(t) = J\alpha(t) = J\frac{d}{dt}\omega(t) = J\frac{d^2}{dt^2}\theta(t)$$



T : Torque J : Inertia α : Angular acceleration
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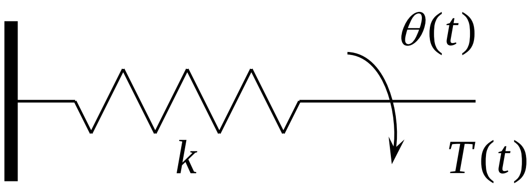
2. *Torsional Spring*

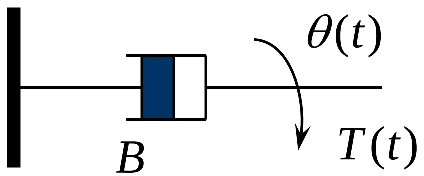
$$T(t) = k\theta(t)$$



k : Torsional spring constant

3. *Friction for rotational motion : viscous, static, coulomb friction*

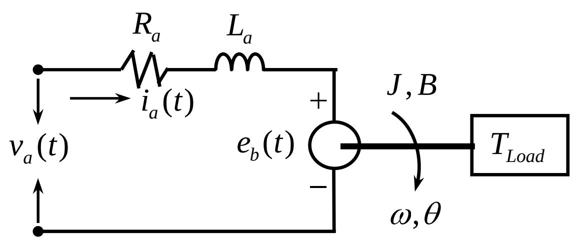
$$T(t) = B\frac{d}{dt}\theta(t)$$



B : Viscous friction coefficient

- *Field-current controlled DC motor* (Homework #1)
- *Armature-controlled DC motor*

Mathematical modeling of PM DC motor:



R_a	Armature resistance	L_a	Armature inductance
$v_a(t)$	Applied voltage	$i_a(t)$	Armature current
$e_b(t)$	Back-emf	k_b	Back-emf constant
k_m	Torque constant	$T_m(t)$	Motor torque
$T(t)$	Load torque	T_d	Disturbance torque
J	Rotor inertia	B	Viscous friction coefficient
$\theta(t)$	Rotor displacement	$\omega(t)$	Rotor angular velocity

The air-gap flux of the dc motor is

$$\phi(t) = k_f i_f(t)$$

and the torque developed by by the dc motor is

$$\begin{aligned} T_m(t) &= k_1 \phi(t) i_a(t) = k_1 k_f i_f(t) i_a(t) \\ &= k_1 k_f I_f i_a(t), \quad i_f(t) = \text{constant} \\ &= k_m i_a(t) \end{aligned}$$

If $L_a/R_a \approx 0$, the approximate model of the dc motor is obtained as

$$\frac{\omega(s)}{V_a(s)} = \frac{k_m}{JR_a s + (BR_a + k_b k_m)}$$

$$\triangleq \frac{k}{s\tau + 1}$$

where

$$\tau = \frac{JR_a}{BR_a + k_b k_m}, \quad k = \frac{k_m}{BR_a + k_b k_m}$$

Relation between k_m and k_b

The power input to the rotor = The power delivered to the shaft

$$e_b(t) i_a(t) = T(t)\omega(t)$$

$$\Leftrightarrow k_b \omega(t) i_a(t) = k_m i_a(t) \omega(t)$$

$$\Leftarrow \quad k_m = k_b$$

The cause and effect equation for the motor circuit are

$$v_a(t) = R_a i_a(t) + L_a \frac{d}{dt} i_a(t) + e_b(t)$$

$$e_b(t) = k_b \omega(t) = k_b \theta(t)$$

$$T(t) = J \frac{d}{dt}\omega(t) + B\omega(t) + T_d$$

$$= J \frac{d^2}{dt^2}\theta(t) + B\frac{d}{dt}\theta(t) + T_d$$

The motor torque is equal to the torque delivered to the load, that is

$$T_m(t) = T(t)$$

Hence the transfer function of the dc motor, with T_d=0, *is*

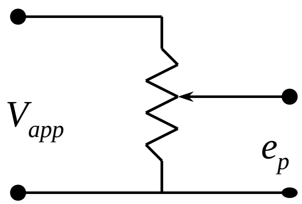
$$\frac{\omega(s)}{V_a(s)} = \frac{k_m}{(R_a + sL_a)(Js + B) + k_b k_m}$$

- Tachometer

$$e_t(t) = k_t\omega(t) = k_t \frac{d}{dt}\theta(t)$$

- Potentiometer

$$e_p(t) = k_p\theta(t), \quad k_p = \frac{V_{app}}{2\pi N}$$



- Incremental encoder ***(Homework #2)***

1. Measuring angular displacement
2. Measuring angular velocity

DC motor Filed-controlled

Home work 2:

$$\frac{\omega(s)}{V_f(s)} = ?$$

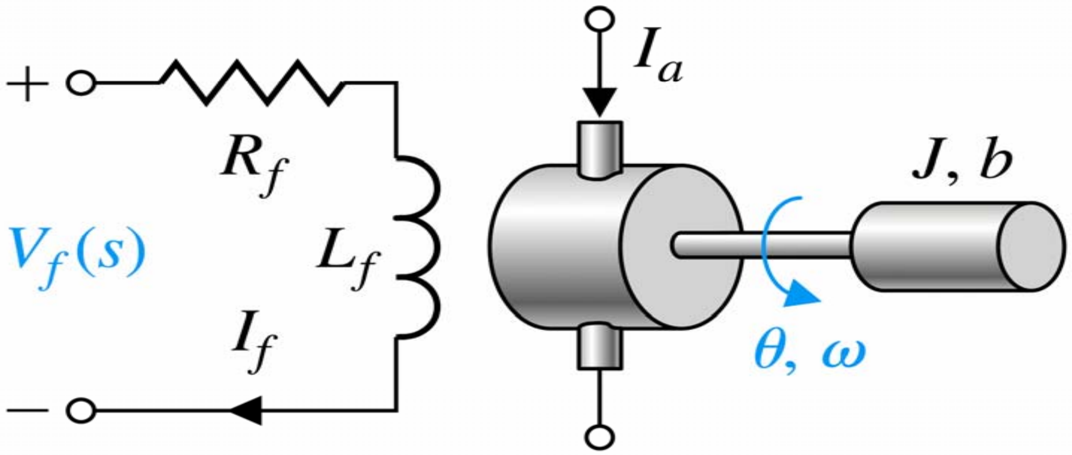


Table 2.5

Transfer Functions of Dynamic Elements and Networks
5. dc motor, field-controlled, rotational actuator

Homework 3

$$\frac{V_2(s)}{V_1(s)} = ?$$

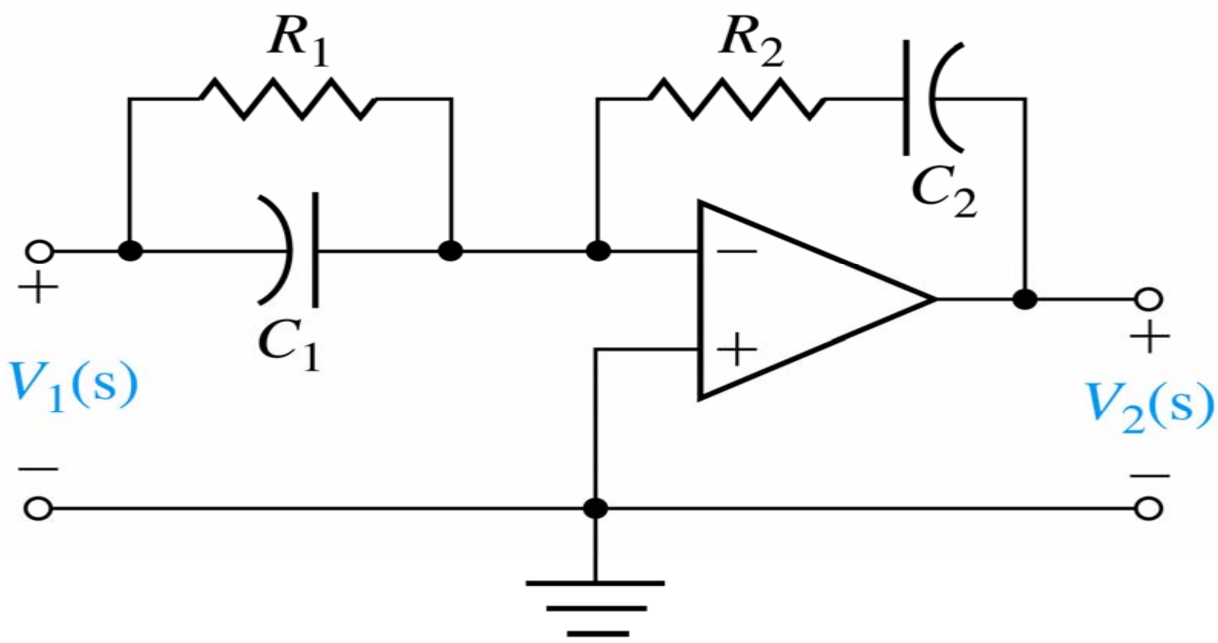


Table 2.5

Transfer Functions of Dynamic Elements and Networks
4. Lead-lag filter circuit

Homework 4:

$$\theta_L = n\theta_m$$

$$\frac{T_m}{T_L} = ?$$

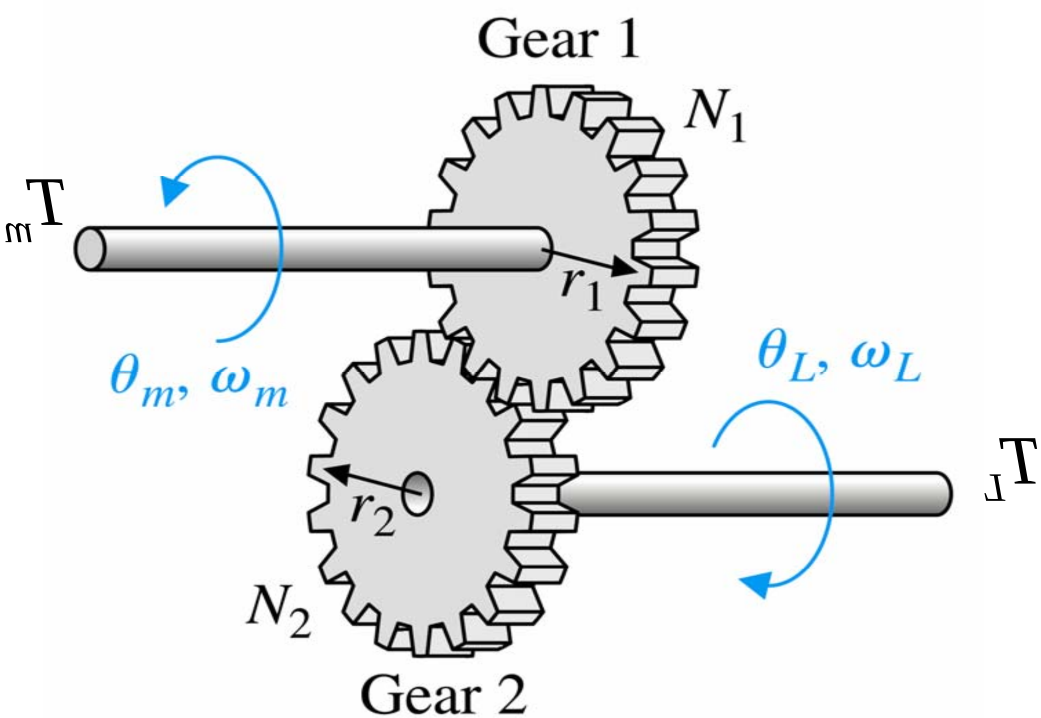


Table 2.5

Transfer Functions of Dynamic Elements and Networks

10. Gear train, rotational transformer

The block diagram

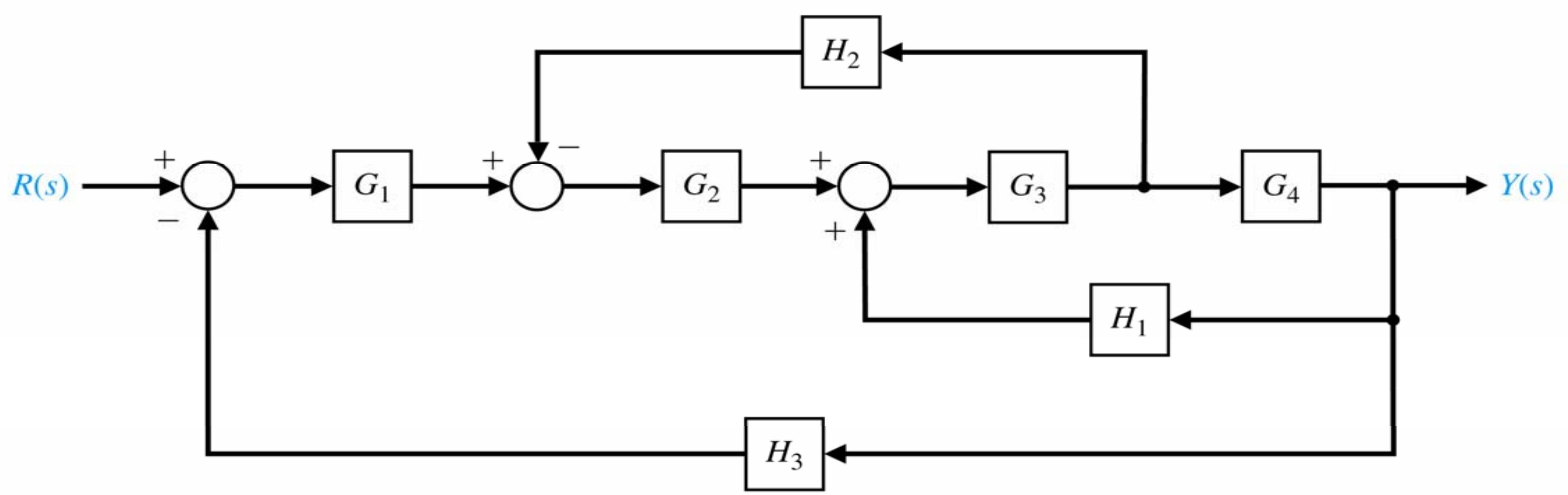
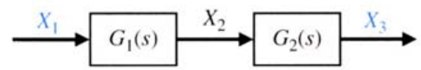
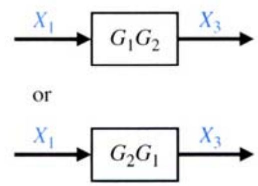
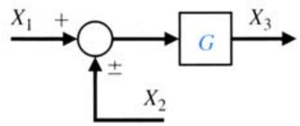
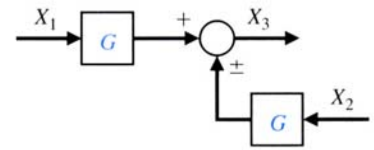
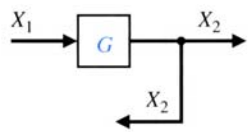
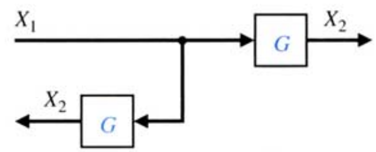
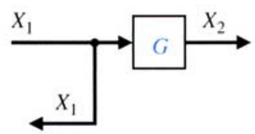
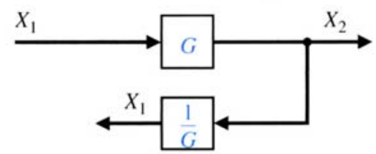
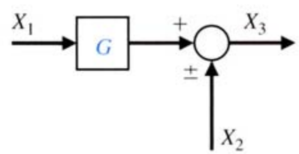
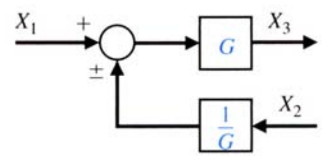
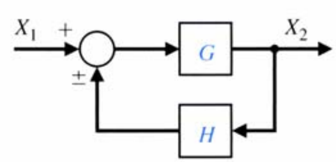
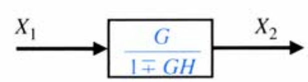


FIGURE 2.26 Multiple-loop feedback control system.

Table 2.6 Block Diagram Transformations

<table>
<tr><th>Transformation</th><th>Original Diagram</th><th>Equivalent Diagram</th></tr>
<tr><td>1. Combining blocks in cascade</td><td></td><td></td></tr>
<tr><td>2. Moving a summing point behind a block</td><td></td><td></td></tr>
<tr><td>3. Moving a pickoff point ahead of a block</td><td></td><td></td></tr>
<tr><td>4. Moving a pickoff point behind a block</td><td></td><td></td></tr>
<tr><td>5. Moving a summing point ahead of a block</td><td></td><td></td></tr>
<tr><td>6. Eliminating a feedback loop</td><td></td><td></td></tr>
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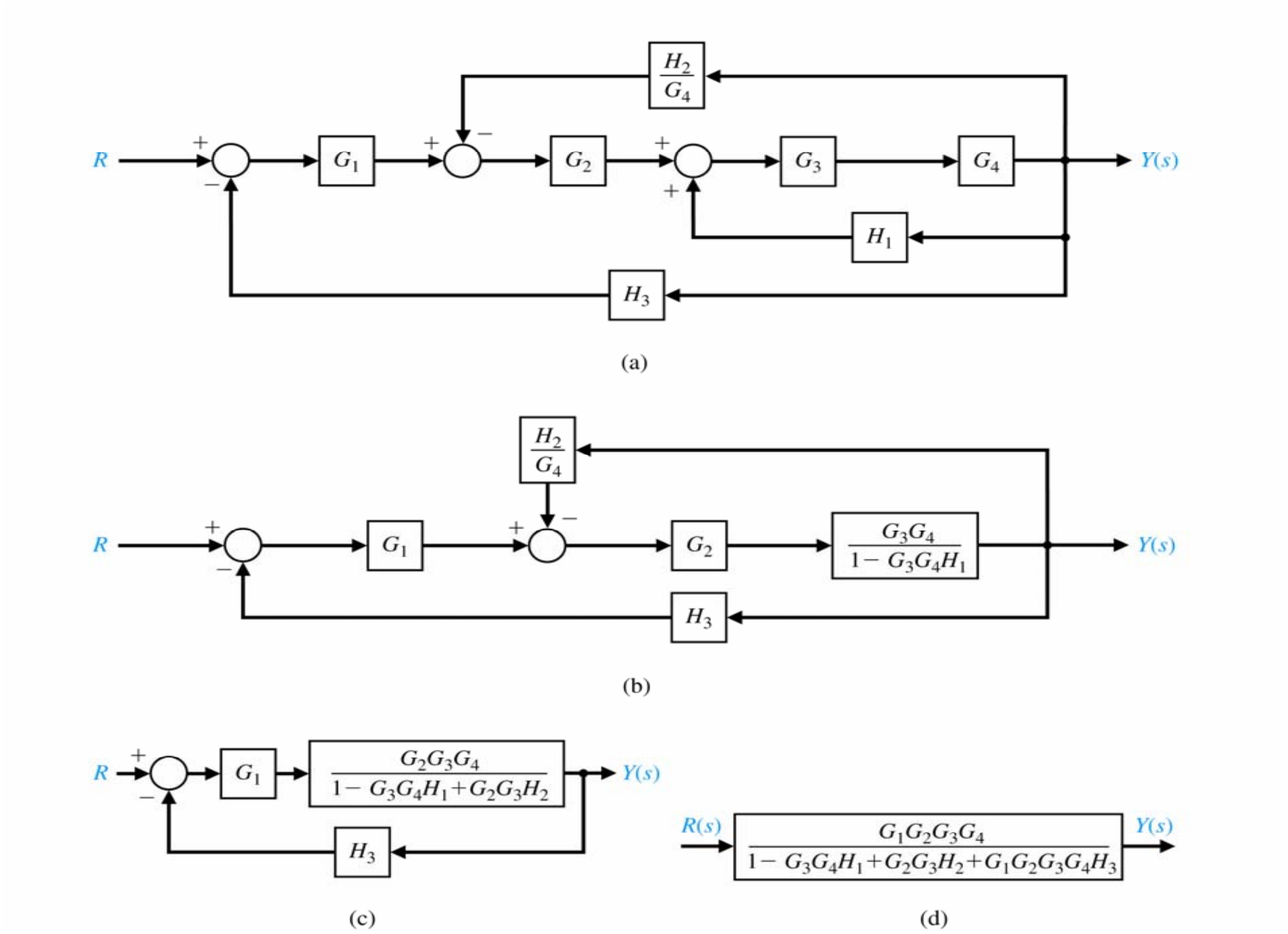
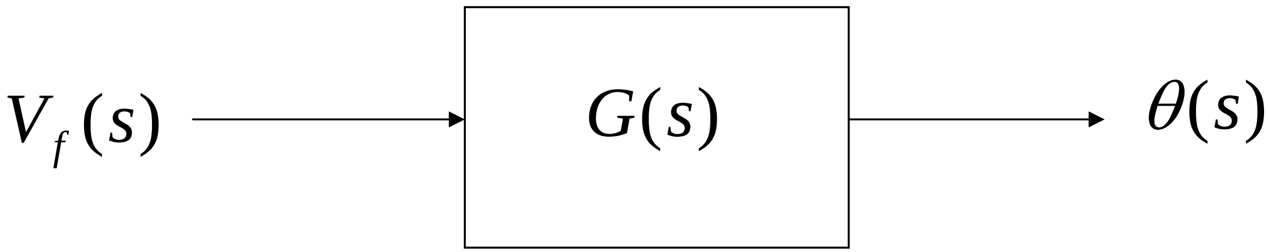


FIGURE 2.27 Block diagram reduction of the system of Fig.2.26

Signal-flow Graph Modes



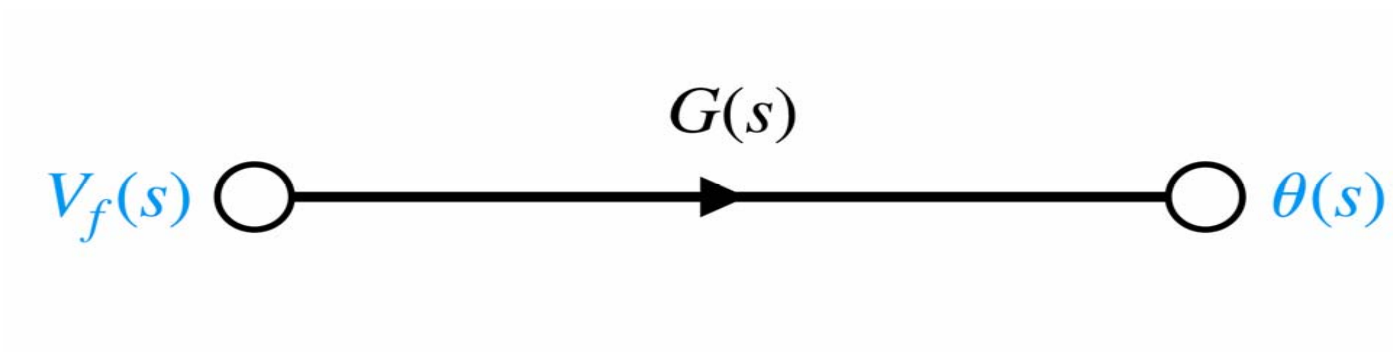


FIGURE 2.28 Signal-flow graph of the dc motor.

Signal-flow Graph Modes

- Branch:
- Nodes:
- Forward-path gain:
- Loop gain:
- Non-touching:

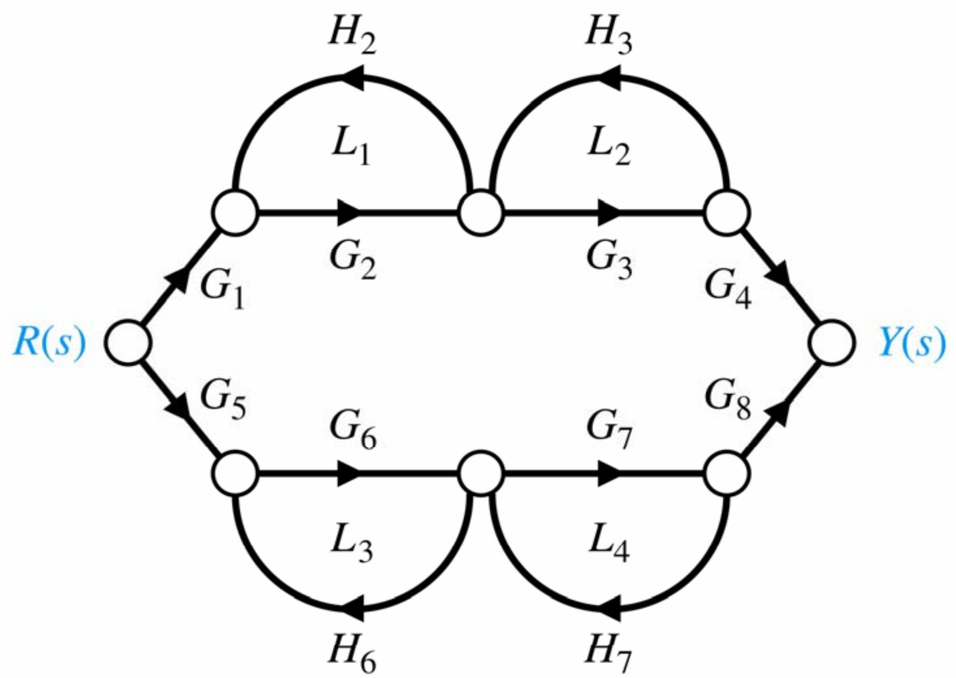


FIGURE 2.31 Two-path interacting system.

Mason's signal-flow gain formula:

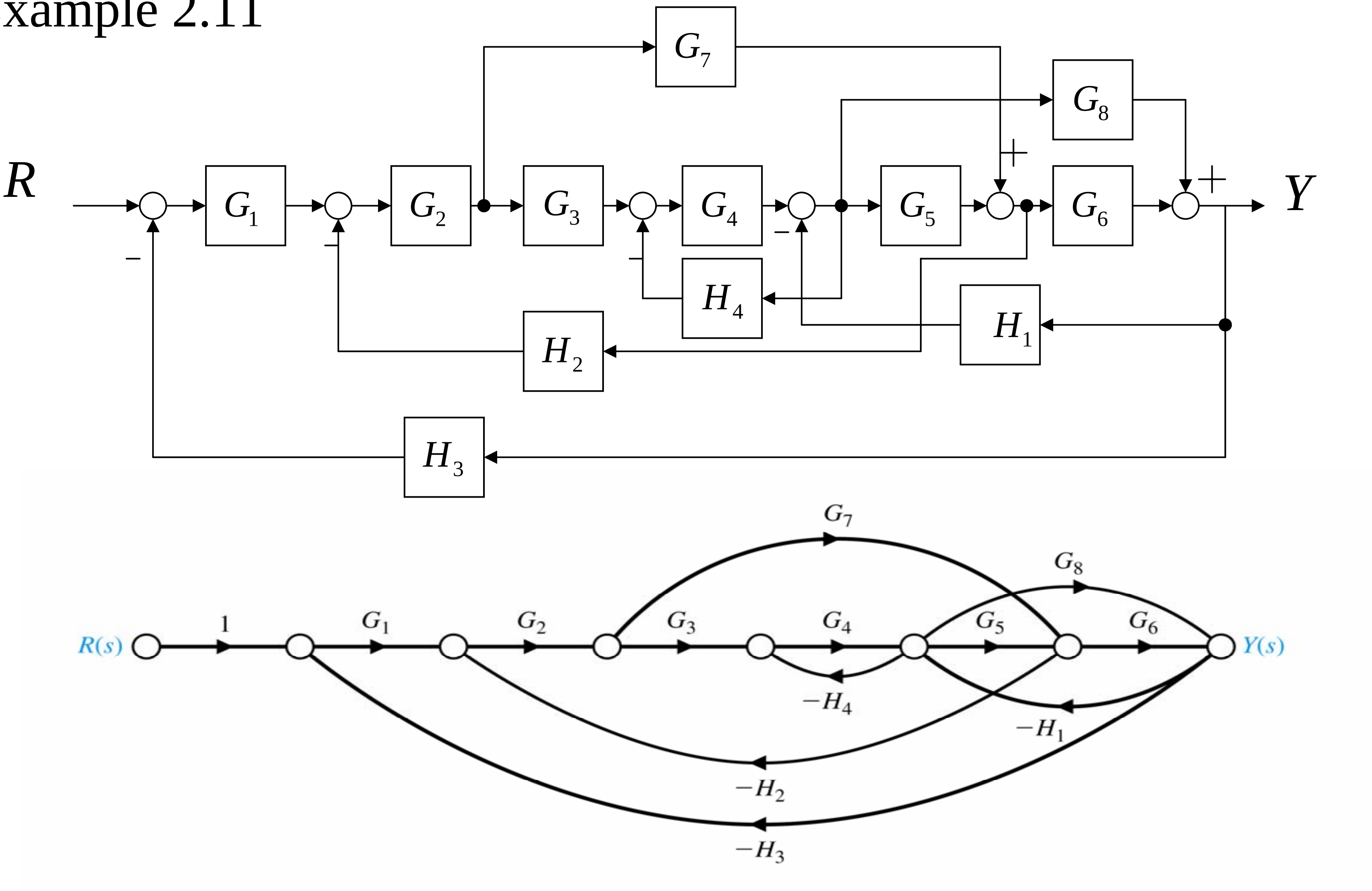
$$T_{ij} = \frac{\sum_k P_{ij}\Delta_{ijk}}{\Delta} = \frac{input\ node}{output\ node}$$

P_{ijk} :

Δ :

Δ_{ijk} :

Example 2.11



The Simulation of System

Assuming that a mode and the simulation are reliably accurate,computer simulation has the following advantages:

1. System performance can be observed under all conceivable conditions.
2. Results of field-system performance can be extrapolated with a simulation model for prediction purposes.
3. Decisions concerning future system presently in a conceptual stage can be examined.
4. Trials of system under test can be accomplished in a much-reduceed period of time.
5. Simulation results can be obtained at lower cost than real experimentation.
6. Study of hypothetical situation can be achieved even when the hypothetical situation would be unrealizable in actual life at the present time.
7. Computer modeling and simulation is often the only feasible or safe technique to analyze and evaluate a system.

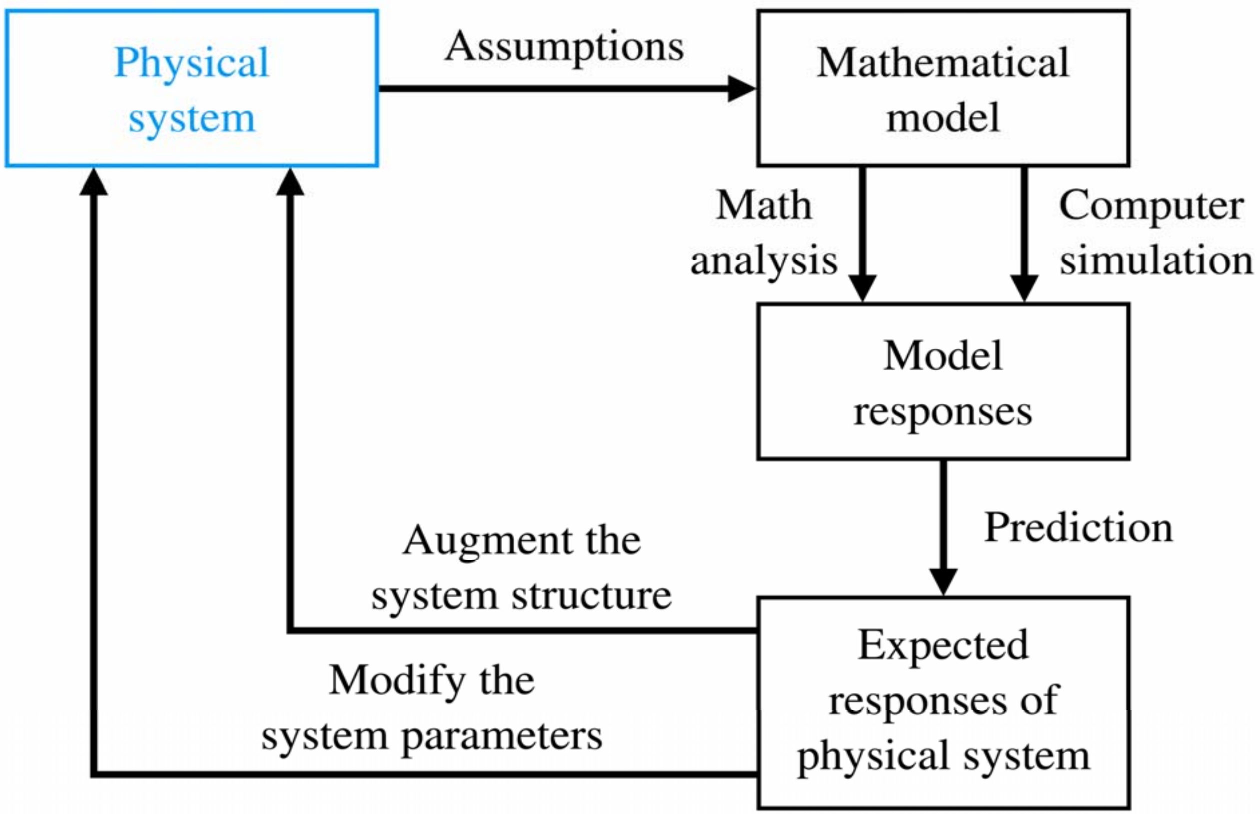


FIGURE 2.34 Analysis and design using a system model.

Example 2.12 Electric traction motor control

$$\frac{\omega(s)}{v_{in}(s)} = ?$$

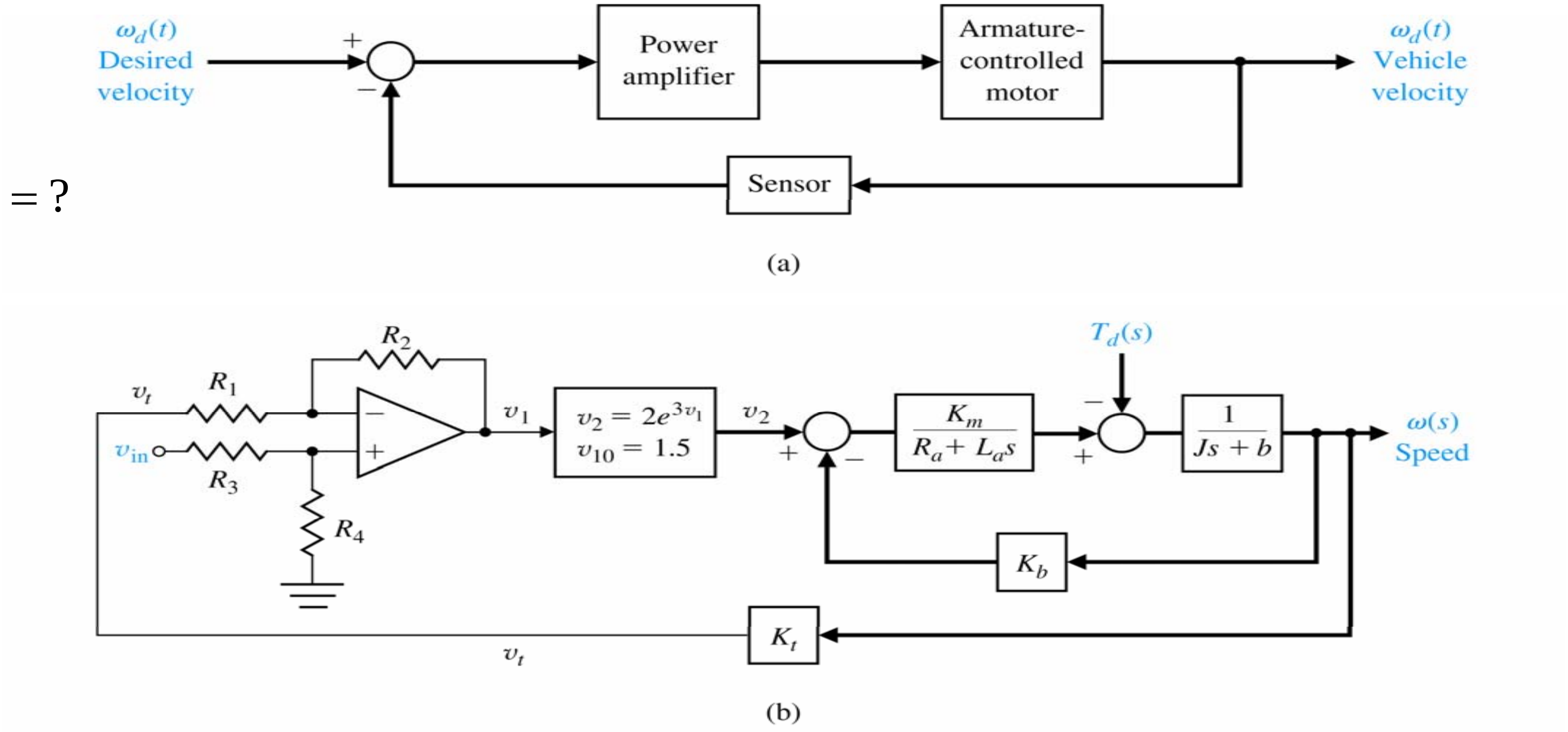


FIGURE 2.35 Speed control of an electric traction motor.

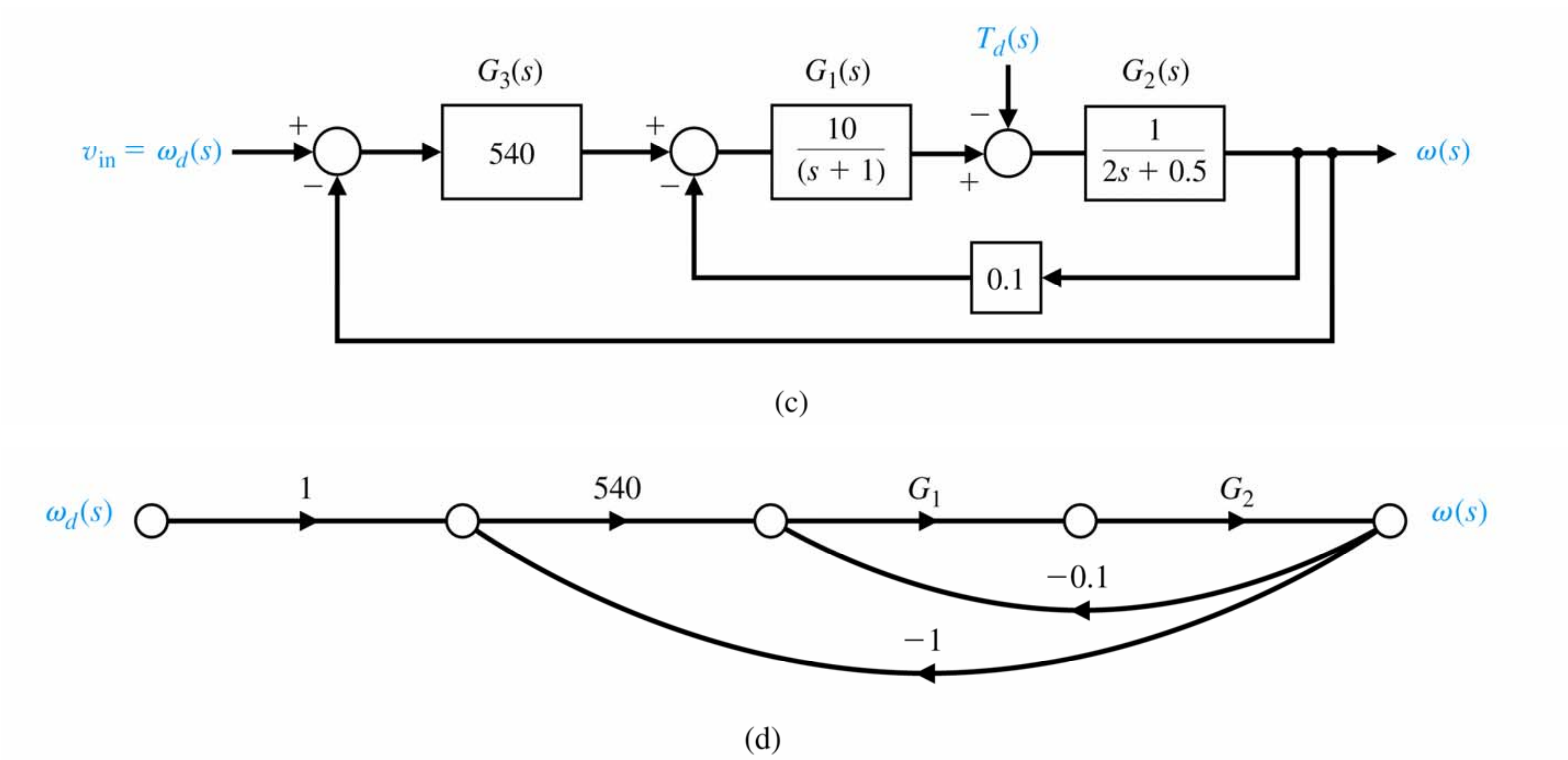


FIGURE 2.35 Speed control of an electric traction motor.

The simulation of system using Matlab

$$M\ddot{y} + b\dot{y} + ky = r(t)$$

The unforced dynamic response,y(t), of the spring-mass-damper mechanical system is

$$y(t) = \frac{y(0)}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin(\omega_n \sqrt{1-\xi^2}\, t + \theta)$$

```
>>y0=0.15;
>>wn=sqrt(2);
>>zeta=1/(2*sqrt(2));
>>t=[0:0.1:10];
>>unforced
```

ω_n

ζ

unforced.m

```
%Compute Unforced Response to an Initial Condition
%
c=(y0/sqrt(1-zeta^2));
y=c*exp(-zeta*wn*t).*sin(wn*sqrt(1-zeta^2)*t+acos(zeta));
%
bu=c*exp(-zeta*wn*t);bl=-bu;
%
plot(t,y,t,bu,'--',t,bl,'--'), grid
xlabel('Time (sec)'), ylabel('y(t) (meters)')
legend(['\omega_n=',num2str(wn),'     \zeta=',num2str(zeta)])
```

$y(0)/\sqrt{1-\zeta^2}$

$e^{-\zeta\omega_n t}$ envelope

FIGURE 2.40 Script to analyze the spring-mass-damper.

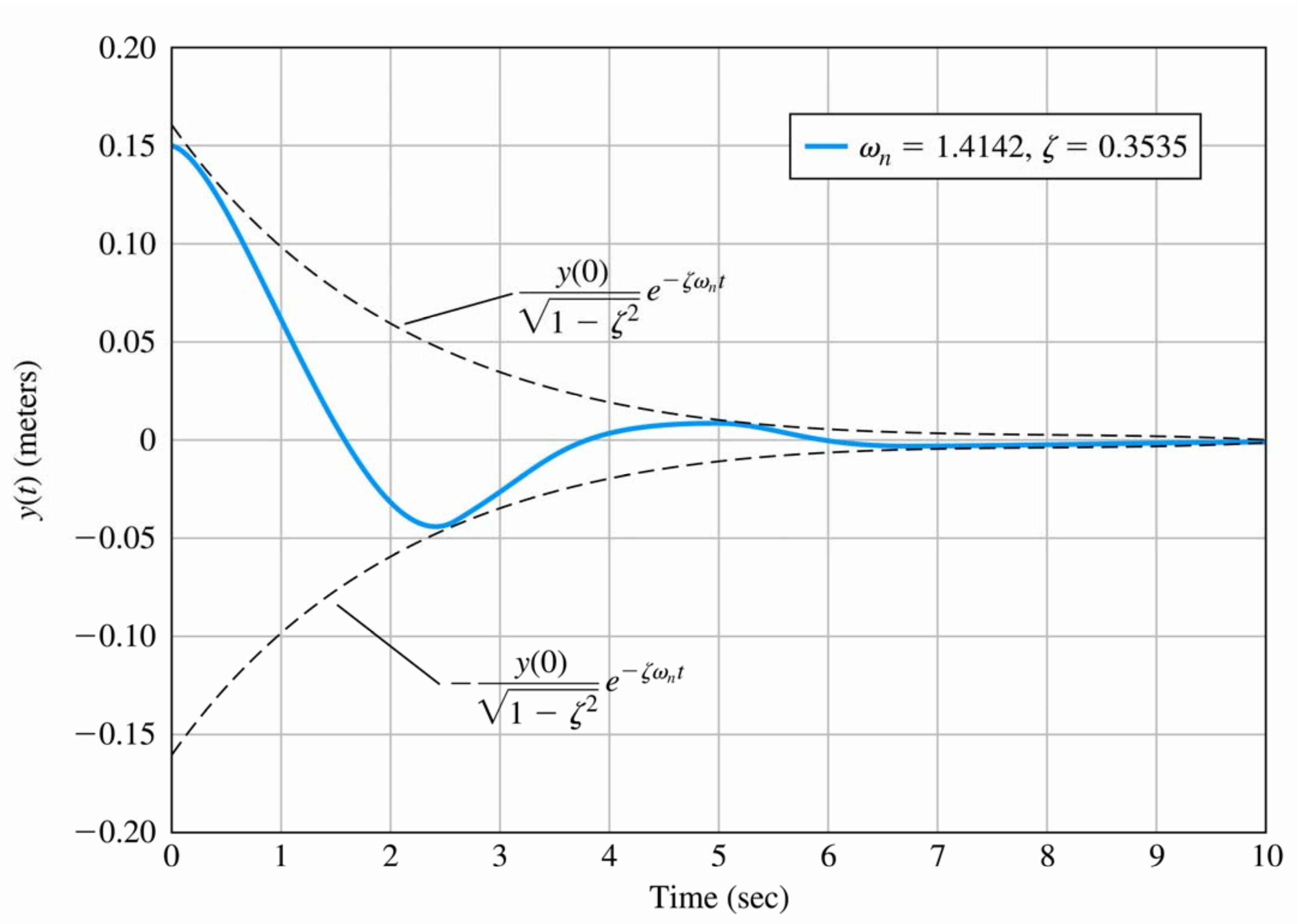


FIGURE 2.41 Spring-mass-damper unforced response.

Exercises

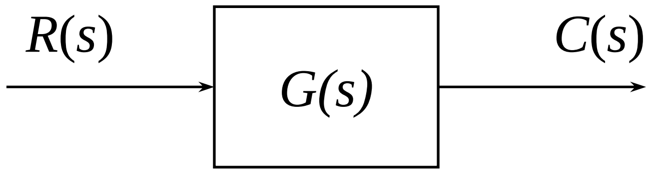
E2.8 E2.9

CHAPTER 4
Feedback Control System Characteristics

- **Open and Closed-Loop Control.**
- **Sensitivity of Control System to Parameter Variations.**
- **Control of the Transient Response of Control system.**
- **Disturbance Signals in a Feedback Control system.**
- **Steady-state Error.**
- **The cost of Feedback**

☑ Open-Loop and Closed-Loop Control Systems

- *Open-loop*



Transfer function: $G(s) = \dfrac{C(s)}{R(s)}$

Error signal: $E(s) = R(s) - C(s)$

Output: $C(s) = G(s)E(s) = G(s)[R(s) - C(s)]$

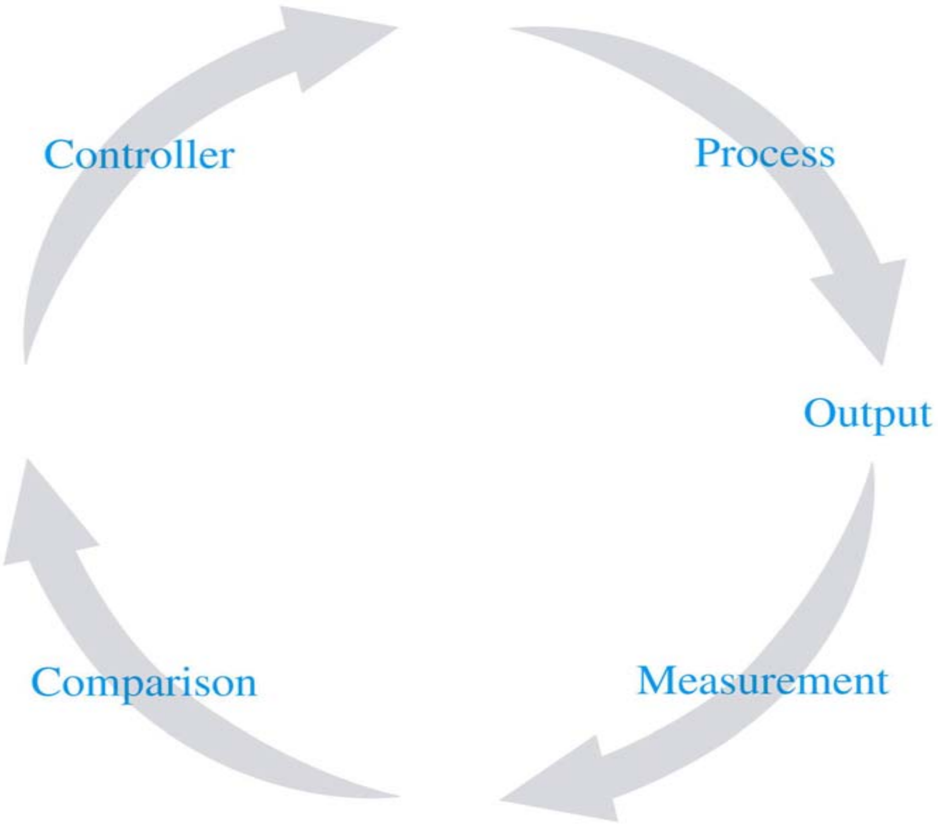
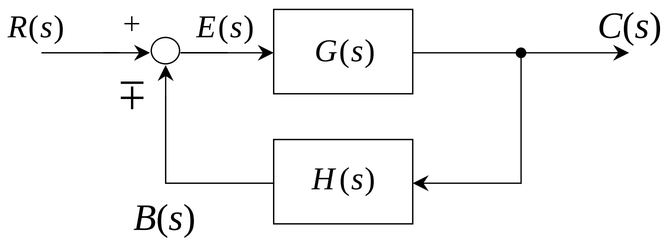


FIGURE 4.1

A closed-loop system.

- *Closed-loop*



$$\textit{Error signal:}\quad E(s) = R(s) - B(s)$$

$$\textit{Feedback signal:}\quad B(s) = H(s)C(s)$$

$$\textit{Output:}\quad C(s) = G(s)E(s) = G(s)[R(s) - B(s)]$$

$$\textit{Transfer function:}\quad T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

$$\textit{Error transfer function:}\quad \frac{E(s)}{R(s)} = \frac{1}{1 \pm G(s)H(s)}$$

Sensitivity of Control Systems to Parameter variations

Definite: ***System sensitivity is the ratio of the change in the system transfer function to the change of a process transfer function(or parameter) for a small incremental change***

$$
\begin{aligned}
\int_{G(s)}^{T(s)} &= \frac{\Delta T(s)/T(s)}{\Delta G(s)/G(s)} = \frac{\Delta T(s)}{\Delta G(s)} \times \frac{G(s)}{T(s)} \\
&= \lim_{\Delta \to 0} \frac{\Delta T(s)}{\Delta G(s)} \times \frac{G(s)}{T(s)} = \frac{\partial T(s)}{\partial G(s)} \times \frac{G(s)}{T(s)} \\
&= \frac{\partial \ln(T(s))}{\partial \ln(G(s))}
\end{aligned}
$$

The chain rule: $\int_{\alpha}^{T(s)} = \int_{G(s)}^{T(s)} \int_{\alpha}^{G(s)}$

- *Open-loop*

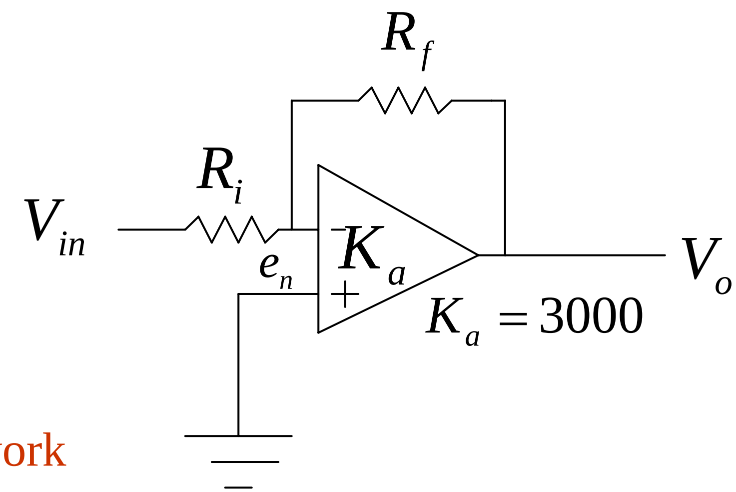
$$\text{Transfer function:} \quad T(s) = \frac{C(s)}{R(s)} = G(s)$$

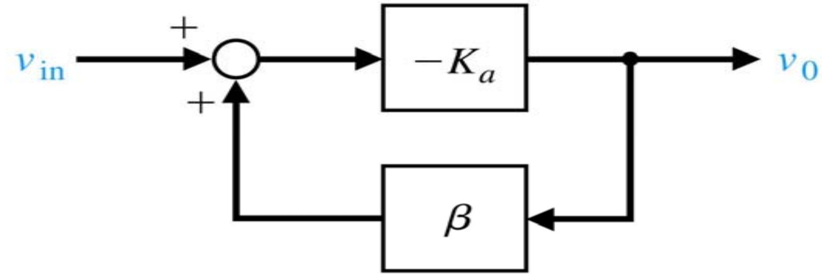
$$\int_{G(s)}^{T(s)} = \frac{\partial T(s)}{\partial G(s)} \cdot \frac{G(s)}{T(s)} = 1$$

- *Closed-loop*

$$\text{Transfer function:} \quad T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$\int_{G(s)}^{T(s)} = \frac{\partial T(s)}{\partial G(s)} \cdot \frac{G(s)}{T(s)} = \frac{1}{1+G(s)H(s)}$$





Homework

Answer the questions:

1. The necessary assumptions, such that the transfer function of the inverse amplifier can be obtained as follows: $\frac{V_o(s)}{V_{in}(s)} = -\frac{R_f}{R_{in}}$

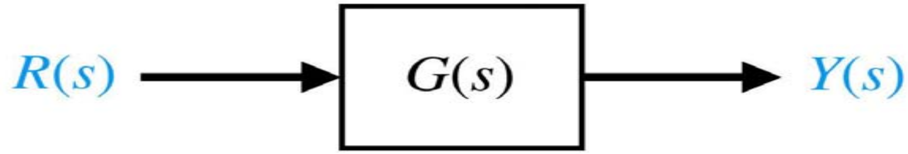
2. The largest ratio of the R_f and R_{in}, why?

3. $S_{A_o}^T$

4. $S_{R_f}^T$

Control of then Transient Response of Control System

- *Open-loop*



Transfer function: $T(s)=\dfrac{Y(s)}{R(s)}=\dfrac{b}{s+a}$

FIGURE 4.18

Open-loop control system (without feedback).

$$G(s)=\frac{b}{s+a},\quad R(s)=\frac{A}{s}$$

Output:

$$Y(s)=G(s)\cdot R(s)=\frac{b}{s+a}\cdot\frac{A}{s}$$

$$y(t)=L^{-1}\{Y(s)\}=\frac{Ab}{a}(1-e^{-at}),\quad \forall t\geq 0$$

Time constant: $\tau=\dfrac{1}{a}$ Settling time: 5τ

Error signal:

$$E(s) = R(s) - Y(s) = \frac{As + (a - Ab)}{s(s+a)}$$

$$e(t) = L^{-1}\{E(s)\} = \frac{a - Ab}{a} + \frac{Aa - (a - Ab)}{a} e^{-at}$$

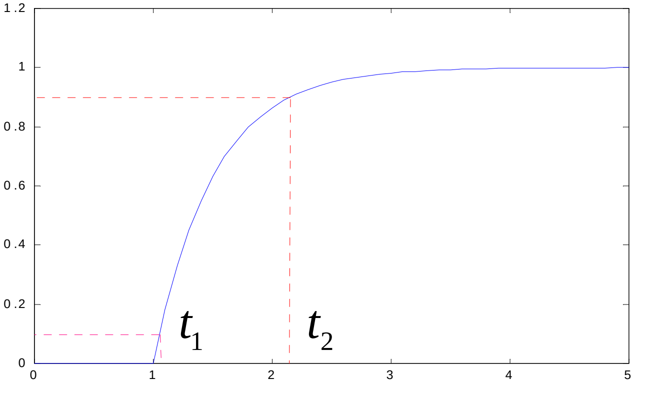
Rise time: $t_r = t_2 - t_1$

$$y(t)\big|_{t=t_2} = 0.9 \cdot Cy(\infty)$$

$$\Rightarrow t_2 = -\frac{1}{a}\ln(0.1)$$

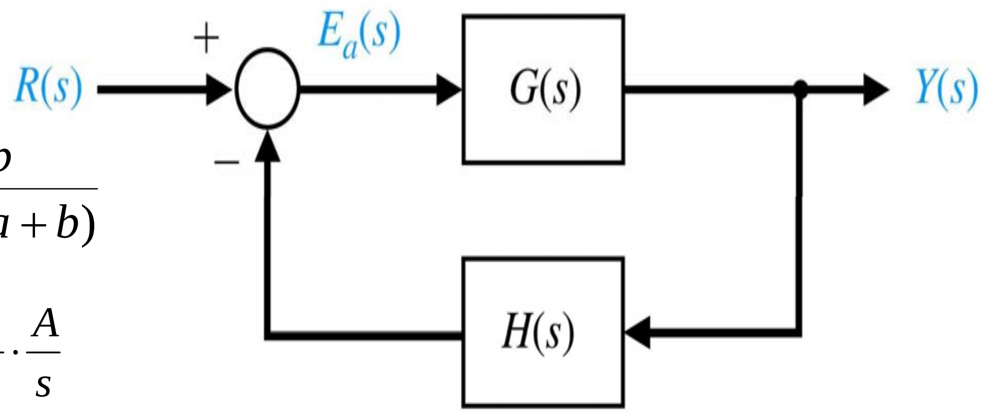
$$c(t)\big|_{t=t1} = 0.1 \cdot y(\infty)$$

$$\Rightarrow t_1 = -\frac{1}{a}\ln(0.9)$$



$$t_r = -\frac{1}{a}\ln(0.1) + \frac{1}{a}\ln(0.9)$$

- *Closed-loop* $(H(s)=1)$



Transfer function: $T(s)=\dfrac{C(s)}{R(s)}=\dfrac{b}{s+(a+b)}$

Output:

$$Y(s)=T(s)\cdot R(s)=\frac{b}{s+(a+b)}\cdot\frac{A}{s}$$

$$y(t)=L^{-1}\{Y(s)\}=\frac{Ab}{a+b}(1-e^{-(a+b)t}),\quad \forall t\ge 0$$

Time constant: $\tau=\dfrac{1}{a+b}$

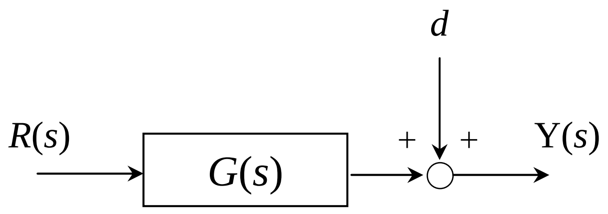
Error signal:

$$E(s)=R(s)-Y(s)=\frac{As+Aa}{s(s+a+b)}$$

$$e(t)=L^{-1}\{E(s)\}=\frac{Aa}{a+b}+\frac{Ab}{a+b}e^{-at}$$

Disturbance Signal in a Feedback Control System

- *Open-loop*

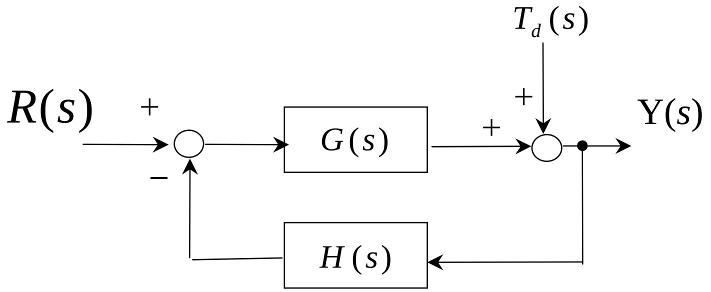


Output due to R(s): $Y_{R(s)}(s)|_{d=0} = G(s)\cdot R(s)$

Output due to d: $Y_d(s)|_{R=0} = 1\cdot d(s)$

Total output: $Y(s) = Y_{R(s)}(s) + Y_{d(s)}(s)$
$$= G(s)\cdot R(s) + d(s)$$

- *Closed-loop*



$$\textit{Output due to R(s):}\quad Y_{R(s)}(s)|_{T_d=0} = \frac{G(s)}{1+G(s)H(s)} \cdot R(s)$$

$$\textit{Output due to } T_d(s)\textit{:}\quad Y_{T_d}(s)|_{R=0} = \frac{1}{1+G(s)H(s)} \cdot T_d(s)$$

$$\textit{Total output:}\quad Y(s) = Y_R(s) + Y_{T_d}(s)$$

$$= \frac{G(s)}{1+G(s)H(s)} \cdot R(s) + \frac{1}{1+G(s)H(s)} \cdot T_d(s)$$

Homework #5

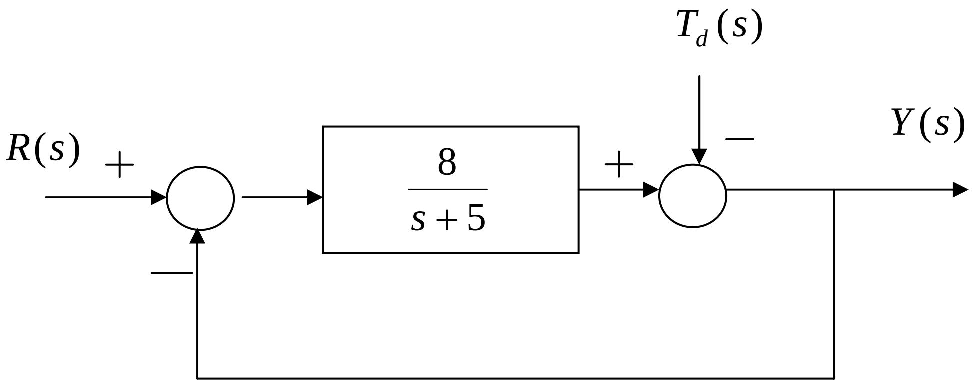
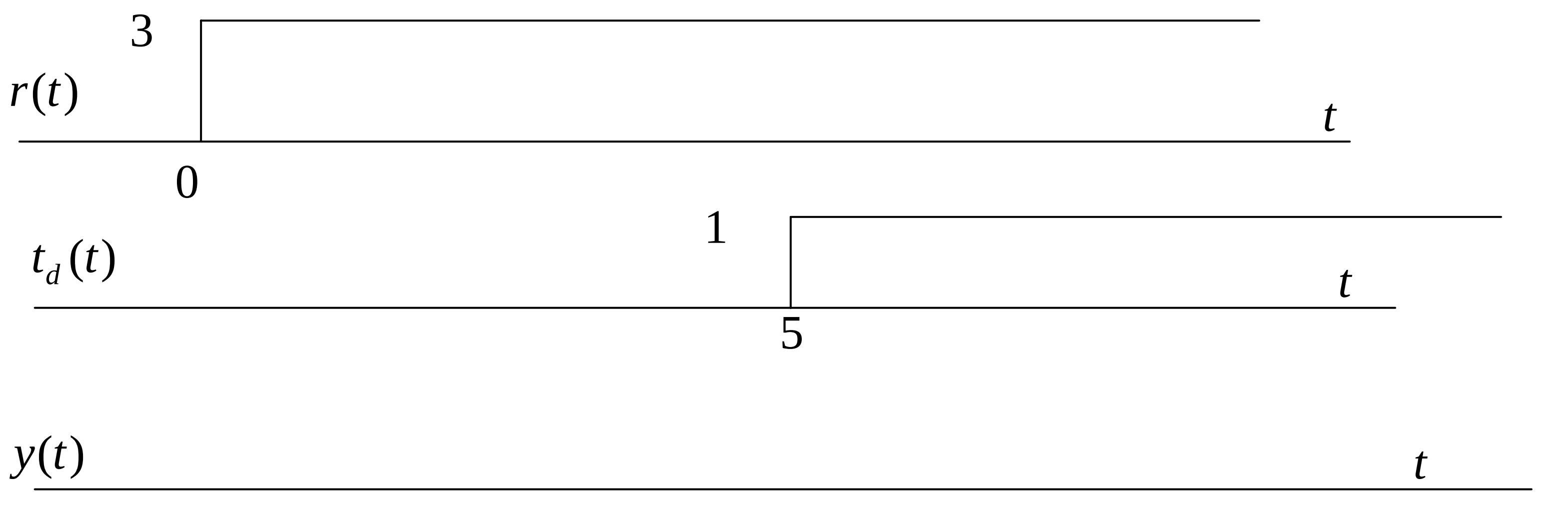


Fig. H5a



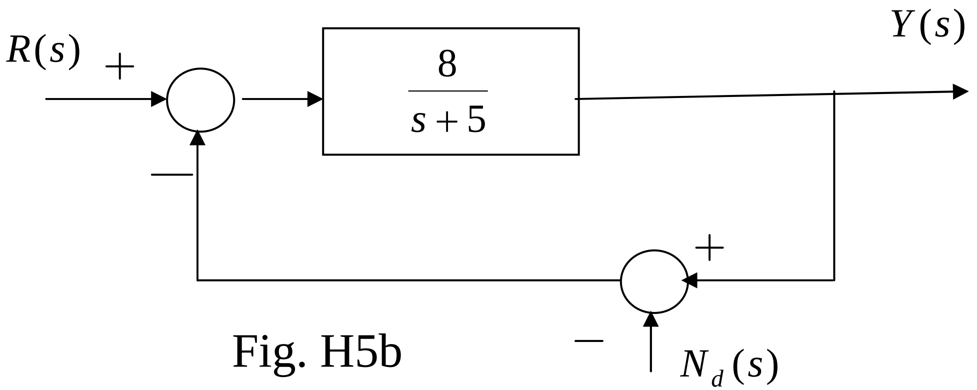
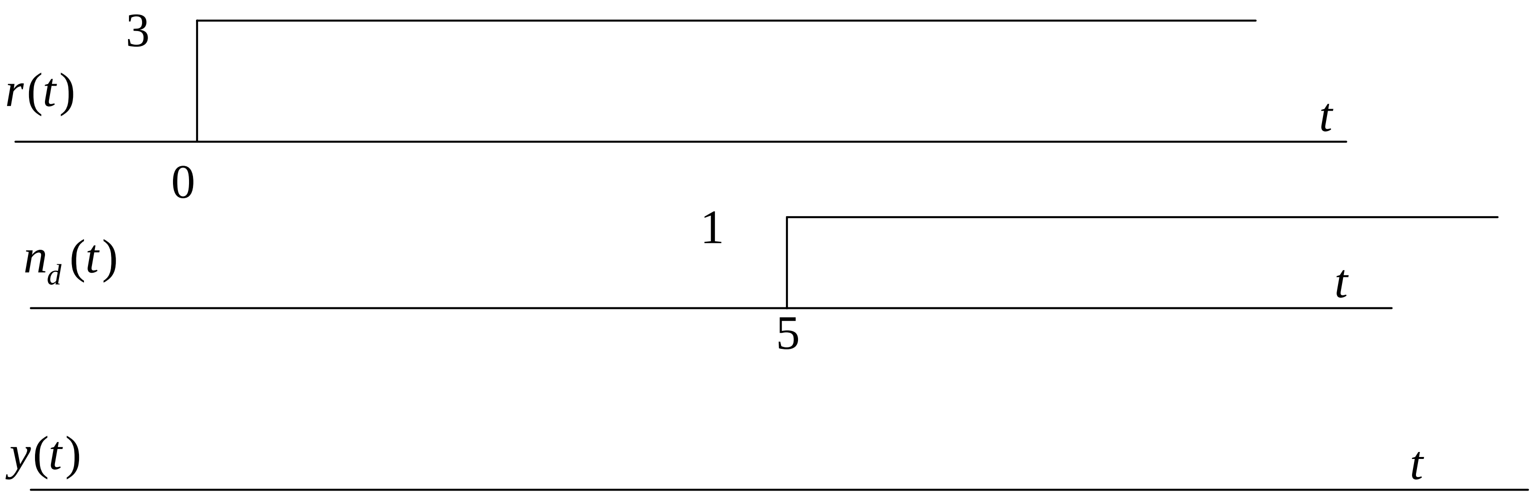
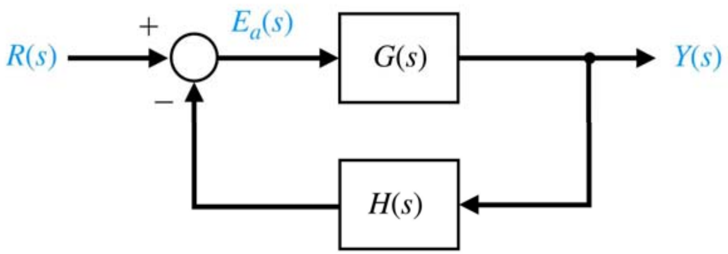


Fig. H5b



Steady-state Error



$$e_{ss} = e(t)\big|_{t\to\infty} = e(\infty)$$
$$= \lim_{t\to\infty} e(t) = \lim_{s\to 0} s \cdot E(s)$$

- *Open-loop*

Error signal: $E(s) = R(s) - Y(s) = (1 - G(s))R(s)$

Steady-state error: *if* $R(s) = \dfrac{A}{s}$

$$e_{ss} = \lim_{s\to 0}(1 - G(s))R(s) = A(1 - G(0))$$

- *Closed-loop*

Error signal: $E(s) = \dfrac{1}{1 + G(s)H(s)} R(s)$

Steady-state error: *if* $R(s) = \dfrac{A}{s}$ *when* $H(s) = 1$ $e_{ss} = \lim_{s\to 0} \dfrac{1}{1+G(s)}$

$$e_{ss} = \lim_{s\to 0}(sE(s)) = \frac{A}{1 + G(0)H(0)}$$

The Cost of Feedback System

1. Loss of gain
2. Increased number of components and complexity
3. Possibility of instability

- **Lab #1**
  - *Introduction to Matlab and Simulink*
  - *Simulation (Lab #3,4,5,6)*
  - *How can the Simulink be applied to measure the angular velocity and angular displacement in which an incremental encoder are used in control systems?*

Exercises:

E4.1 E4.3 E4.6 E4.7 P4.4 P4.14 AP4.6 AP4.7

CHAPTER 5

The Performance of Feedback Control Systems

- Introduction
- Test Input Signal
- Performance of a Second-order System
- The S-Plane Root Location and The Transient Response
- The Steady-state Error of Feedback Control System
- Performance Indices

Introduction

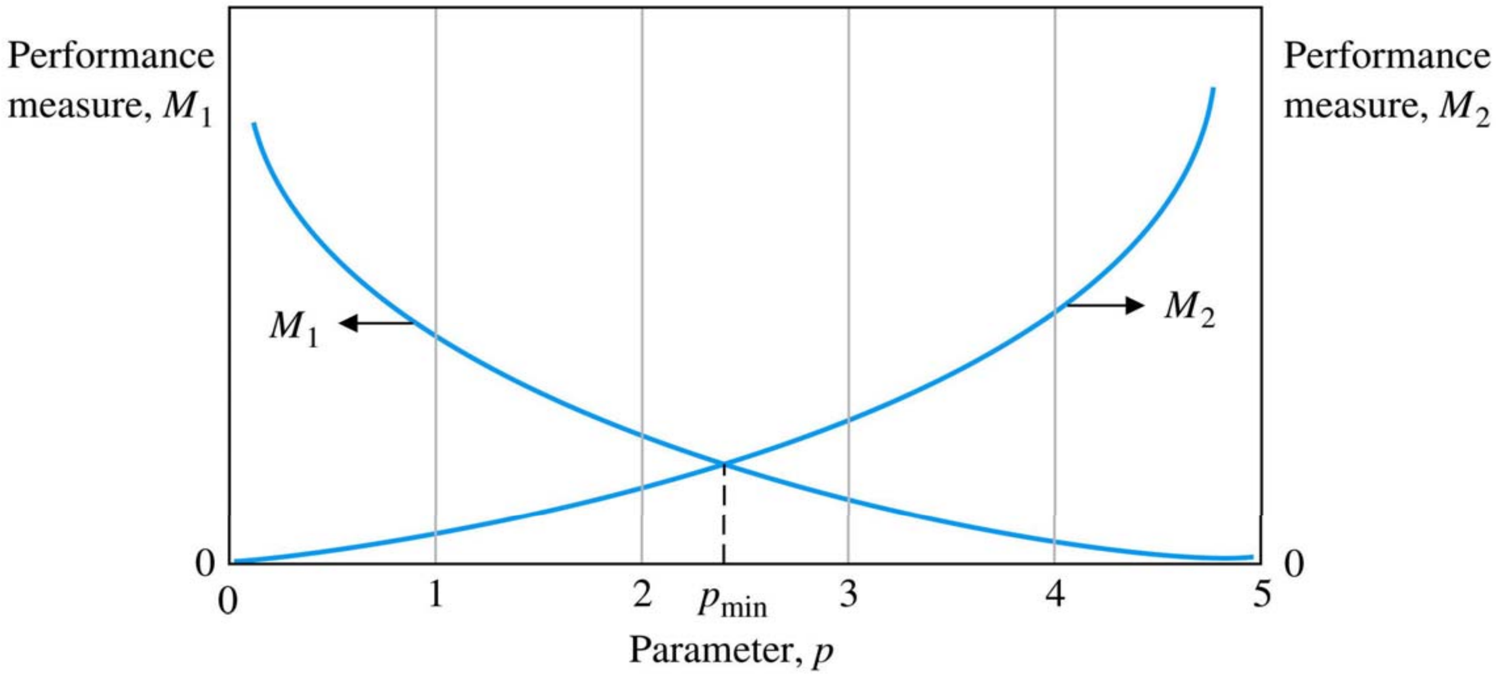


FIGURE 5.1

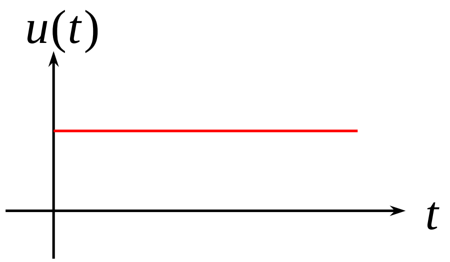
Two performance measures versus parameter p.

Test Input Signal

- *Step-function*

$$u(t) = \begin{cases} A & t \geq 0 \\ 0 & t < 0 \end{cases} \quad or \quad u(t) = A \cdot u_s(t)$$

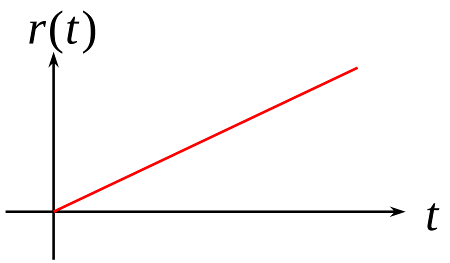
$$U(s) = L\{u(t)\} = \frac{A}{s}$$



- *Ramp-function*

$$r(t) = \begin{cases} At & t \geq 0 \\ 0 & t < 0 \end{cases} \quad or \quad r(t) = At \cdot u_s(t)$$

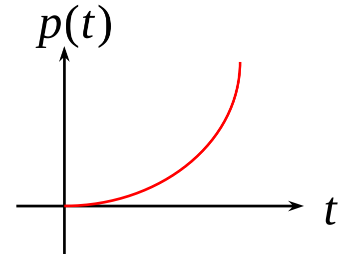
$$R(s) = L\{r(t)\} = \frac{A}{s^2}$$



- *Parabolic-function*

$$p(t) = \begin{cases} \frac{1}{2}At^2 & t \geq 0 \\ 0 & t < 0 \end{cases} \quad or \quad p(t) = \frac{1}{2}At^2 \cdot u_s(t)$$

$$P(s) = L\{p(t)\} = \frac{A}{s^3}$$



General Form

$$r(t) = A \cdot \frac{t^n}{n!} \cdot u_s(t) \Rightarrow R(s) = \frac{A}{s^{n+1}}$$

Performance of a Second-order System

$$Y(s)=\frac{G(s)}{1+G(s)}R(s)=\frac{K}{s^2+ps+K}R(s)=\frac{\omega_n^2}{s^2+2\xi\omega_n s}$$

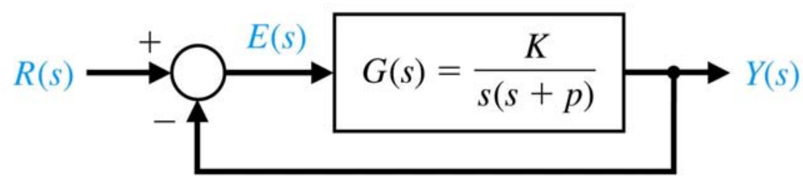


FIGURE 5.4

Closed-loop control system (with feedback).

With a unit step input, we obtain

$$Y(s)=\frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)}$$

ω_n is held constant while the damping ratio ζ is varied

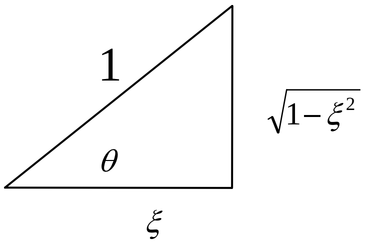
- $\zeta>1$: ***Overdamped*** $\Rightarrow s_{1,2}=-\zeta\omega_n\pm\omega_n\sqrt{\zeta^2-1}$
- $\zeta=1$: ***Critically damped*** $\Rightarrow s_{1,2}=-\omega_n$
- $0<\zeta<1$: ***Underdamped*** $\Rightarrow s_{1,2}=-\zeta\omega_n\pm j\omega_n\sqrt{1-\zeta^2}$
- $\zeta=0$: ***Undamped*** $\Rightarrow s_{1,2}=\pm j\omega_n$
- $\zeta<0$: ***Negatively damped*** $\Rightarrow s_{1,2}=-\zeta\omega_n\pm j\omega_n\sqrt{1-\zeta^2}$

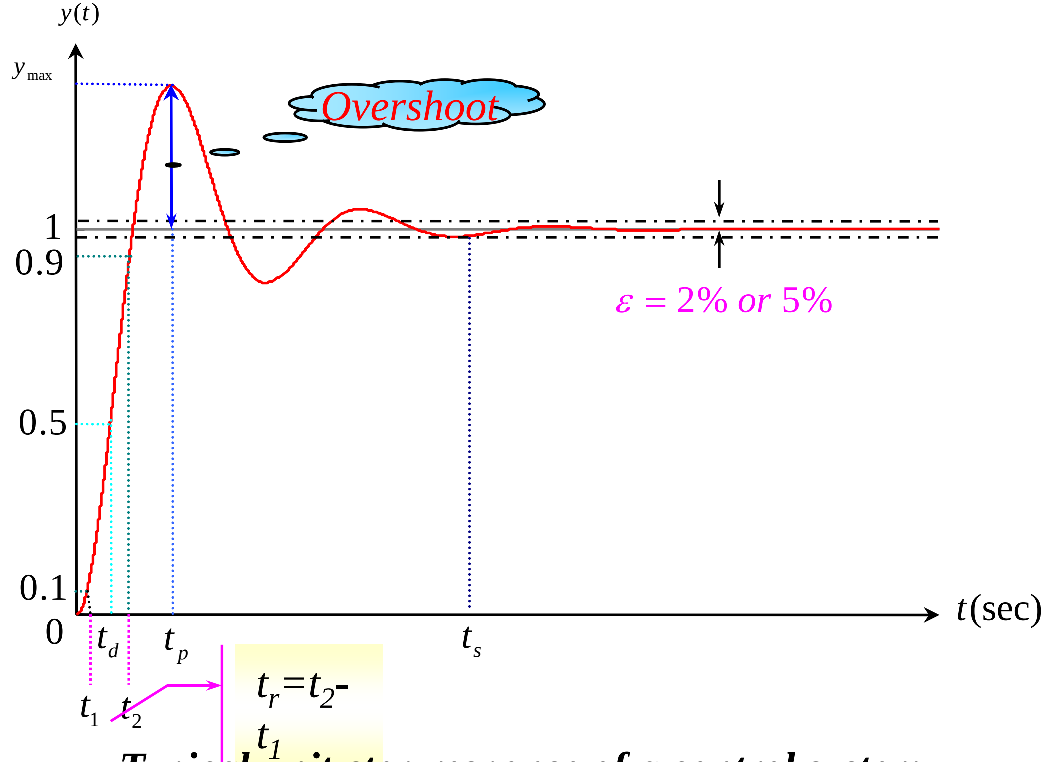
Performance of a Second-order System

$$Y(s)=\frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)}$$

$$when \quad 0<\xi<1$$

$$y(t)=1-\frac{1}{\beta}e^{-\xi\omega_n t}\sin(\omega_n\beta t+\theta) \quad \beta=\sqrt{1-\xi^2}$$





Typical unit-step response of a control system

$$\frac{dy(t)}{dt} = \frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin \omega_n \sqrt{1-\xi^2}\, t = 0$$

$$e^{-\xi\omega_n t} = 0 \quad \Rightarrow t \to \infty$$

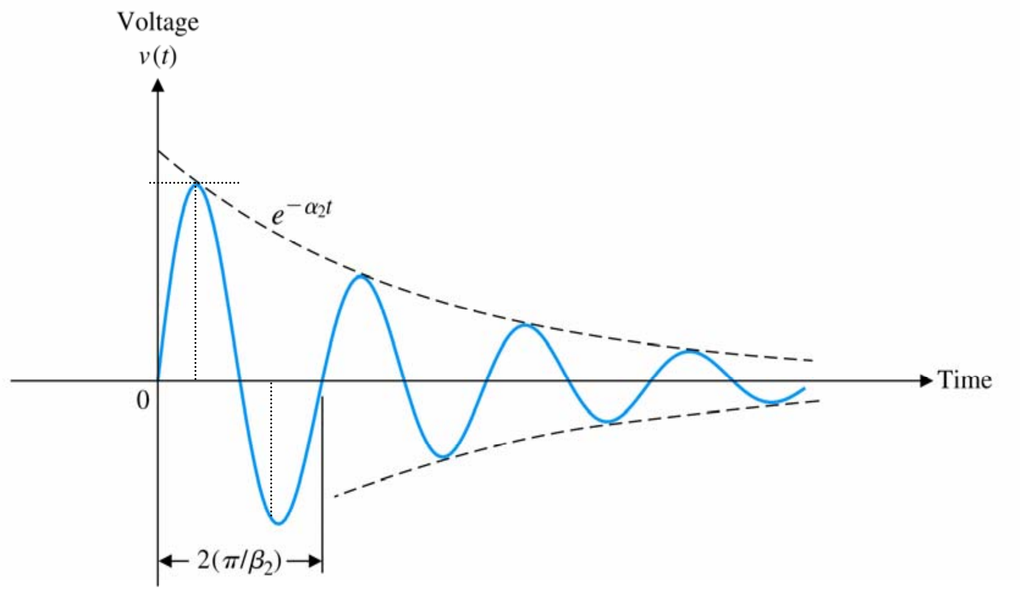
$$\sin \omega_n \sqrt{1-\xi^2}\, t = 0 \quad \Rightarrow \omega_n \sqrt{1-\xi^2}\, t = n\pi$$

$$t = \frac{n\pi}{\omega_n \sqrt{1-\xi^2}} \qquad n = 1 \to t = t_p$$

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\xi^2}}$$

$$M_p = 1 + e^{-\xi\pi/\sqrt{1-\xi^2}}$$

$$P.O = \frac{y_{\max} - y_{fin}}{y_{fin}} \times 100\% = e^{-\xi\pi/\sqrt{1-\xi^2}} \times 10$$

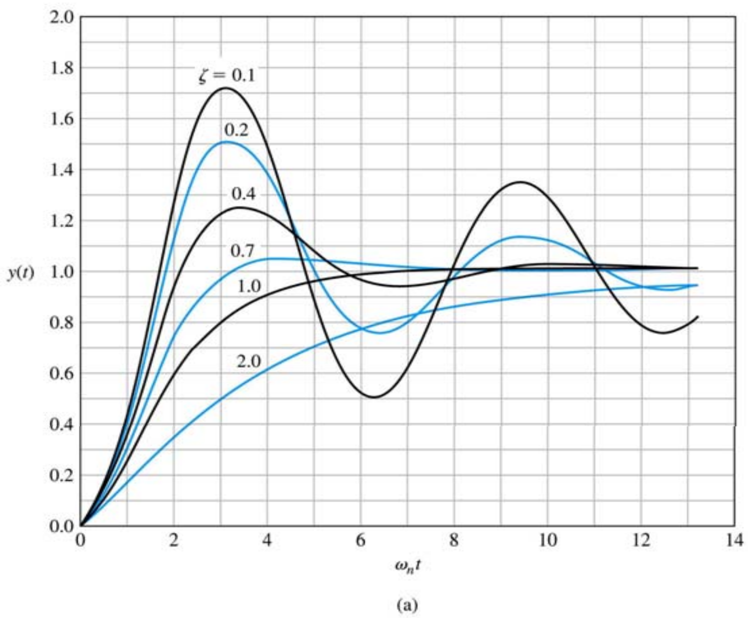


Settling Time:

$$\frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t_s} < 0.02$$

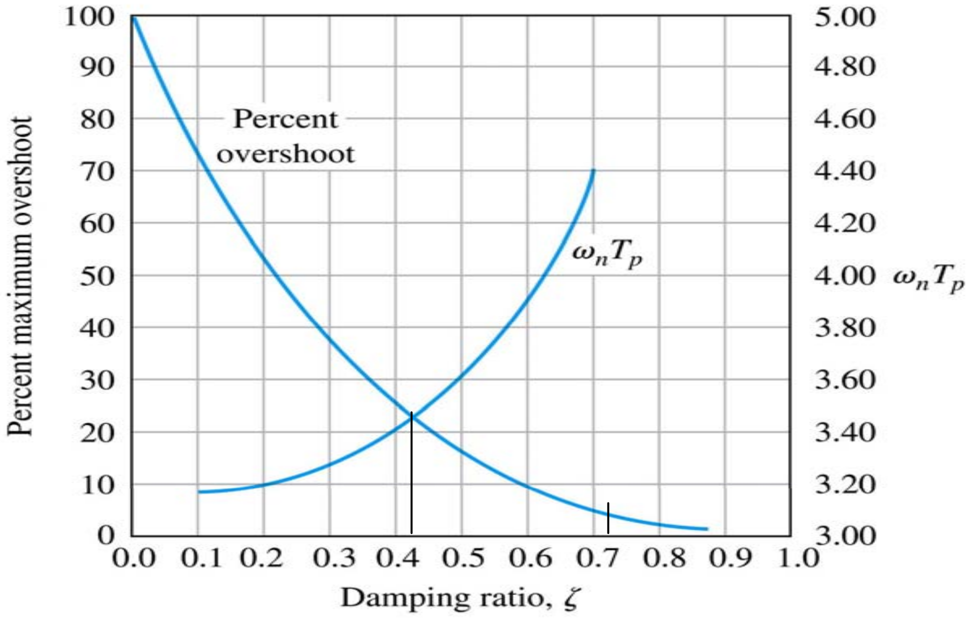
$$t_s = -\frac{1}{\xi\omega_n} \ln(0.02\sqrt{1-\xi^2}) \approx \frac{4}{\xi\omega_n}, \quad \text{for} \quad e_{ss} < 0.02$$

$$Damping \quad Factor: \alpha = \xi\omega_n$$



$\zeta = 0.1$
0.2
0.4
0.7
1.0
2.0
y(t)
$\omega_n t$

(a)



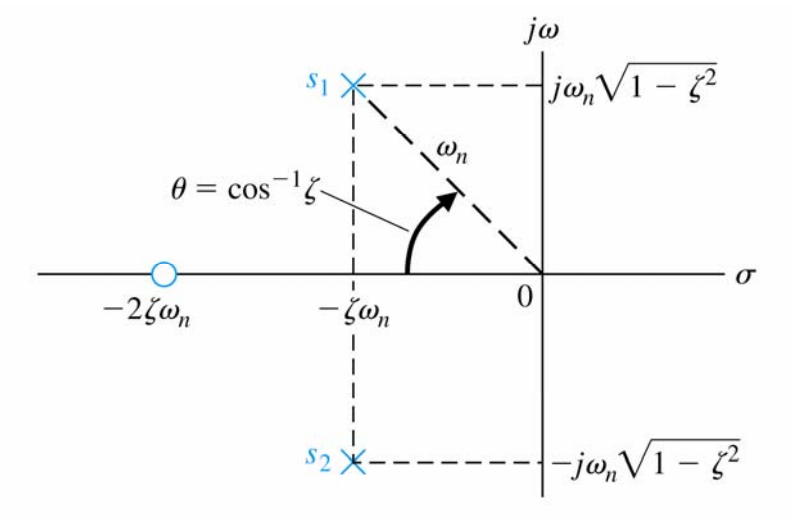
Percent overshoot
$\omega_n T_p$
Percent maximum overshoot
Damping ratio, ζ

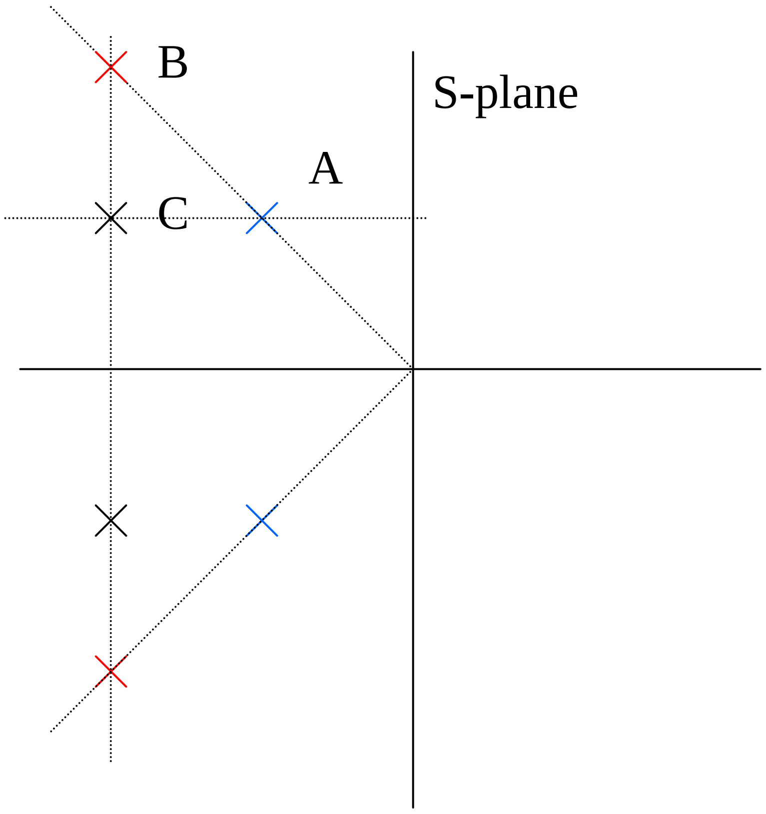
The S-plane Root Location and the Transient Response

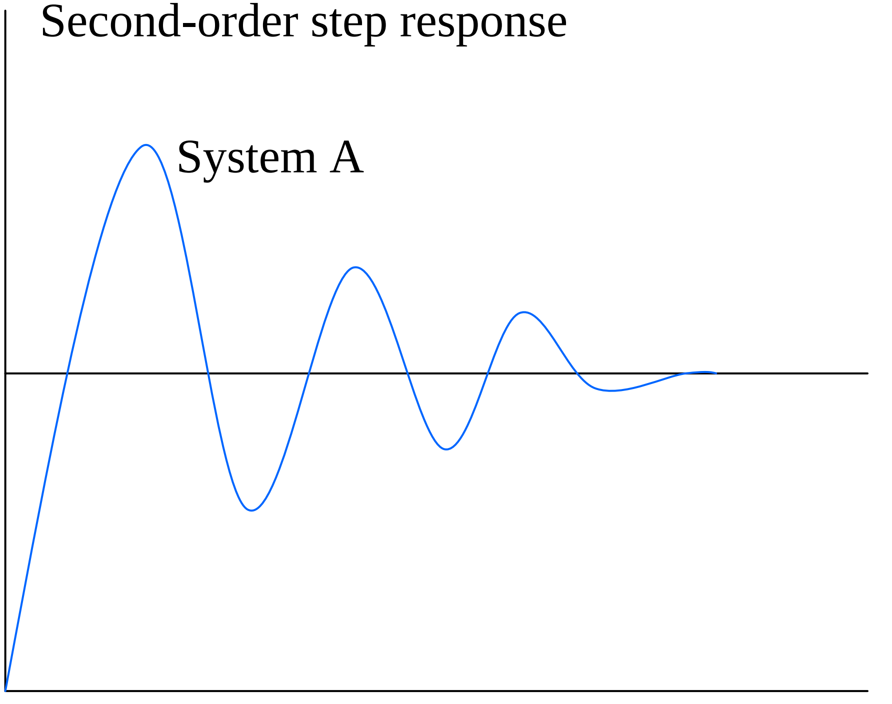
$\theta \uparrow \quad \downarrow \quad \uparrow \quad \Leftrightarrow$

$\alpha \uparrow \quad \downarrow \quad \Leftrightarrow$

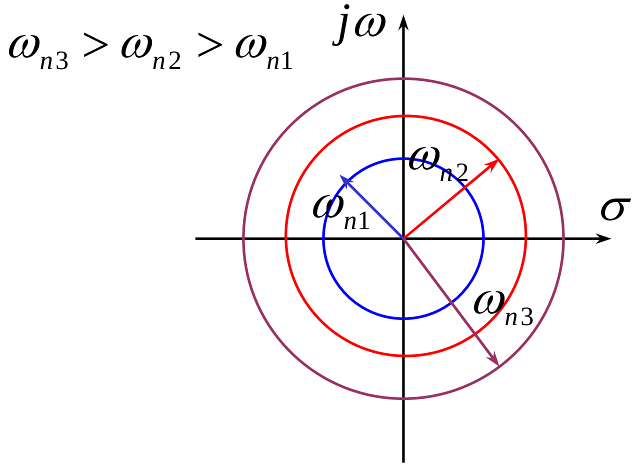
$\omega_d = \omega_n \sqrt{1-\xi^2} \uparrow \quad \uparrow \quad \Leftrightarrow$



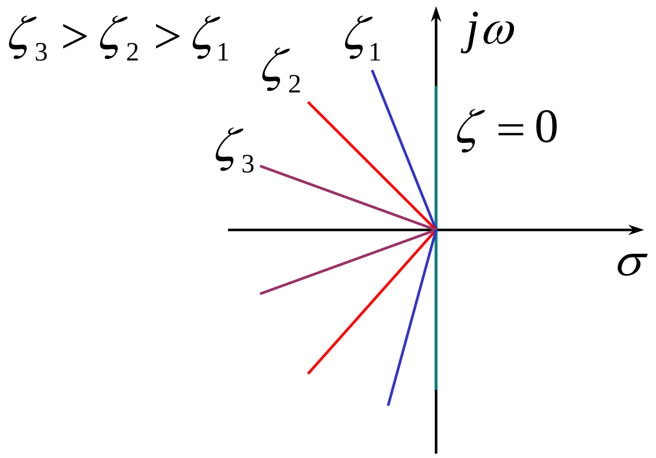




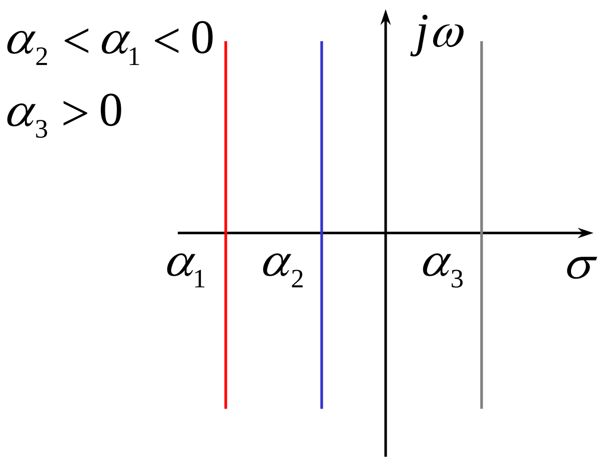
Draw the step responses of the system B and C,. respectively



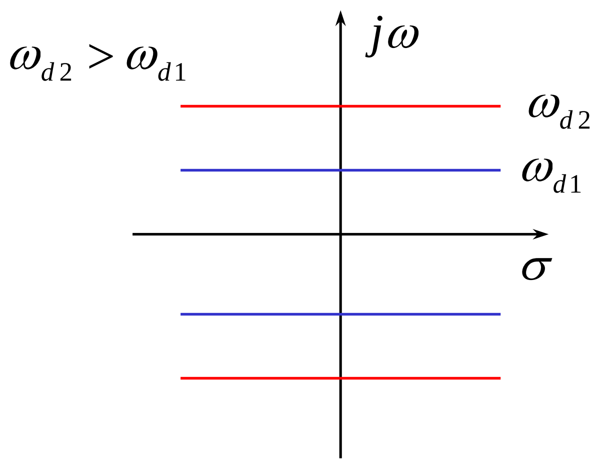
constant ω_n



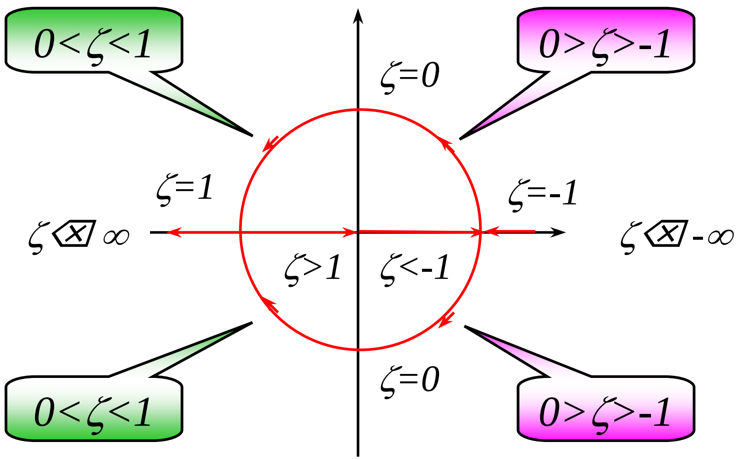
constant ζ

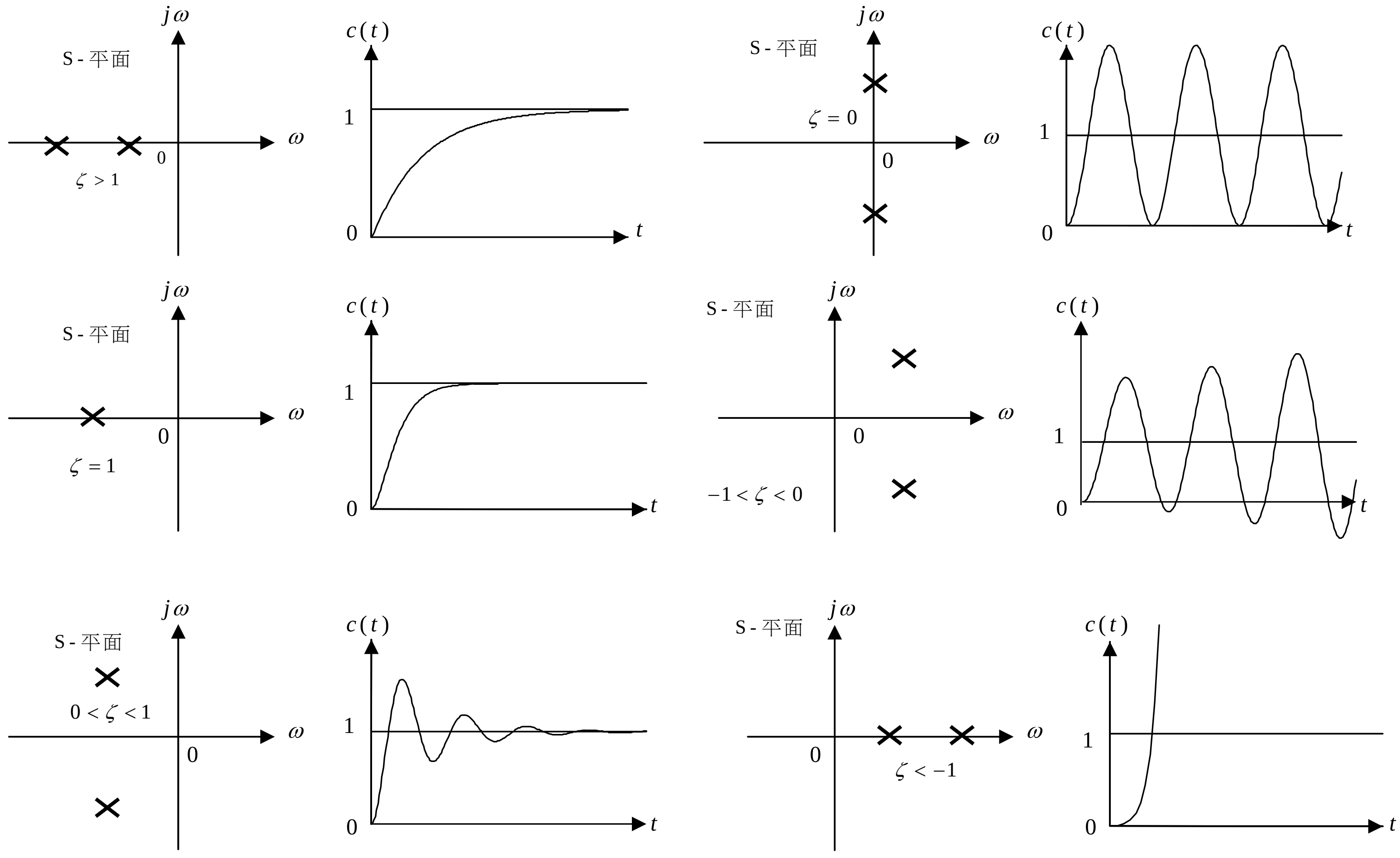


constant α



constant ω_d


0<ζ<1
0>ζ>-1
ζ=0
ζ=1
ζ=-1
ζ⌫∞
ζ⌫-∞
ζ>1
ζ<-1
ζ=0
0<ζ<1
0>ζ>-1



Step response

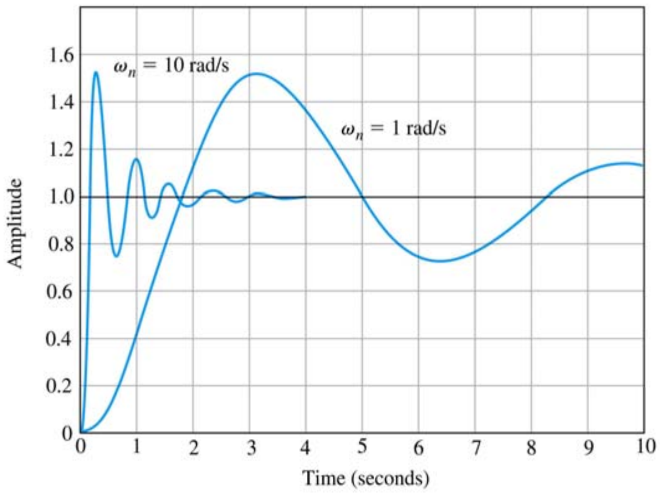


FIGURE 5.10

The step response for $\zeta = 0.2$ for $\omega_n = 1$ and $\omega_n = 10$.

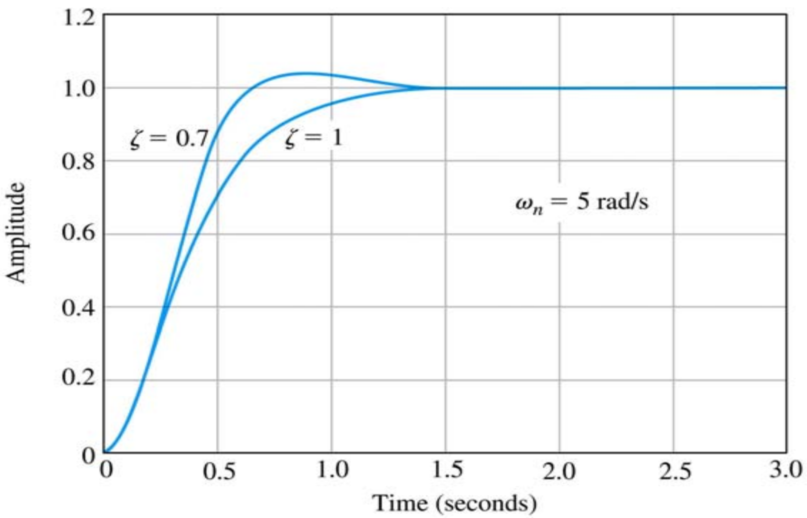
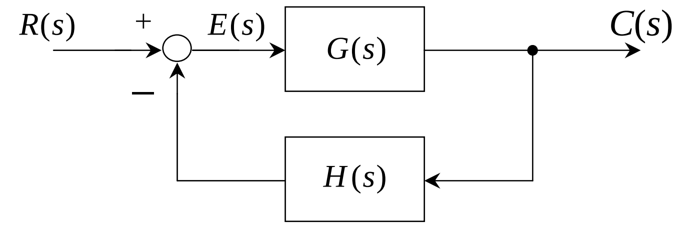


FIGURE 5.11

The step response for $\omega_n = 5$ with $\zeta = 0.7$ and $\zeta = 1$.

The Steady-state Error of Feedback Control System

- *Type of Control Systems*



Loop transfer function:

$$G(s)H(s)=\frac{Q(s)}{P(s)}$$

$$=\frac{b_m s^m+b_{m-1}s^{m-1}+\cdots+b_1 s+b_0}{a_n s^n+a_{n-1}s^{n-1}+a_{n-2}s^{n-2}+\cdots+a_1 s+a_0}$$

$$=\frac{K\prod_{j=1}^{m}(s+z_j)}{s^N\prod_{i=1}^{n-N}(s+p_i)}$$

- The loop transfer function as s approaches zero depends on the number of integrations *N*.
- The number of integrations is often indicated by labeling a system with a *type number* that simply is equal to *N*.

Error function: $E(s) = \dfrac{1}{1+G(s)H(s)} \cdot R(s)$

Steady-state error: $e_{ss} = \lim_{s\to 0} s \cdot E(s)$

if $H(s) = 1$

☞ ***Step-function input***

$$e_{ss} = \lim_{s\to 0} s \cdot \frac{1}{1+G(s)} \cdot \frac{A}{s}$$

$$= \frac{A}{1+\lim_{s\to 0} G(s)}$$

Define the position error constant as: $K_p = \lim_{s\to 0} G(s)$

$$\Rightarrow \quad e_{ss} = \frac{A}{1+K_p}$$

☞ ***Ramp-function input***

$$e_{ss} = \lim_{s\to 0} s \cdot \frac{1}{1+G(s)} \cdot \frac{A}{s^2} = \frac{A}{\lim_{s\to 0} sG(s)}$$

Define the velocity error constant as: $K_v = \lim_{s\to 0} sG(s)$

$$\Rightarrow \quad e_{ss} = \frac{A}{K_v}$$

☞ ***Parabolic-function input***

$$e_{ss} = \lim_{s\to 0} s \cdot \frac{1}{1+G(s)} \cdot \frac{A}{s^3} = \frac{A}{\lim_{s\to 0} s^2 G(s)}$$

Define the acceleration error constant as: $K_a = \lim_{s\to 0} s^2 G(s)$

$$\Rightarrow \quad e_{ss} = \frac{A}{K_a}$$

◎ Summary of steady-state errors (unit feedback)

Type of system N	Error Constant			Steady-state error e_{ss}		
	K_p	K_v	K_a	$\frac{A}{1+K_p}$	$\frac{A}{K_v}$	$\frac{A}{K_a}$
0	K	0	0	Constant	∞	∞
1	∞	K	0	0	Constant	∞
2	∞	∞	K	0	0	Constant
3	∞	∞	∞	0	0	0

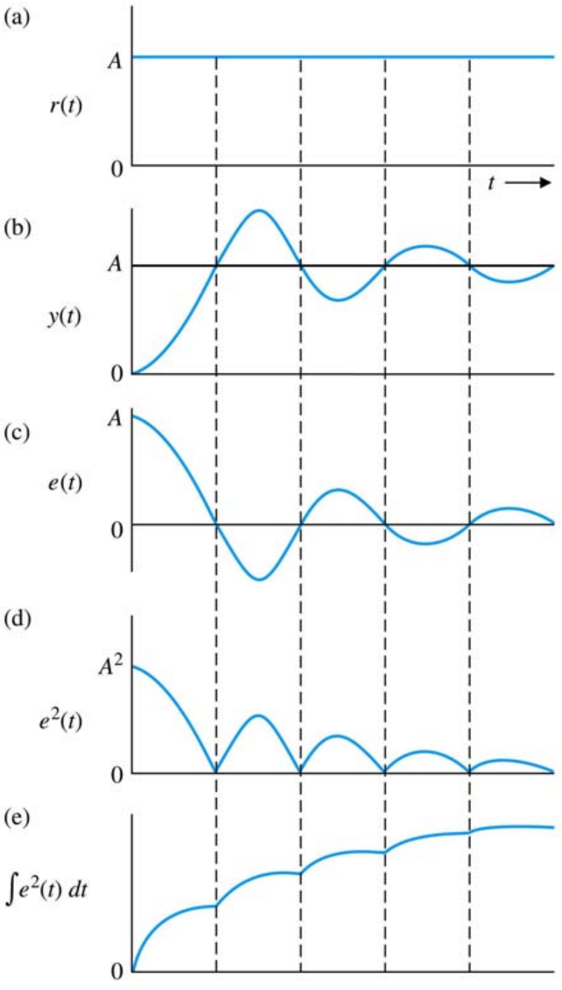
Performance indices

A performance index is a quantitative measure of the performance of a system and is chosen so that emphasis is given to the important system specifications.

A system is consider an optimum control system when the system parameters are adjusted so that the index reaches an extreme value, commonly a minimum value.

The Integral of the Square of the Error:

$$ISE = \int_0^T e^2 dt$$



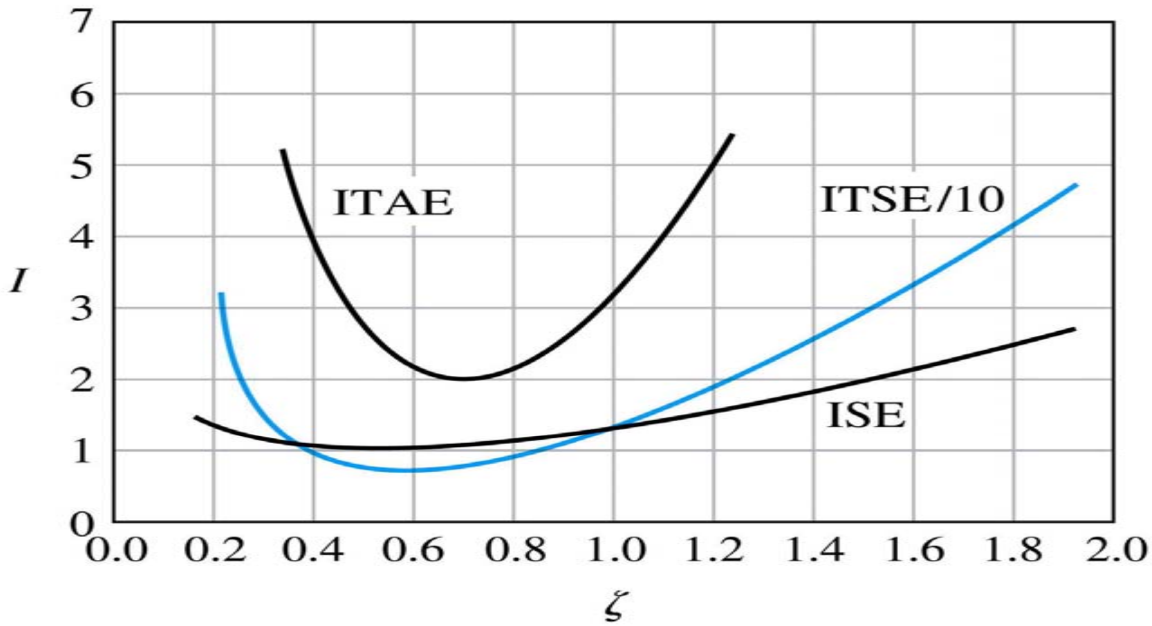


FIGURE 5.27

Three performance criteria for a second-order system.

CHAPTER 6

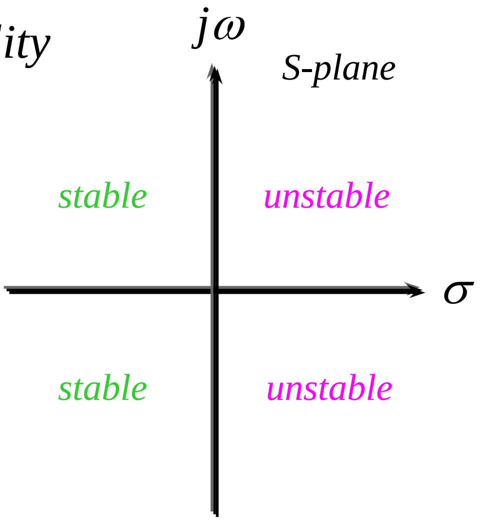
The Stability of Linear Feedback Systems

- The Concept of Stability
- The Routh-Hurwitz Stability Criterion
- The Relative Stability of Feedback Control Systems

The Concept of Stability

- *Stability* →
  - *Absolute stability*
  - *Relative stability*

- *The methods for the determination of stability of linear continuous-time systems:*
  - *Routh-Hurwitz criterion*
  - *Nyquist criterion*
  - *Bode diagram*



A stable system is a dynamic system with a bounded response to a bounded input

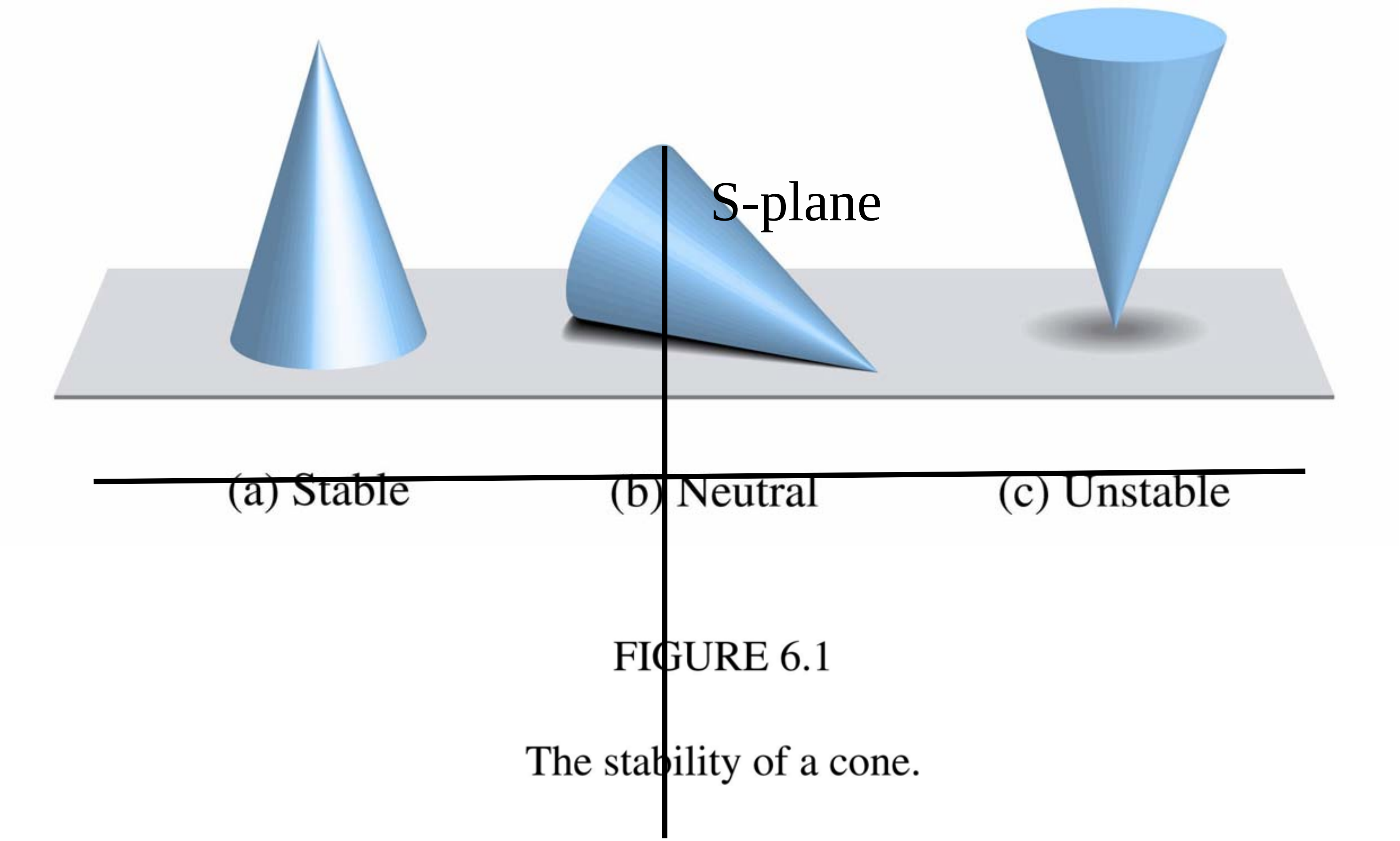


FIGURE 6.1

The stability of a cone.

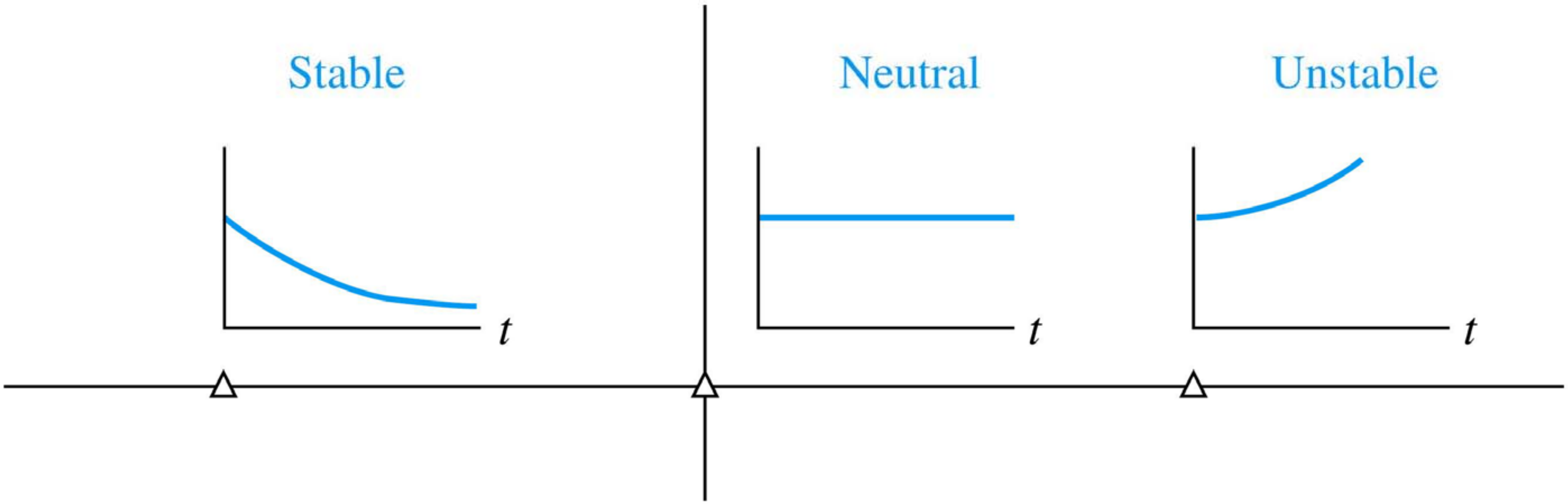
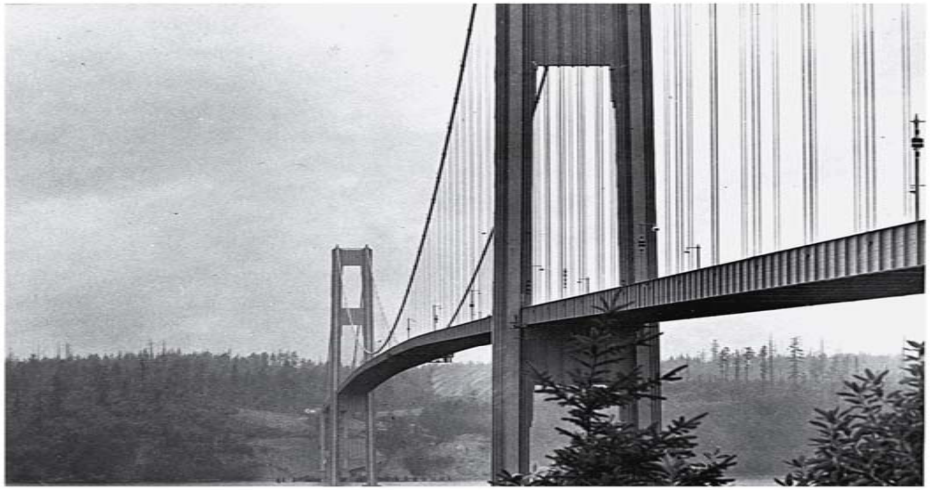


FIGURE 6.2

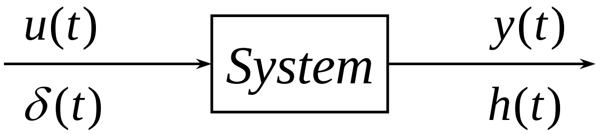
Stability in the *s*-plane.



(a)

(b)

- *Bounded-input bounded-output (**BIBO**) stability:*



$$y(t) = \int_0^{\infty} u(t-\tau)\cdot h(\tau)d\tau$$

$$|y(t)| = \left|\int_0^{\infty} u(t-\tau)\cdot h(\tau)d\tau\right| \Rightarrow |y(t)| \le \int_0^{\infty} |u(t-\tau)|\cdot|h(\tau)|d\tau$$

If u(t) is bounded, $|u(t)| \le M \quad \Rightarrow |y(t)| \le M\int_0^{\infty} |h(\tau)|d\tau$

Thus, if y(t) is to be bounded, or $|y(t)| \le N < \infty$

$$\Rightarrow M\int_0^{\infty} |h(\tau)|d\tau \le N < \infty$$

$$\Leftrightarrow \int_0^{\infty} |h(\tau)|d\tau \le Q < \infty$$

M, N *and* Q *are finite positive number*

☑ The *Routh-Hurwitz* Stability Criterion

Consider the characteristic equation of a linear SISO system

$$q(s) = a_n(s-r_1)(s-r_2).....(s-r_n) = 0$$

$$= a_n s^n - a_n(r_1 + r_2 + ... + r_n)s^{n-1} + a_n(r_1 r_2 + r_2 r_3 + ...)s^{n-2}$$

$$+ a_n(r_1 r_2 r_3 + r_1 r_2 r_4 + ...)s^{n-3} + ... + a_n(-1)^n r_1 r_2 ... = 0$$

$$q(s) = s^n + \frac{a_{n-1}}{a_n}s^{n-1} + \cdots + \frac{a_1}{a_n}s + \frac{a_0}{a_n} = 0$$

$$\frac{a_{n-1}}{a_n} = -\sum \textit{all roots}$$

$$\frac{a_{n-2}}{a_n} = \sum \textit{products of the roots taken two at a time}$$

$$\frac{a_{n-3}}{a_n} = -\sum \textit{products of the roots taken three at a time}$$

$$\vdots$$

$$\frac{a_0}{a_n} = (-1)^n \textit{ products of the roots}$$

The necessary condition to guarantee that all roots of q(s)=0 with negative real part are:

- ***All the coefficients of the equation have the the same sign.***
- ***None of the coefficients vanishes.***

Necessary but not sufficient!

$$q(s) = a_n s^n + a_{n-1} s^{n-1} + \dots\dots + a_1 s + a_0 = 0$$

+	s^n	a^n	a^{n-2}	a^{n-4}	a^{n-6}......
+	s^{n-1}	a^{n-1}	a^{n-3}	a^{n-5}	a^{n-7}....
+	s^{n-2}	b_1	b_2	b_3	
—	s^{n-3}	c_1			
+	.	.			
	.	.			
	s	.			
		.			
	s^0				

$$b_1 = \frac{a^{n-1}a^{n-2} - a^n a^{n-3}}{a_n} \qquad b_2 = \frac{a^{n-1}a^{n-4} - a^n a^{n-5}}{a_n}$$

$$c_1 = \frac{b_1 a^{n-3} - a^{n-1} b_2}{b_1}$$

The Routh-Hurwitz criterion states that the number of roots of q(s) with positive real parts is equal to the number of changes in sign of the first column of the Routh array

Case1:No element in the first column is zero

Second-order system

$$q(s)=a_2s^2+a_1s+a_0=0$$

s^2	a_2	a_0
s^1	a_1	0
s^0	$b_1=\dfrac{a_1a_0-a_2 0}{a_1}=a_0$	

The system is stable if all the coefficients have the same sign.

Third-order system

$$q(s)=a_3s^3+a_2s^2+a_1s+a_0=0$$

s^3	a_3	a_1
s^2	a_2	a_0
s^1	$b_1=\dfrac{a_2a_1-a_3a_0}{a_2}$	0
s^0	a_0	

set $a_3 \sim a_0 > 0$

The system is stable if $a_2a_1 > a_3a_0$

Case 2: Zeros in the first column while some other elements of the row containing a zero in the first column are nonzero.

$$q(s)=s^5+2s^4+2s^3+4s^2+11s+10=0$$

$\varepsilon>0$

+	s^5	1	2	11
+	s^4	2	4	10
+	s^3	$0 \to \varepsilon$	6	0
−	s^2	c_1	10	0
+	s^1	d_1	0	0
+	s^0	10	0	0

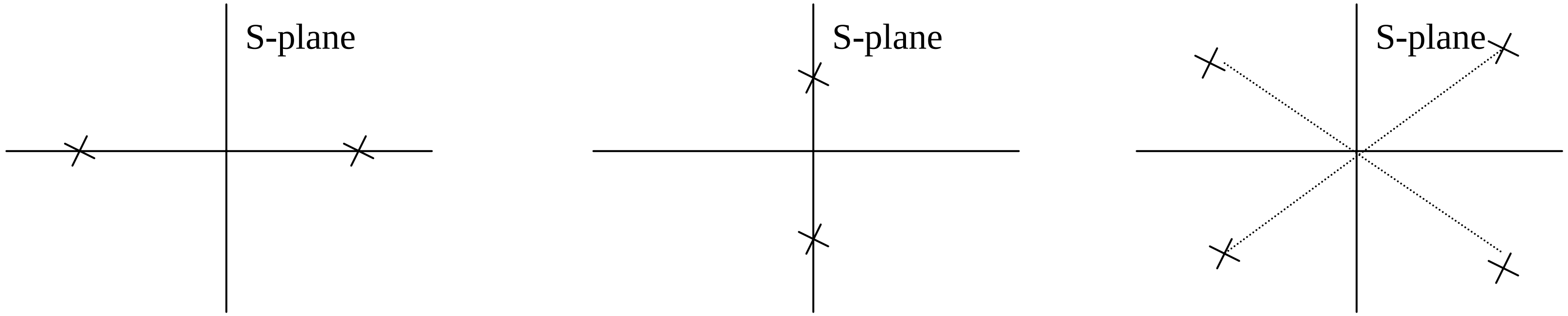
$$c_1=\frac{4\varepsilon-12}{\varepsilon}\approx-\frac{12}{\varepsilon}\approx-\infty$$

$$d_1\frac{6c_1-10\varepsilon}{c_1}\approx 6$$

$$F(s)\Big|_{s=\frac{1}{x}}=\tfrac{1}{x^5}+2\tfrac{1}{x^4}+2\tfrac{1}{x^3}+4\tfrac{1}{x^2}+11\tfrac{1}{x^1}+10$$
$$=\tfrac{1}{x^5}\cdot(1+2x^1+2x^2+4x^3+11x^4+10x^5)$$
$$\equiv\tfrac{1}{x^5}\cdot F(x)$$

Case 3: Zeros in the first column, and the other element of the row containing the zero are also zero.

This condition occurs when the polynomial contains singularities that are symmetrically located about the origin of the s-plane.



$$q(s) = s^3 + 2s^2 + 4s + 8 = 0$$

s^3	1	4
s^2	2	8
s^1	0	0
s^0		

$$A(s) = 2s^2 + 8 = 0$$
$$= s^2 + 4 = 0$$
$$s = \pm j2$$

(1)

$$\begin{array}{r|l} & s+2 \\ \hline s^2+4 & s^3 + 2s^2 + 4s + 8 \\ & \underline{s^3 \qquad\quad + 4s} \\ & \quad 2s^2 \qquad + 8 \\ & \quad \underline{2s^2 \qquad + 8} \end{array}$$

$$q(s) = (s+2)(s+j2)(s-j2) = 0$$

The system has not any pole on the RHP,
but system is marginally stable.

(2)

$$\frac{dA(s)}{ds} = 2s$$

s^3	1	4
s^2	2	8
s^1	2	0
s^0	8	

Case 4: Repeated roots of the characteristic equation on the imaginary axis.

(2)

$$q(s) = s^5 + s^4 + 2s^3 + 2s^2 + s + 1 = 0$$

s^5	1	2	1
s^4	1	2	1
s^3	0	0	0
s^2			

$$A(s) = s^4 + 2s^2 + 1 = (s^2+1)^2 = 0$$

$$s = j, j, -j, -j$$

.

s^5	1	2	1
s^4	1	2	1
s^3	4	4	0
s^2	1	1	0
s^1	0	0	0

$$\frac{dA(s)}{ds} = 4s^3 + 4s$$

.

The resulted of Routh array is Falsely indicated.

(1)

$$A(s) = s^4 + 2s^2 + 1 = (s^2+1)^2 = 0$$

$$s = j, j, -j, -j$$

$$\begin{array}{r|l} & s+1 \\ \hline s^4+2s^2+1 & s^5 + s^4 + 2s^3 + 2s^2 + s + 1 \\ & \underline{s^5 \quad + 2s^3 \quad + s} \\ & s^4 \quad + 2s^2 \quad + 1 \\ & \underline{s^4 \quad + 2s^2 \quad + 1} \end{array}$$

The system has not any root on the RHP, but the system is unstable.

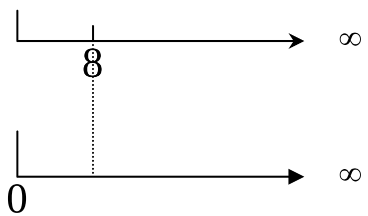
Repeated roots on the imaginary axis

$$q(s) = s^3 + 2s^2 + 4s + k = 0$$

s^3	1	4
s^2	2	k
s^1	$\dfrac{8-k}{2}$	0
s^0	k	

(1) $k > 8$

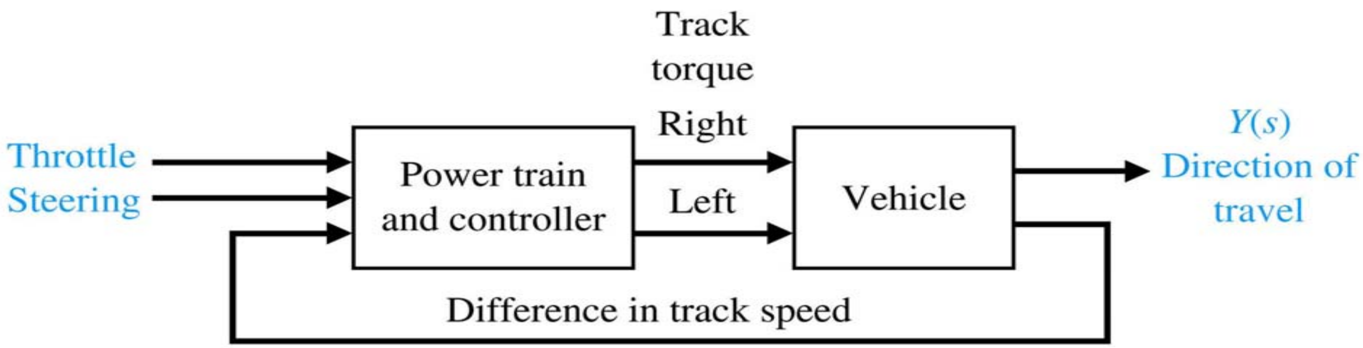
(2) $k > 0$



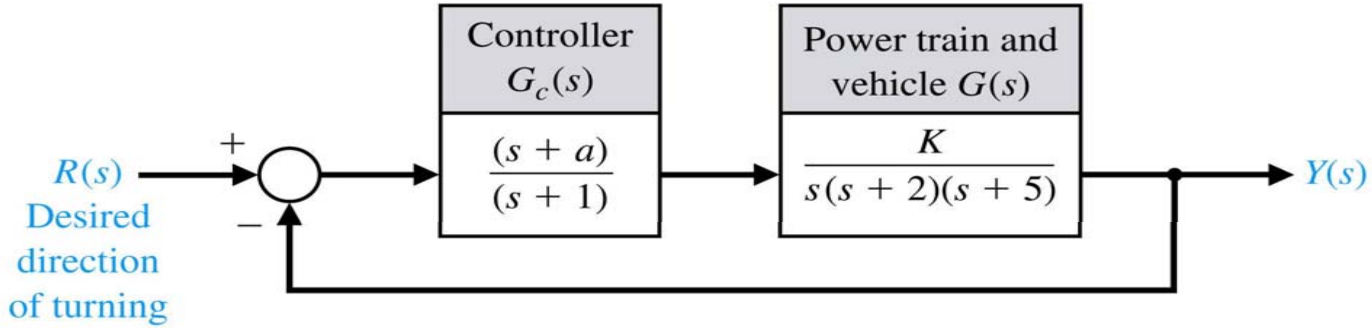
Ans: k>8, system is stable.

$$q(s) = s^3 + 3ks^2 + (k+2)s + 4 = 0$$

Ans: k>0.528, system is stable.


Track
torque
Right
Left
Throttle
Steering
Power train
and controller
Vehicle
$Y(s)$
Direction of
travel
Difference in track speed

(a)


Controller
$G_c(s)$
$\frac{(s+a)}{(s+1)}$
Power train and
vehicle $G(s)$
$\frac{K}{s(s+2)(s+5)}$
$R(s)$
Desired
direction
of turning
+
−
$Y(s)$

(b)

$$q(s) = s^4 + 8s^3 + 17s^2 + (k+10)s + ka = 0$$

$$\begin{array}{llll}
s^4 & 1 & 17 & ka \\
s^3 & 8 & (k+10) & 0 \\
s^2 & b_1 & ka & \\
s^1 & c_1 & & \\
s^0 & ka & &
\end{array}$$

$$b_1 = \frac{126-k}{8}, \text{ and } c_1 = \frac{b_1(k+10) - 8ka}{b_1}$$

$k < 126$

$ka > 0$

$(k+10)(126-k) - 64ka > 0$

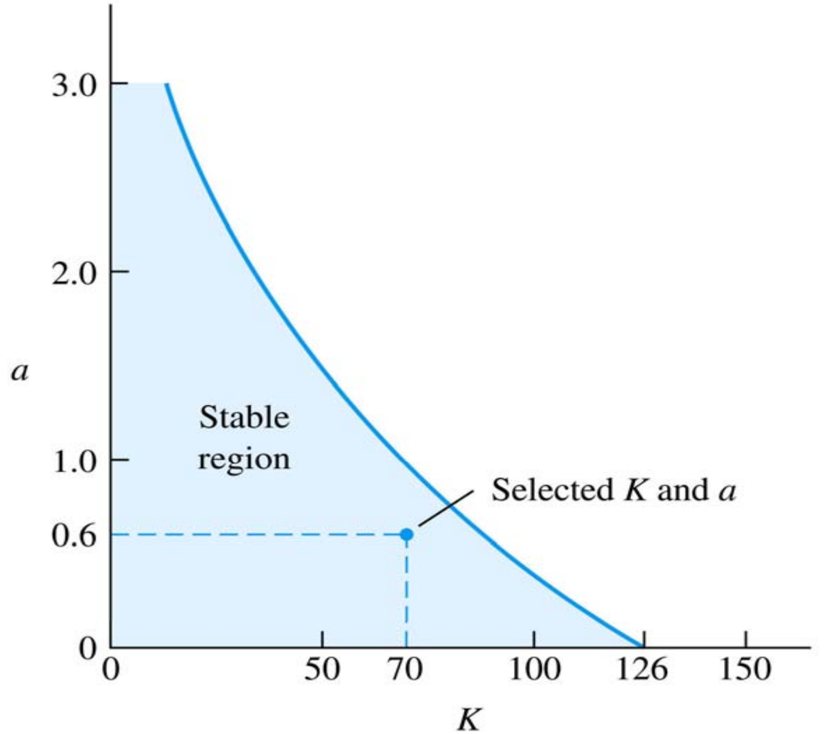


FIGURE 6.9

The stable region.

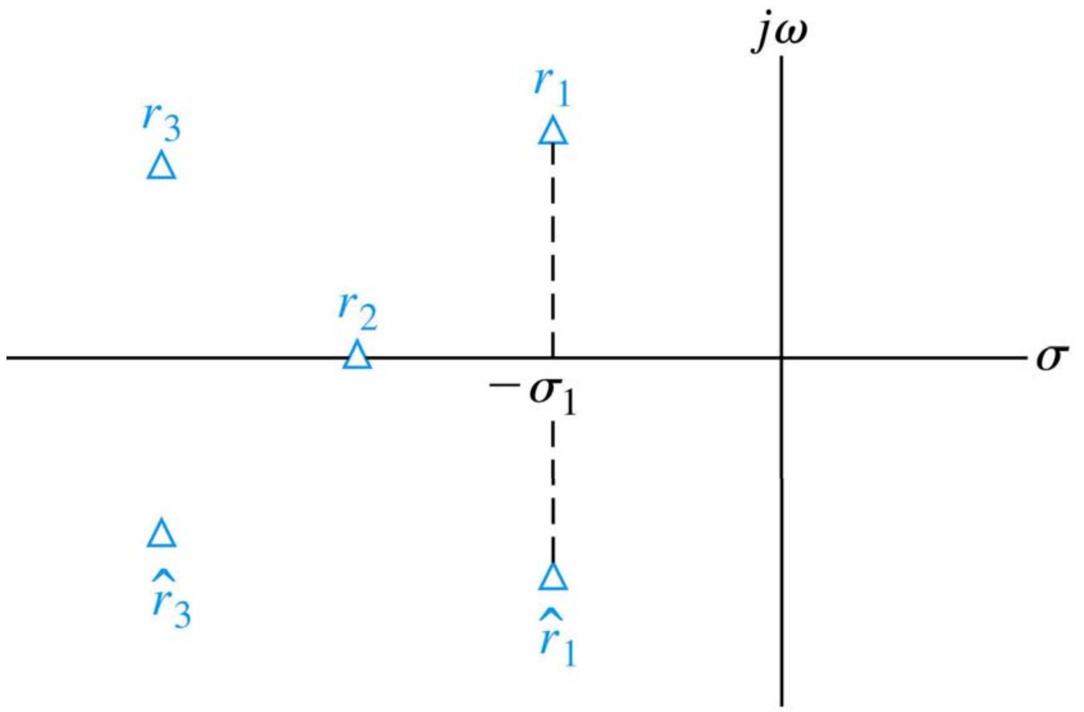


FIGURE 6.6

Root locations in the *s*-plane.

Exercises and Problems

E6.1,E6.2,E6.6,E6.17

P6.6,P6.12,P6.15,P6.18

AP6.4,DP6.1

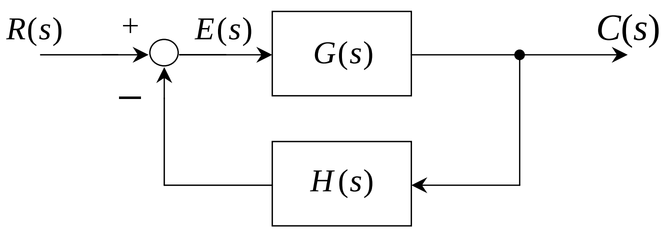
CHAPTER 7

The Root Locus Method

- The Root Locus Concept
- The Root Locus Procedure
- The Root contour
- The Root Locus Using Matlab

$$T(s)=\frac{Y(s)}{R(s)}=\frac{p(s)}{q(s)}$$

Roots ⇨ to find out the solution of the characteristic equation.



$$q(s)=1+G(s)H(s)=0$$

$$q(s)=1+KG_1(s)H_1(s)=0$$

$$G_1(s)H_1(s)=-1\bullet\frac{1}{K}$$

$$|G_1(s)H_1(s)|=\frac{1}{|K|}$$

$$\angle G_1(s)H_1(s)=\left\{\begin{matrix}180\pm 2k\pi, & \text{for } K>0\\ 0^0\pm 2k\pi, & \text{for } K<0\\ where\ \ k=0,1,2\ldots & \end{matrix}\right\}$$

$$\begin{aligned}q(s)&=s^3+2s^2+Ks+4K=0\\&=s^3+2s^2+K(s+4)=0\\&=1+K\frac{s+4}{s^2(s+2)}=0\end{aligned}$$

$$G_1(s)H_1(s)=\frac{s+4}{s^2(s+2)}$$

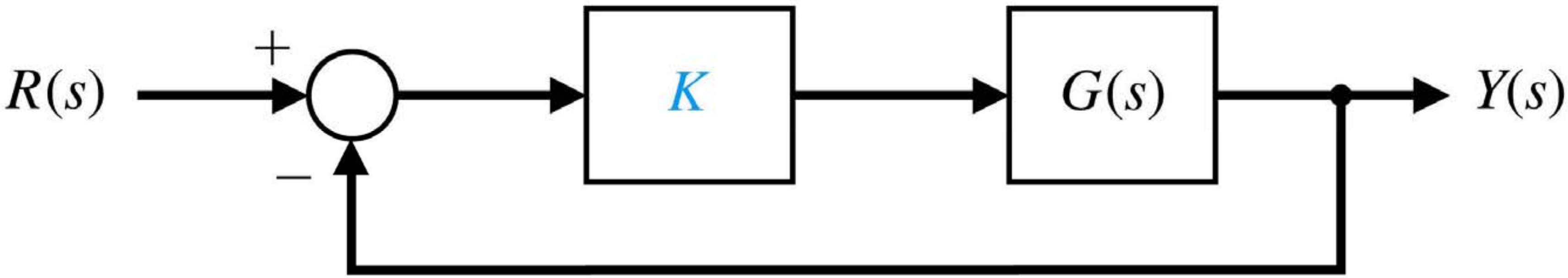
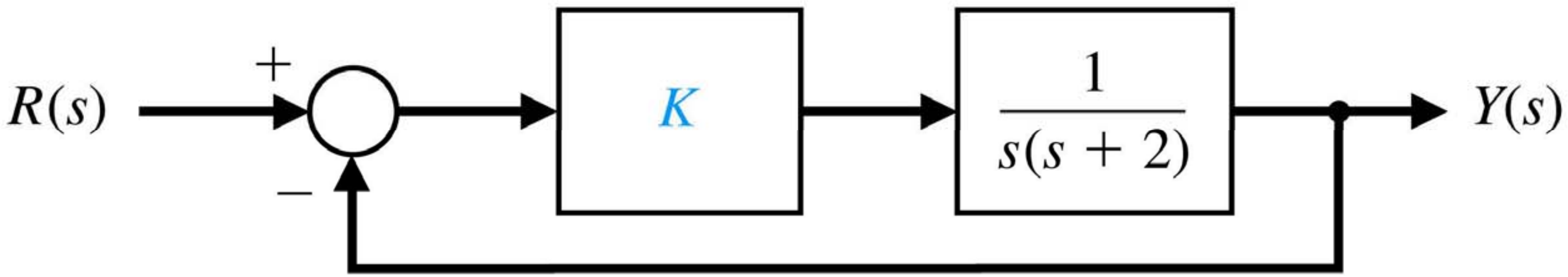


FIGURE 7.1

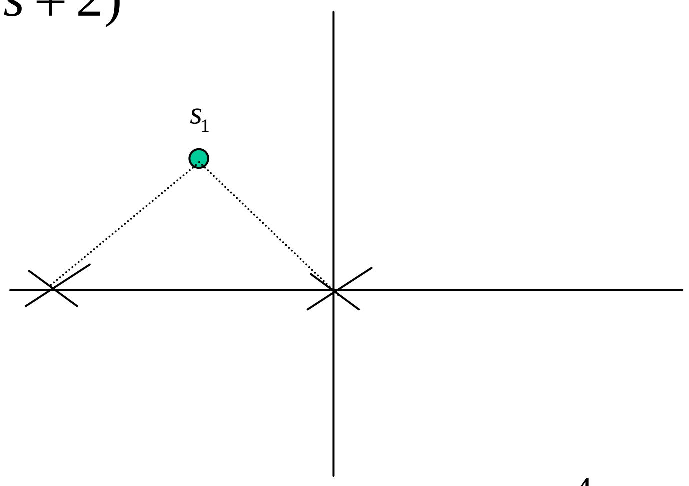
Closed-loop control system with a variable parameter *K*.



$$q(s) = 1 + G(s)H(s) = 1 + K\frac{1}{s(s+2)} = 0$$

$$|G_1(s)H_1(s)|_{s=s_1} = \left|\frac{1}{s(s+2)}\right|_{s=s_1} =$$

$$\angle G_1(s)H_1(s)|_{s=s_1}$$



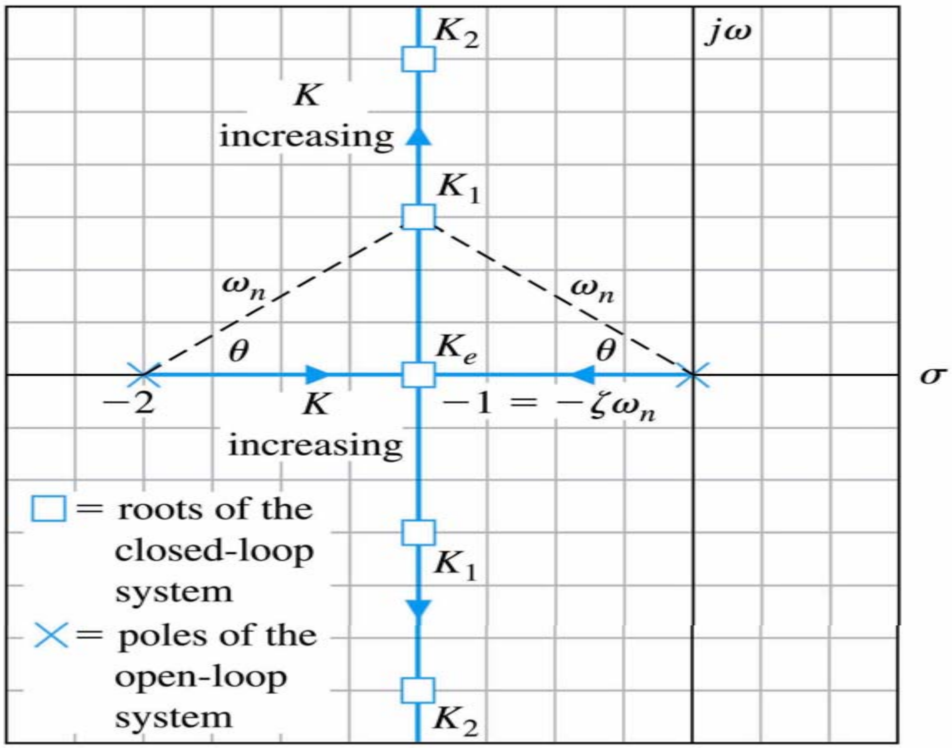


FIGURE 7.3

Root locus for a second-order system when $K_e < K_1 < K_2$. The locus is shown in color.

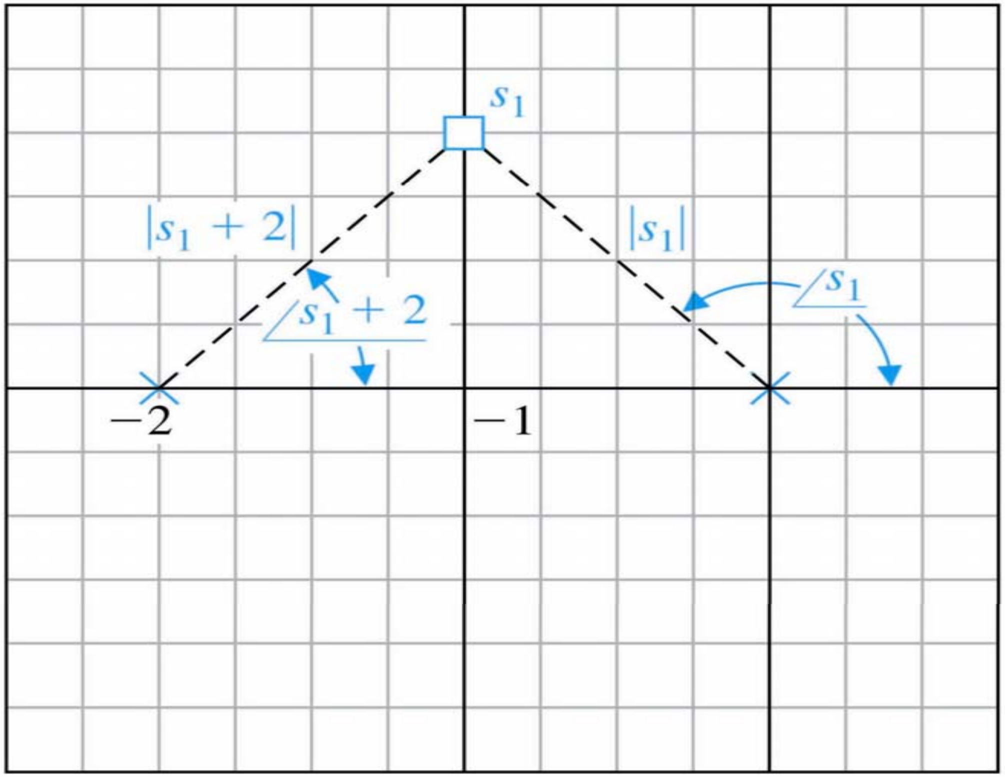


FIGURE 7.4

Evaluation of the angle and gain at s_1, for gain $K = K_1$.

☑ The Root Locus Procedure

Step	Related equation or Rule
1. Write the characteristic equation so that the parameter of interest K appears as a multiplier.	$F(s) = 1 + KG_1(s)H_1(s) = 0$
2. Factor $G_1(s)H_1(s)$ in terms of n poles and m zeros.	$G_1(s)H_1(s) = \dfrac{\prod_{j=1}^{m}(s+z_j)}{\prod_{i=1}^{n}(s+p_i)}$
3. Locate the open-loop poles and zeros of $F(s)$ in the *s-plane* with selected symbols.	× : poles, ○: zeros, Δ or □: roots of characteristic equation
4. Locate the segments of the real axis that are root locus.	a). Locus begins at a pole and ends at zero. b). Locus lies to left of an odd number of poles and zeros ($K \geq 0$).
5. The number of branch on the root loci, ρ.	$\rho = n, \quad when\ n \geq m;$ n: number of finite poles, m: number of finite zeros
6. The root loci are symmetrical with respect to the horizontal real axis.	
7. Intersect of the asymptotes (Centroid)	$\sigma = \dfrac{\sum p_i - \sum z_j}{n-m}$ or $\sigma = \dfrac{\sum \mathrm{Re}(p_i) - \sum \mathrm{Re}(z_j)}{n-m}$
8. Angles of asymptotes of the root loci.	$\theta_k = \begin{cases} \dfrac{(2k+1)\pi}{n-m}, & \forall K \geq 0 \\ \dfrac{2k\pi}{n-m}, & \forall K < 0 \end{cases} ;\ k = 0,1,2,\cdots,\lvert n-m-1 \rvert$

9. Breakaway points (saddle points) on the root loci.	Roots of $\frac{d}{ds}G_1(s)H_1(s)=0$ or $\frac{d}{ds}K=0$
10. Intersection of root loci with imaginary axis.	*Routh-Hurwitz* criterion.
11. Angles of departure and angles of arrival of the root loci.	$\angle G_1(s)H_1(s)=\begin{cases}(2k+1)\pi & \forall K\geq 0\\ 2k\pi & \forall K<0\end{cases},\ k=0,1,2,\cdots$ at $s=p_i$ or $s=z_j$.
12. Calculation of K at a specific root s_i.	$K=\left.\frac{\prod_{i=1}^{n}\lvert(s+p_i)\rvert}{\prod_{j=1}^{m}\lvert(s+z_j)\rvert}\right\rvert_{s=s_i}$

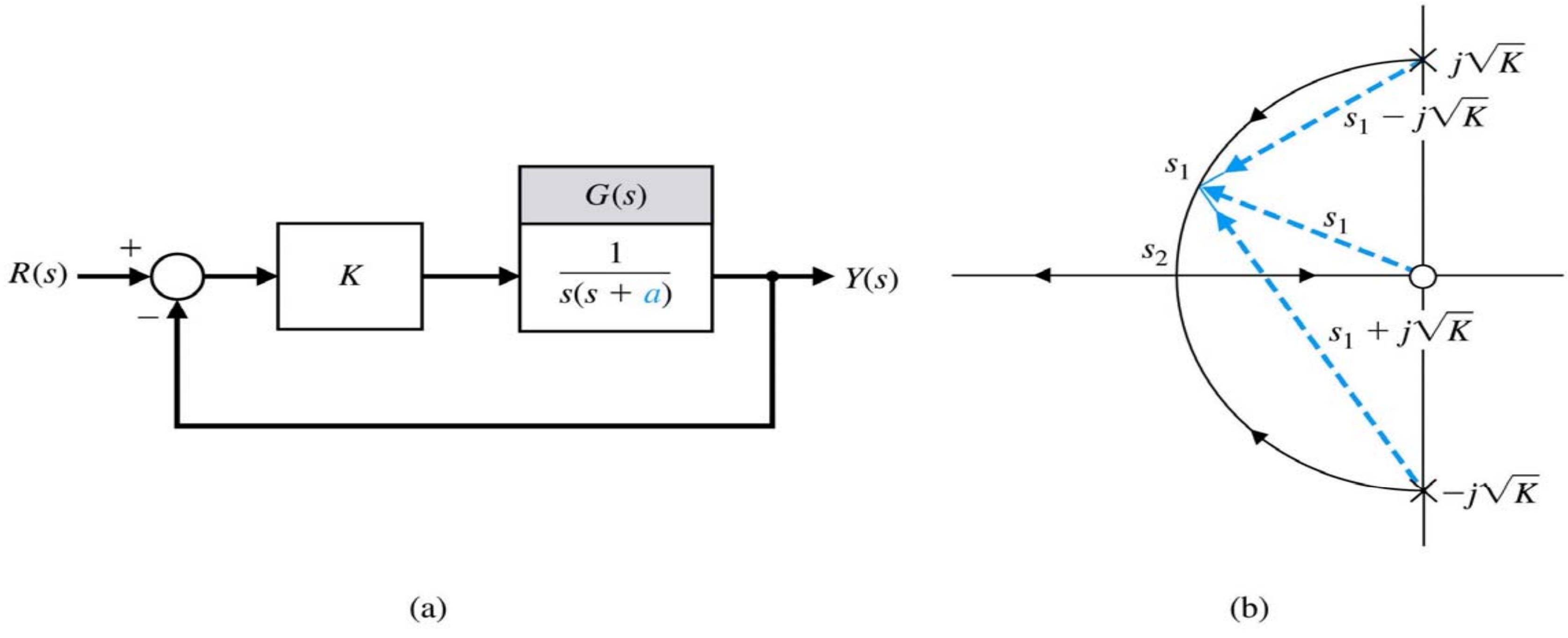
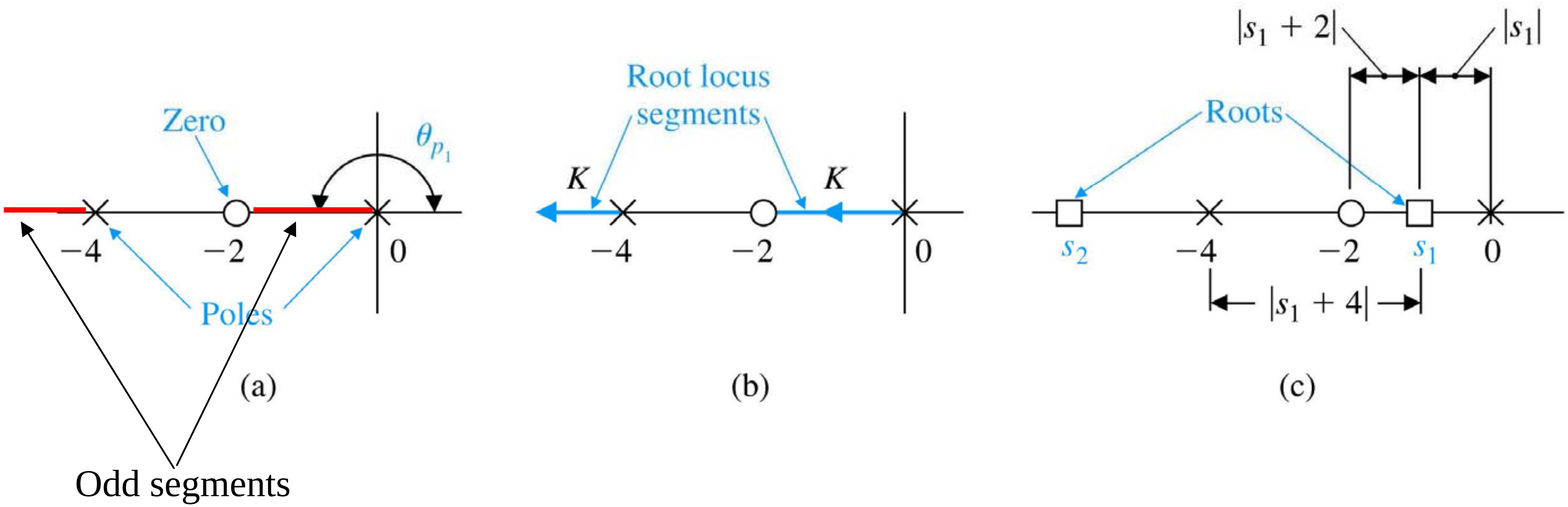


FIGURE 7.5

(a) Single-loop system. (b) Root locus as a function of the parameter a.

Step 4: The root locus on the real axis always lies in a section of the real axis to the left of an **odd number** of poles and zeros.

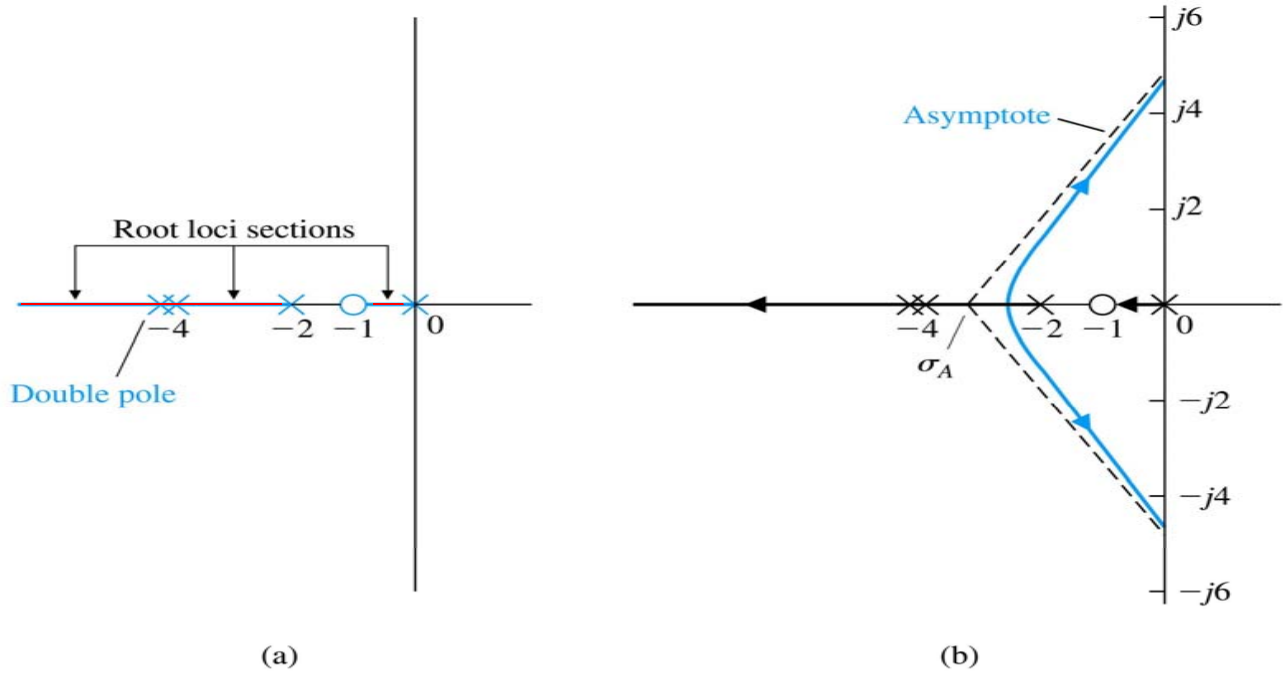


(a) (b) (c)

Step 5: Determine the number of separate loci,SL.The number of separate loci is equal to the number of poles

Step 6: The root loci must be symmetrical with respect to the horizontal real axis.

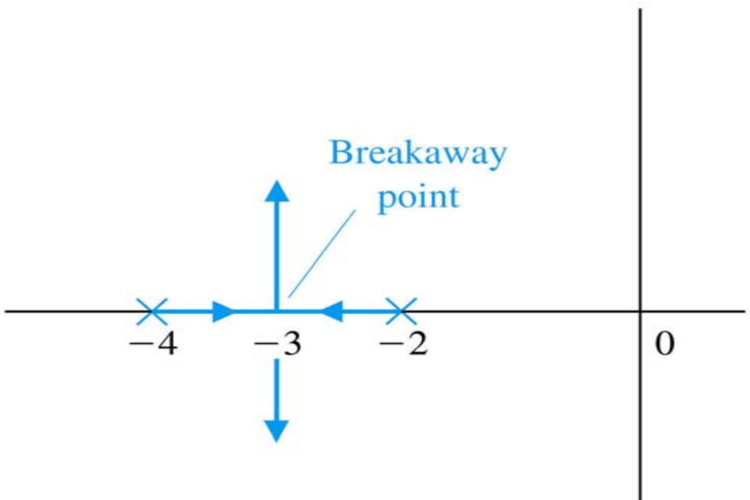
Step 7: The linear asymptotes are centered at a point on the real axis given by $\sigma_A = \frac{\sum poles - \sum zeros}{n-m}$ The angle of the asymptotes with respect to the real axis is $\phi_A = \frac{(2q+1)}{n-m}\times 180^0$ $n=4, m=1, \phi_1 = \frac{(2q+1)}{4-1}\times 180^0, q=0,1,2, n-m-1$



Step 8: The actual point at which the root locus crosses the imaginary axis is readily evaluated by utilizing the Routh-Hurwitz criterion.

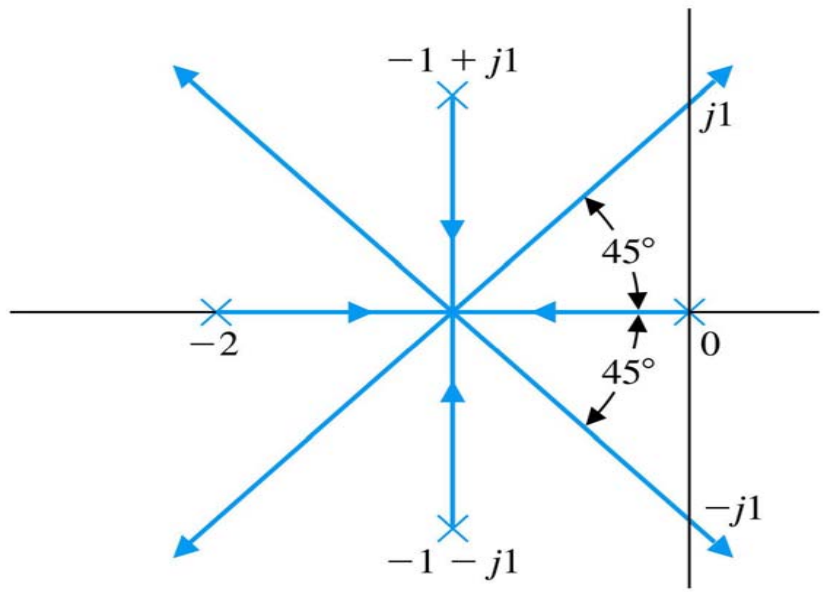
Step 9: Determine the breakaway point $\frac{dK}{ds} = 0$

the tangents to the loci at the breakaway point are equally over 360^0



$$1 + KG_1(s) = 1 + K\frac{1}{(s+2)(s+4)} = 0$$

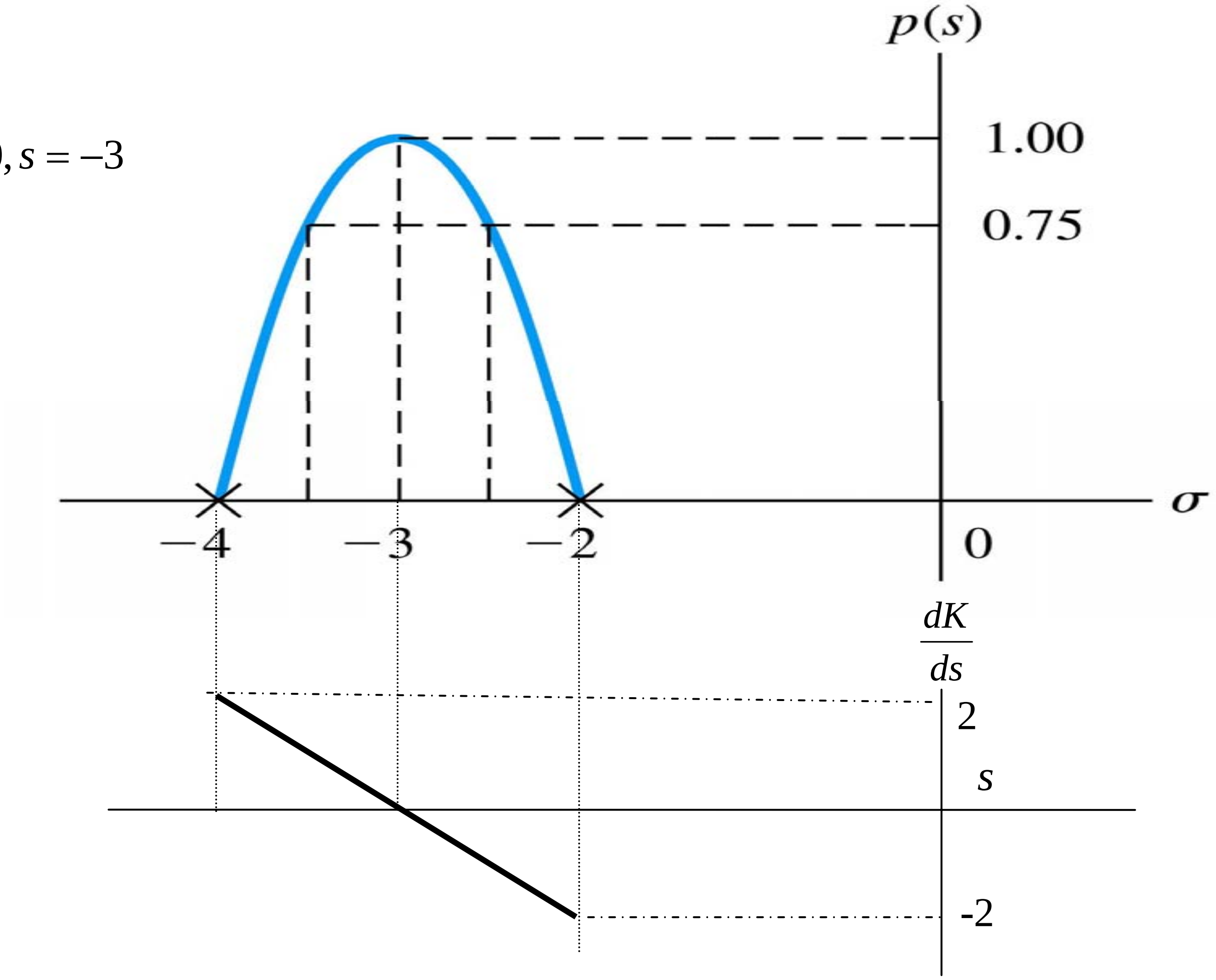
(a)



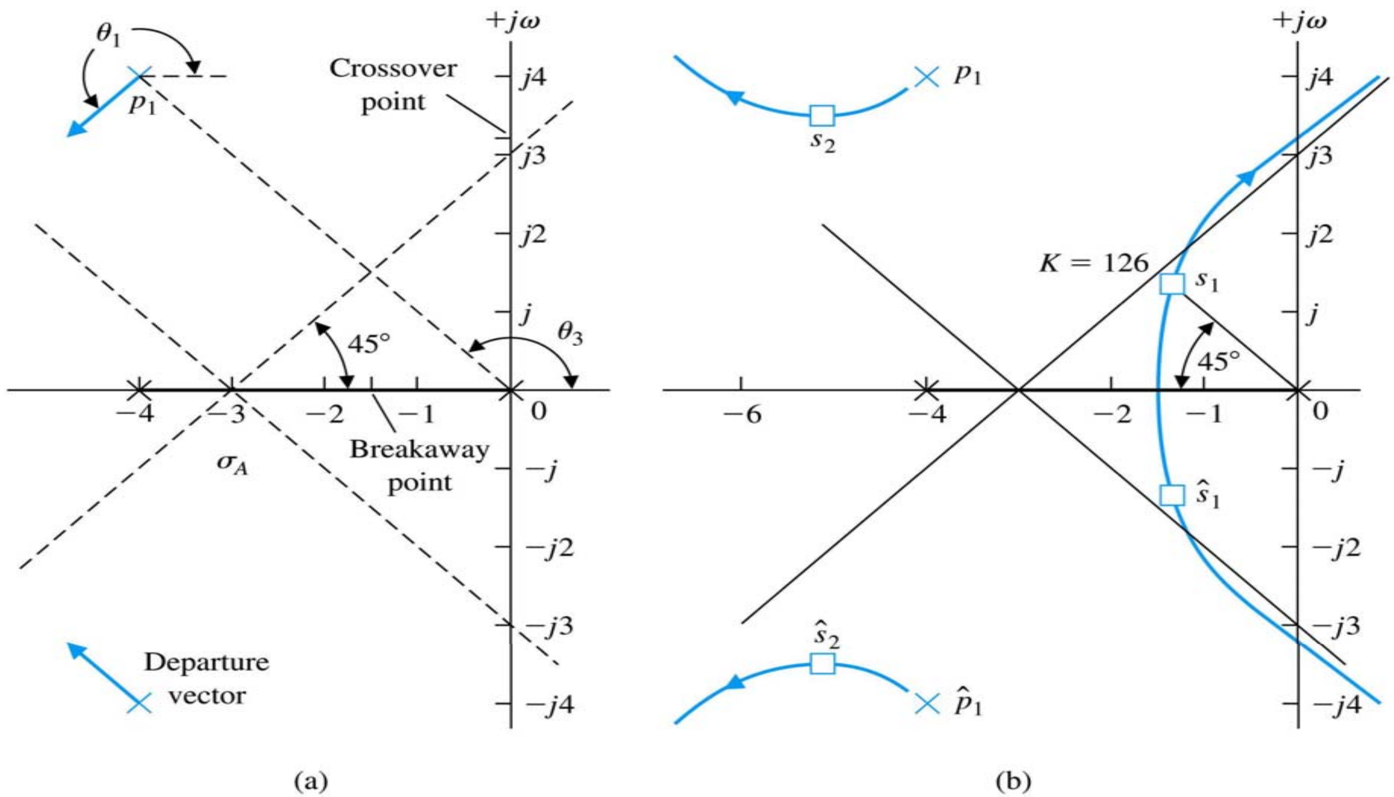
(b)

$$K = -(s+2)(s+4)$$

$$\frac{dK}{ds} = -(2s+6) = 0, s = -3$$



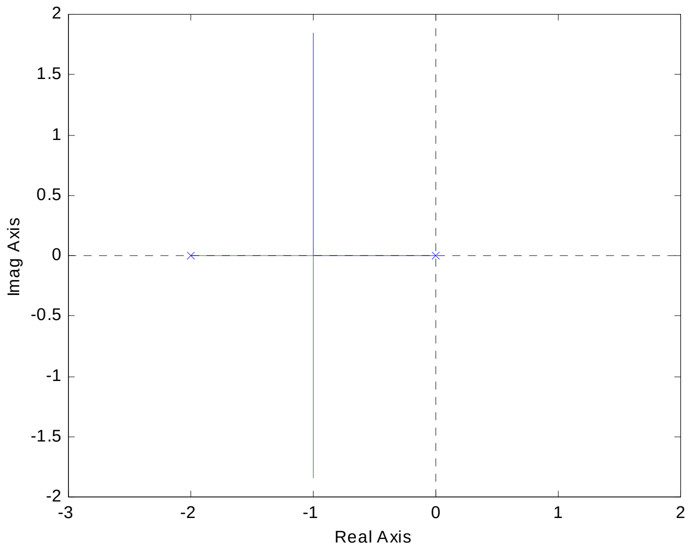
Step 10: Determine the angle of departure of the locus from a pole and the angle of arrival of the locus at a zero, using the phase angle criterion.

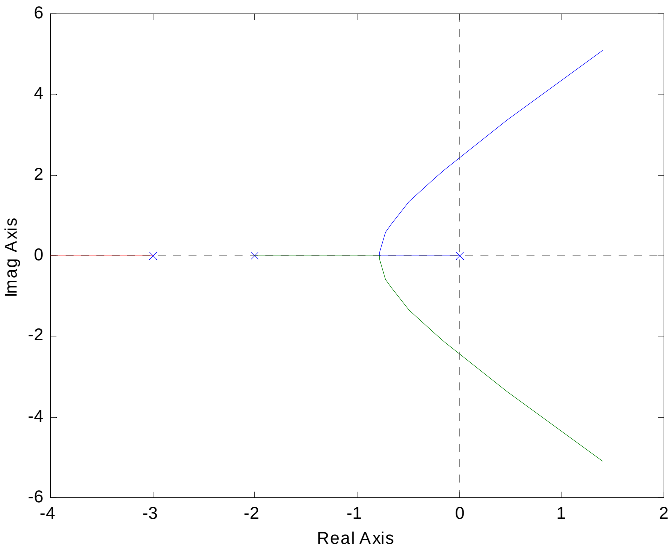


☑ Examples

$$F(s) = 1 + K\frac{1}{s(s+2)}$$

$$F(s) = 1 + K\frac{1}{s(s+2)(s+3)}$$

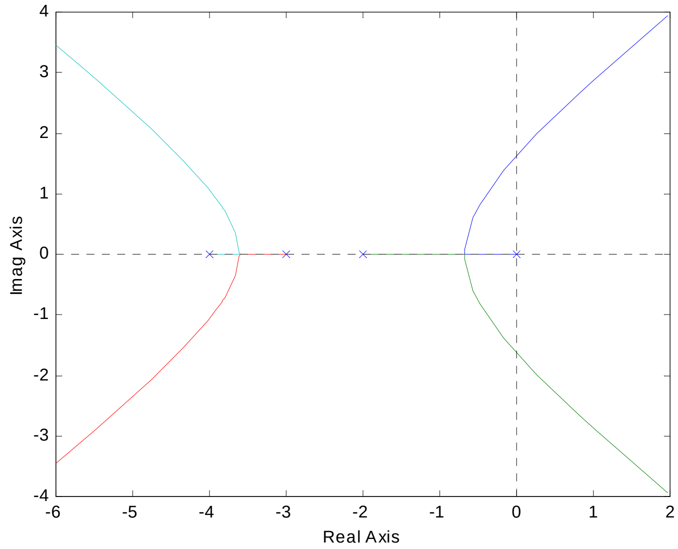




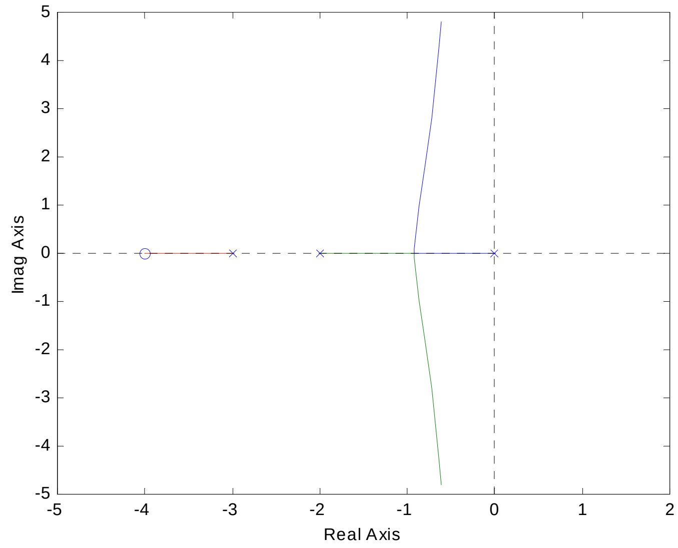
Effects adding poles to $G_1(s)H_1(s)$

$$F(s) = 1 + K\frac{1}{s(s+2)(s+3)(s+4)}$$

$$F(s) = 1 + K\frac{s+4}{s(s+2)(s+3)}$$



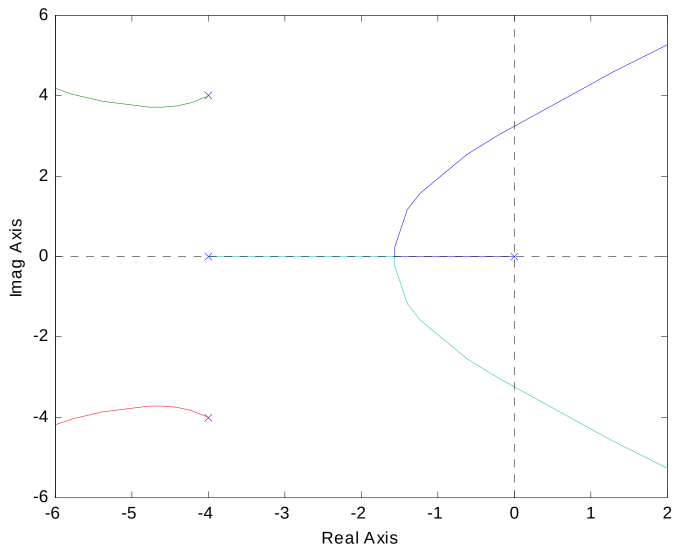
Effects adding poles to $G_1(s)H_1(s)$



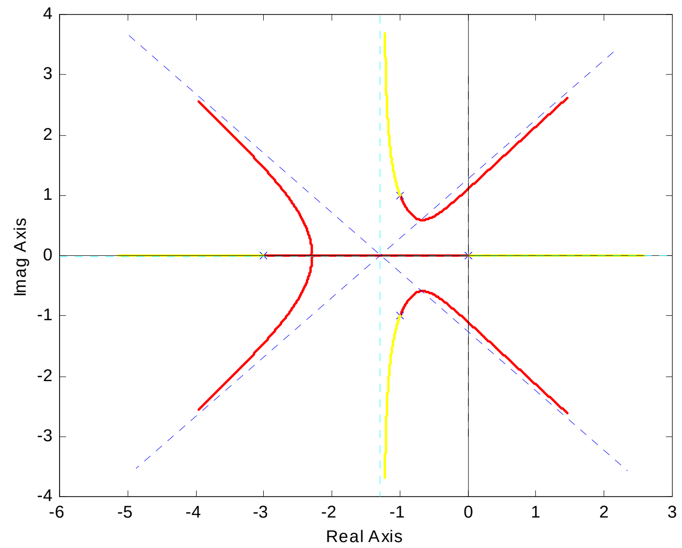
Effects adding zeros to $G_1(s)H_1(s)$

$$F(s) = 1 + K\frac{1}{s^4 + 12s^3 + 64s^2 + 32s}$$

$$F(s) = 1 + K\frac{1}{s(s+3)(s^2+2s+2)}$$



Effects adding poles to $G_1(s)H_1(s)$



$-\infty < K < \infty$

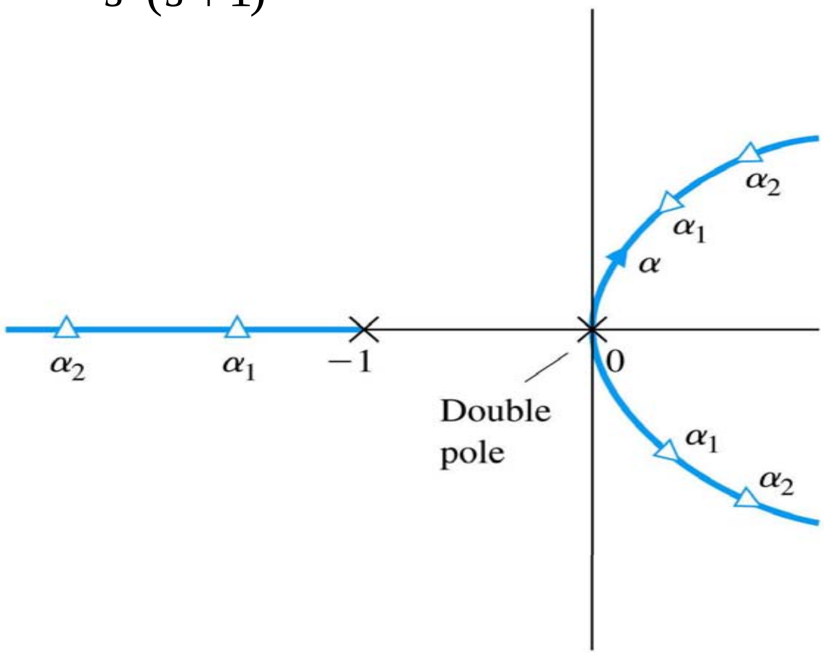
Lab #3

Written a M-file to plot the root loci step by step. (rlocfind)

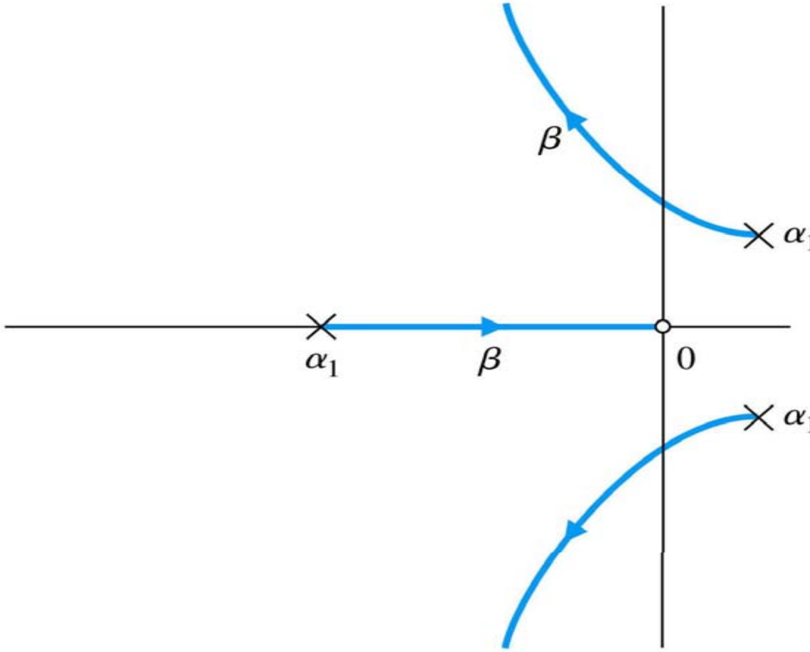
$$q(s) = s^3 + s^2 + \beta s + \alpha = 0 \quad \Rightarrow \quad 1 + \frac{\beta s}{s^3 + s^2 + \alpha} = 0$$

$$s^3 + s^2 + \alpha = 0$$

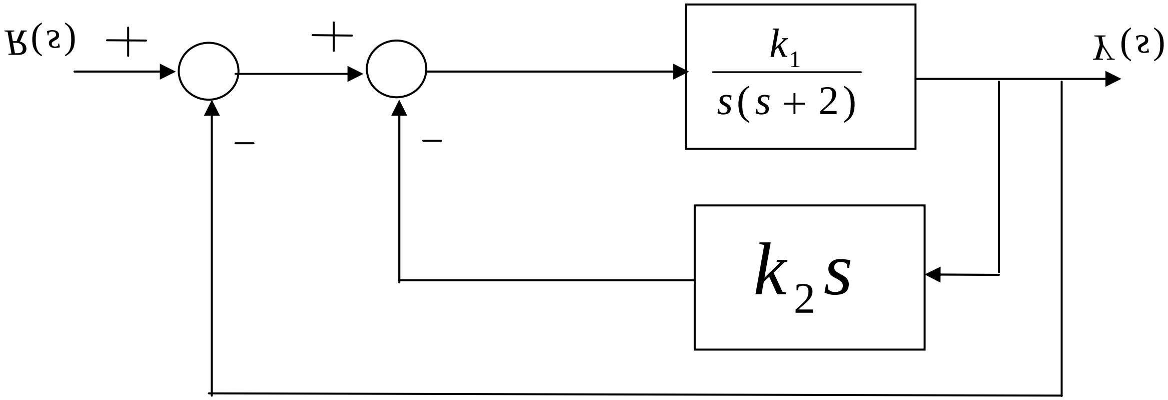
$$1 + \frac{\alpha}{s^2(s+1)} = 0$$

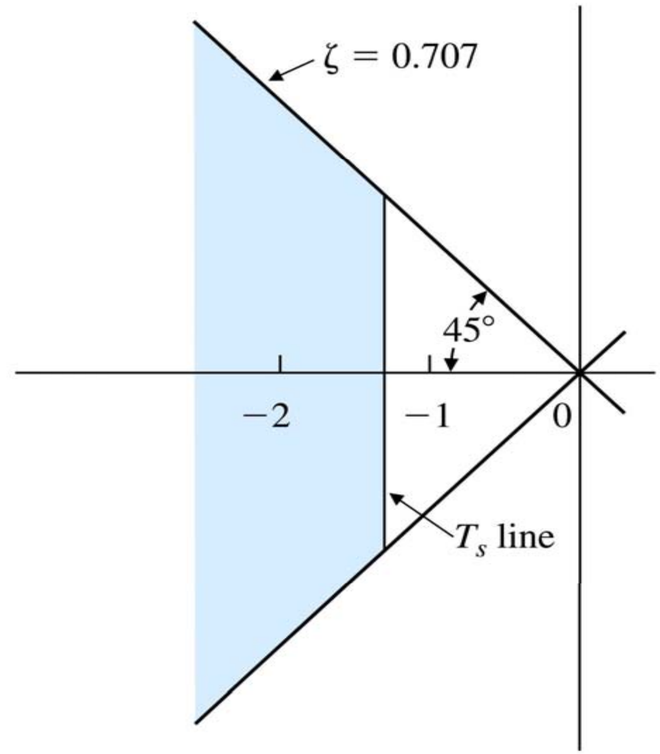


(a)



(b)





Specifications:

1. Steady-state error for a ramp input $\leq 35\%$
2. Damping ratio of dominant roots ≥ 0.707 sec.
3. Settling time to within 2 % of the final value ≤ 3 sec.

$$1 + GH(s) = s^2 + 2s + \beta s + \alpha = 0$$

$$\beta = k_2 k_1, \alpha = k_1$$

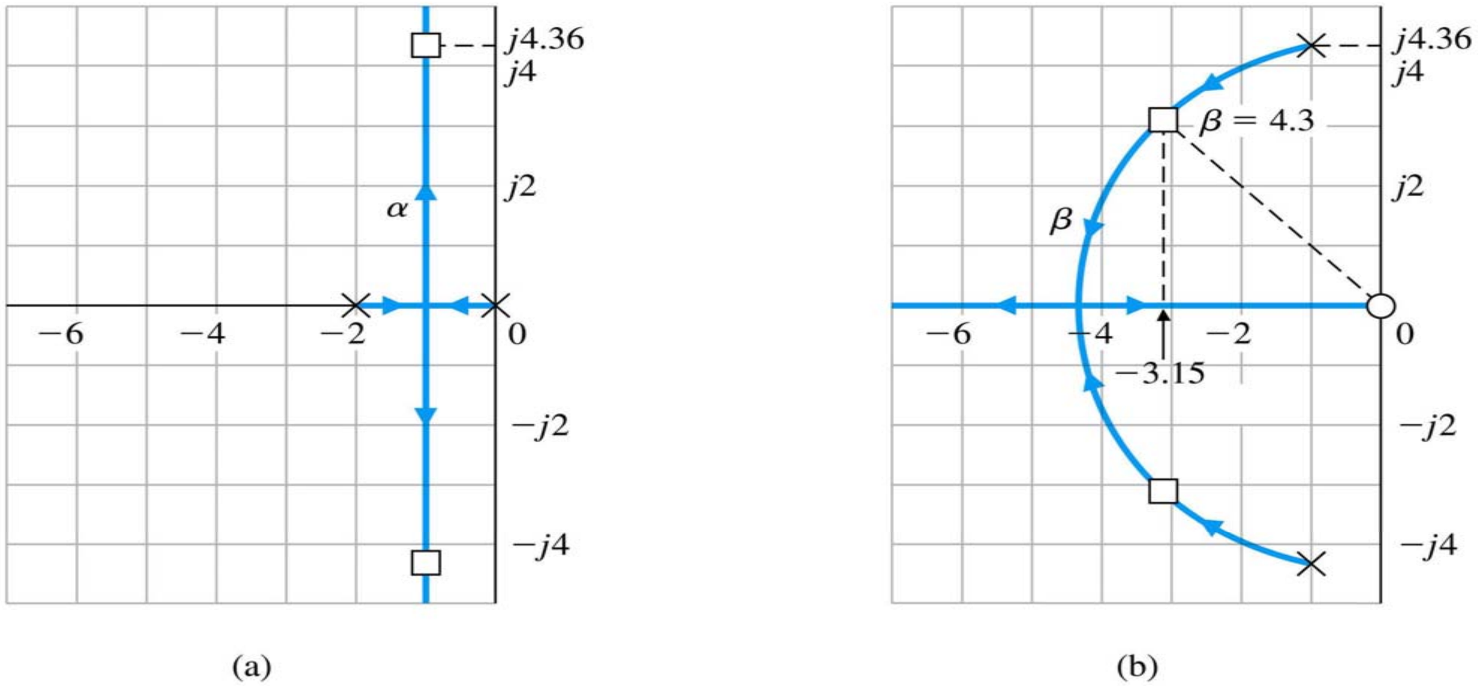


FIGURE 7.21

Root loci as a function of (a) α and (b) β.

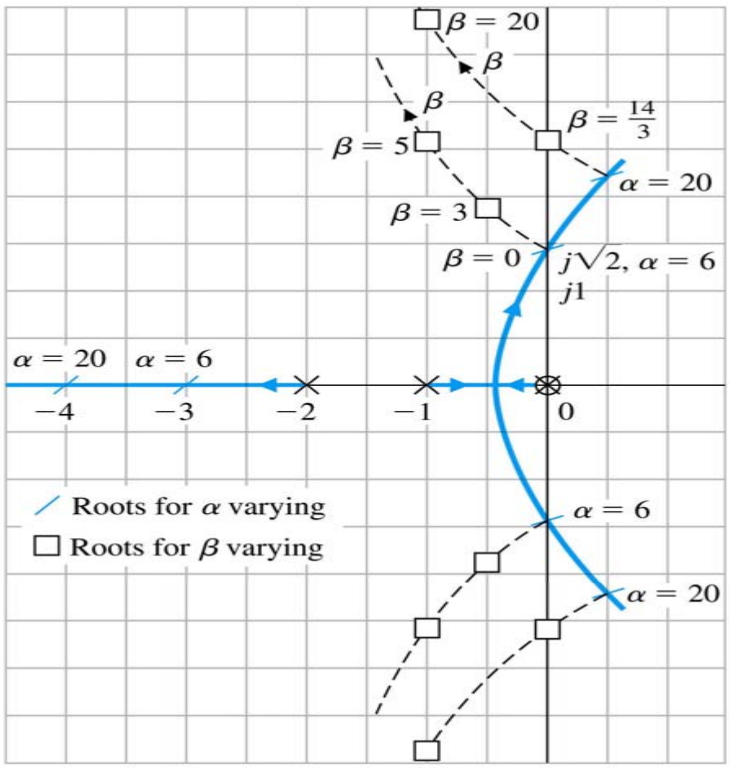
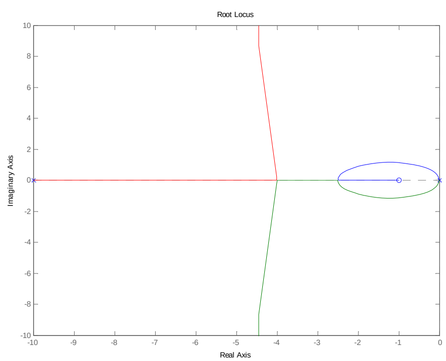


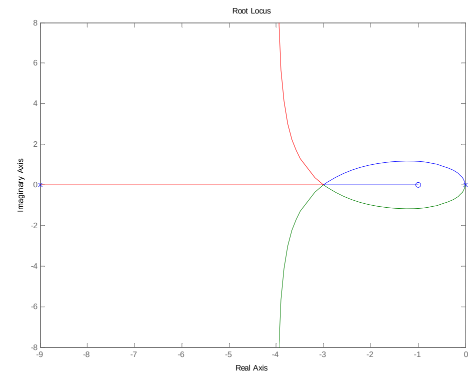
FIGURE 7.22

Two-parameter root locus. The loci for α varying are in color.

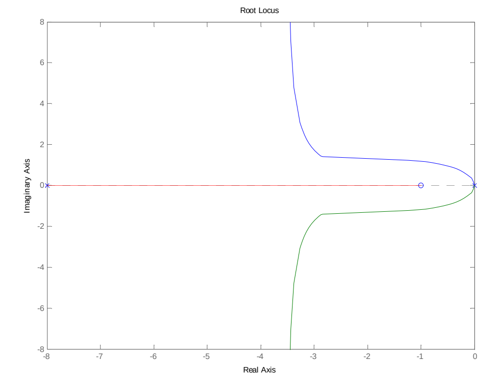
$$1+G_1(s)H_1(s)=1+K\frac{s+1}{s^2(s+a)}=0$$



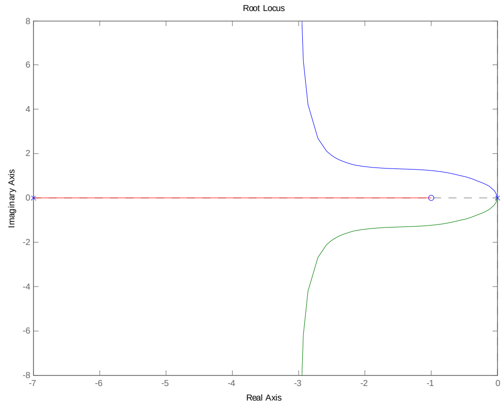
$a=10$



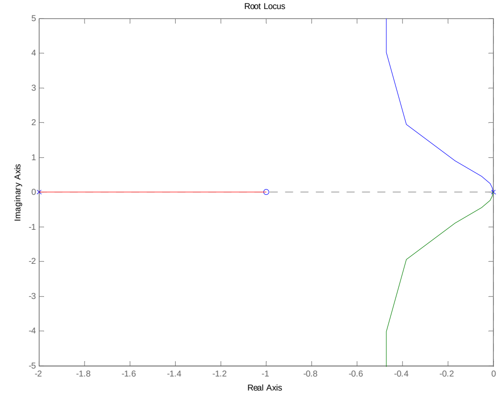
$a=9$



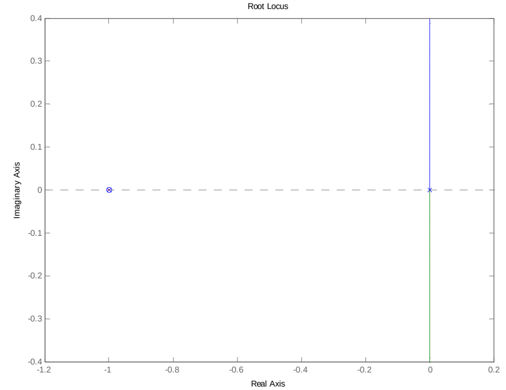
$a=8$



$a=7$



$a=2$



$a=1$

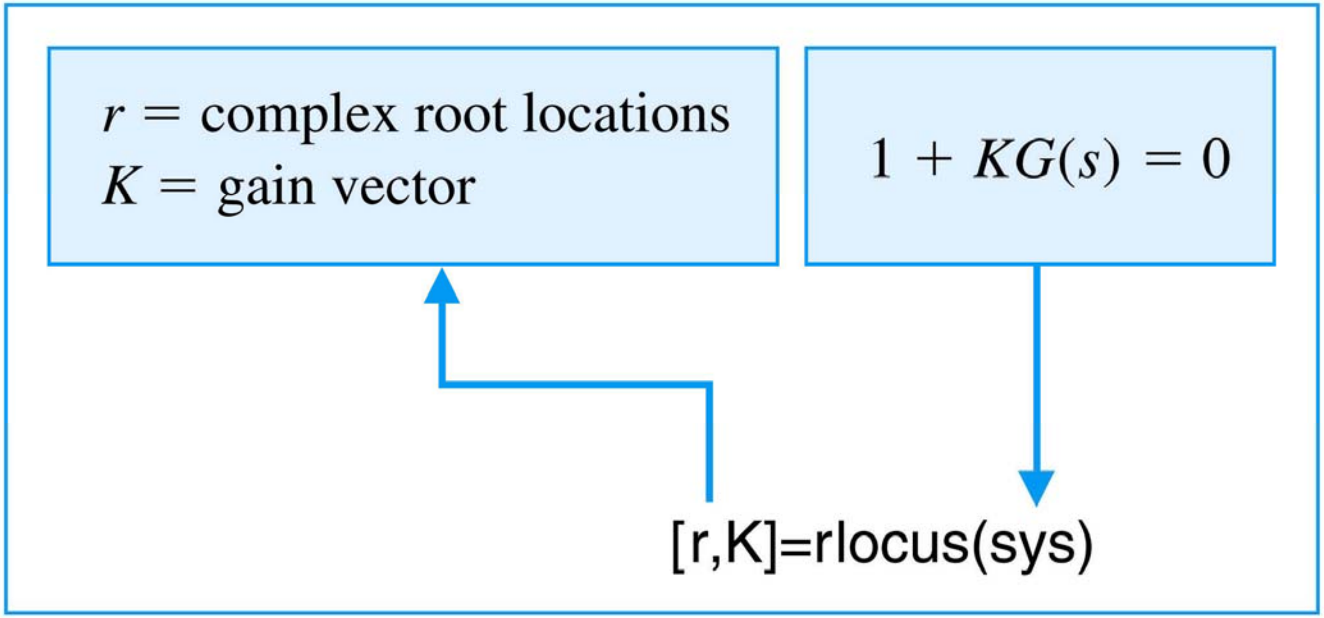


FIGURE 7.37

The rlocus function.

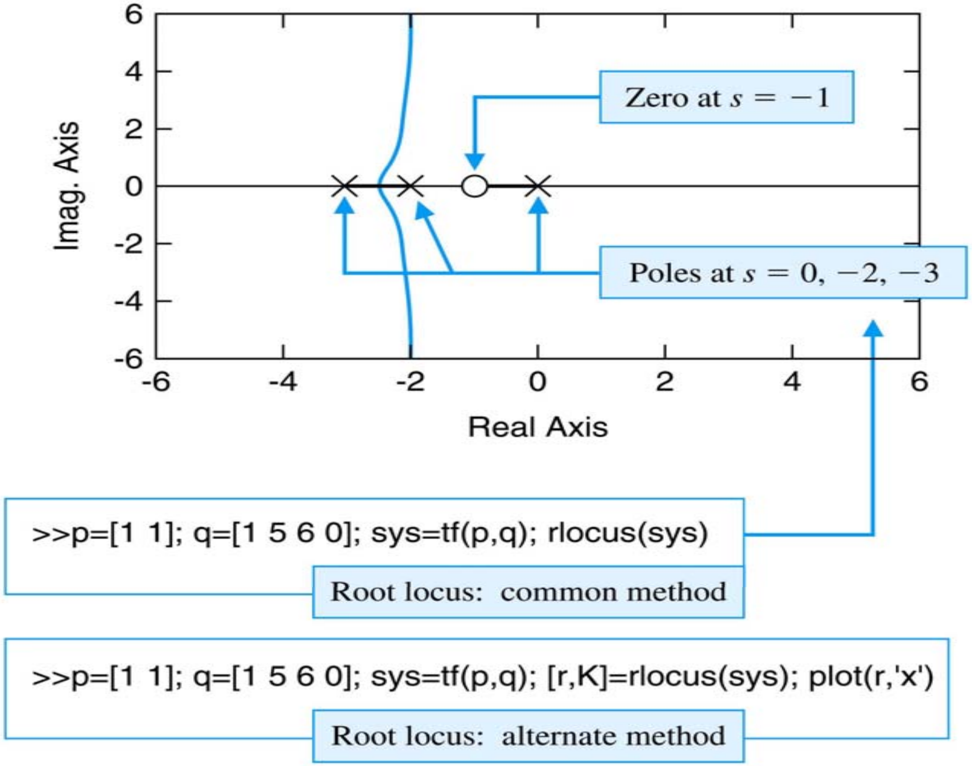


FIGURE 7.38

The root locus for the characteristic equation (7.122).

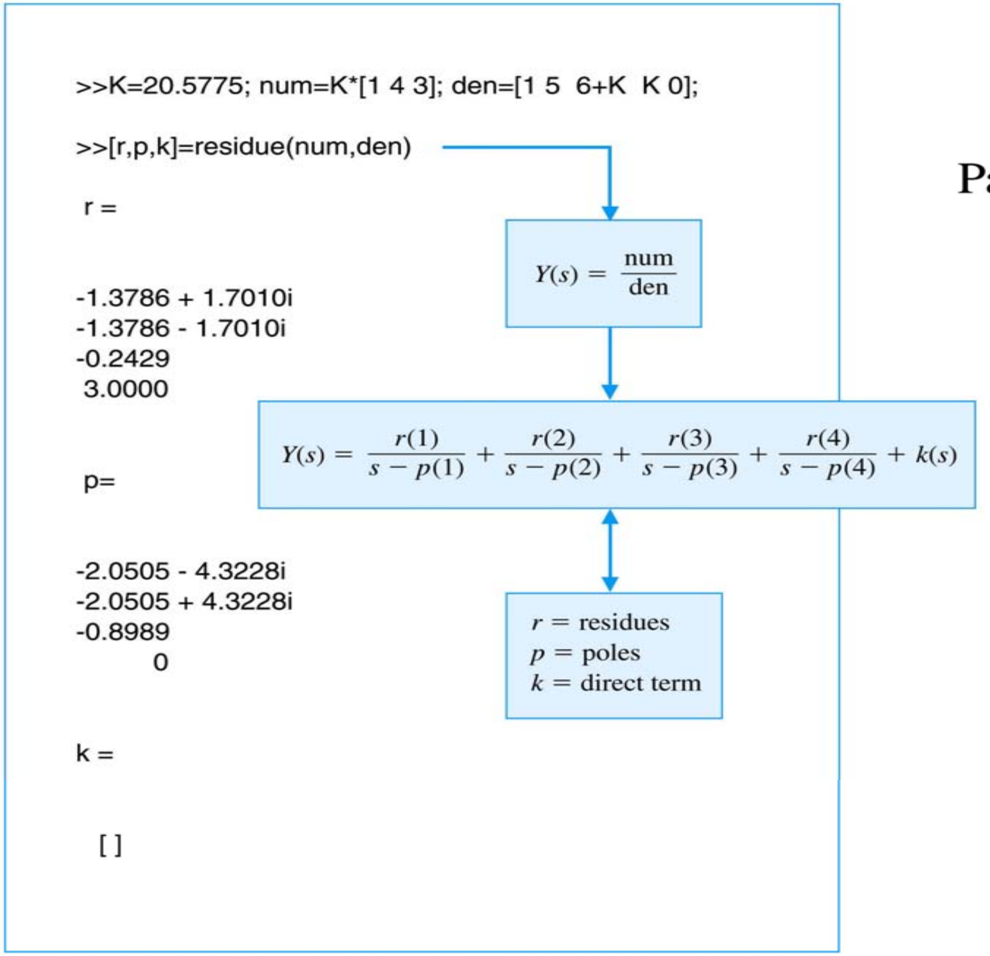


FIGURE 7.40

Partial fraction expansion of Eq. (7.124).

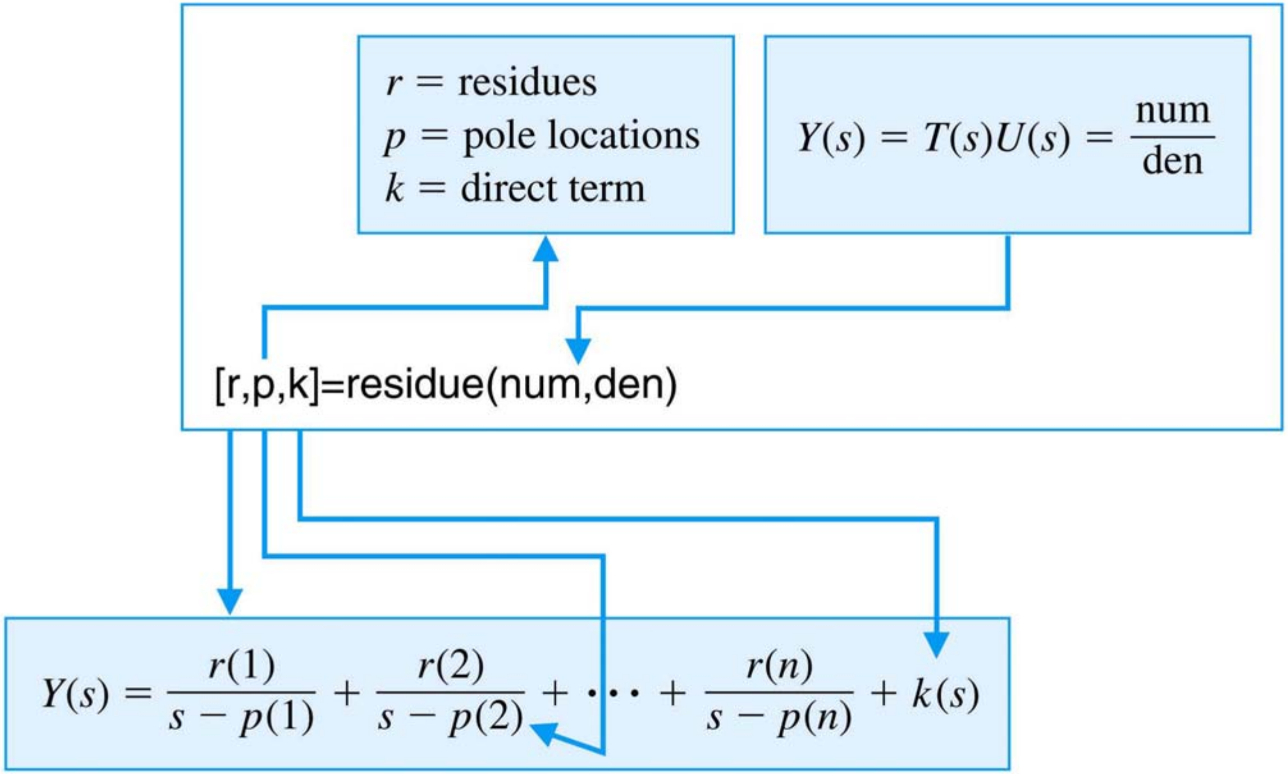


FIGURE 7.41

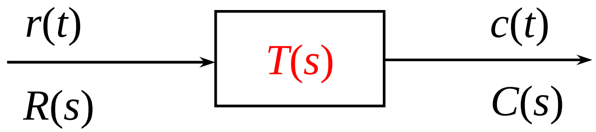
The residue function.

CHAPTER 8

Frequency Response Methods

- **Introduction**
- **Frequency Response Plots**
  - *Polar Plot*
  - *Bode Plot*
  - *Log Magnitude and Phase Plot*
- **Frequency Response Measurements**
- **Performance Specifications in The Frequency Domain**

☑ Introduction



Output: $C(s) = T(s)R(s)$

If $r(t) = A\sin(\omega \cdot t) \Rightarrow c(t) = C\sin(\omega \cdot t + \phi)$

For sinusoidal steady-state analysis, we replace s by $j\omega$, and output c(t) becomes

$$C(j\omega) = T(j\omega)\cdot R(j\omega) = |C(j\omega)|\angle C(j\omega) = |C(j\omega)|e^{j\angle C(j\omega)}$$

For a closed-loop system

$$T(j\omega) = T(s)\big|_{s=j\omega} = \frac{G(j\omega)}{1+G(j\omega)H(j\omega)}$$

$$= |T(j\omega)|\angle T(j\omega)$$

$$\underline{\underline{or}}\ R(w) + jX(w)$$

where

$$|T(j\omega)| = \left|\frac{G(j\omega)}{1+G(j\omega)H(j\omega)}\right|$$

$$\angle T(j\omega) = \angle G(j\omega) - \angle 1 + G(j\omega)H(j\omega)$$

and

$$R(\omega) = \mathrm{Re}[T(j\omega)]$$

$$X(\omega) = \mathrm{Im}[T(j\omega)]$$

$$T(j\omega) = \mathrm{Re}[T(j\omega)] + j\,\mathrm{Im}[C(j\omega)]$$

$$\underline{\underline{\Delta}} R(\omega) + jX(\omega)$$

$$|T(j\omega)| = \sqrt{R^2(\omega) + X^2(\omega)}$$

$$\angle T(j\omega) = \tan^{-1}\frac{X(\omega)}{R(\omega)}$$

The Laplace transform pair

$$C(s) = L\{c(t)\} = \int_0^{\infty} c(t)e^{-st}dt$$

$$c(t) = L^{-1}\{C(s)\} = \frac{1}{2\pi j}\int_{\sigma - j\infty}^{\sigma + j\infty} C(s)e^{st}ds$$

The Fourier transform pair

$$C(j\omega) = F\{c(t)\} = \int_{-\infty}^{\infty} c(t)e^{-j\omega t}dt$$

and

$$c(t) = F^{-1}\{C(j\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{j\infty} C(j\omega)e^{j\omega t}d\omega$$

The Fourier transform exists for f(t) when

$$\int_{-\infty}^{\infty} |c(t)|dt < \infty$$

☑Frequency Response Plots

- ***Polar plot***

A plot of the magnitude versus phase in the polar coordinates as ω is varied from $0 \to \infty$.

- *Find $T(j\omega)$; $|T(j\omega)|$, $\angle T(j\omega)$, $R(j\omega)$ and $X(j\omega)$.*
- *To Determine the behavior of the magnitude and phase of $T(j\omega)$ at $\omega \to 0$ and $\omega \to \infty$.*

$$\lim_{\omega\to 0}|T(j\omega)|;\quad \lim_{\omega\to 0}\angle T(j\omega)$$

$$\lim_{\omega\to \infty}|T(j\omega)|;\quad \lim_{\omega\to \infty}\angle T(j\omega)$$

- *To Determine the the intersection*

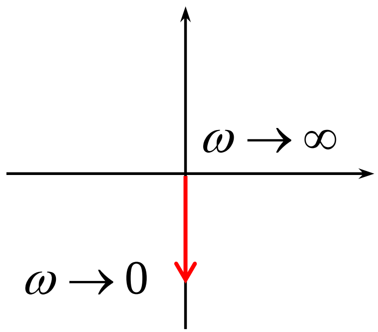
$$R(\omega)\big|_{\omega=\omega_i} = 0 \Rightarrow X(\omega)\big|_{\omega=\omega_i} = \beta$$

$$X(\omega)\big|_{\omega=\omega_r} = 0 \Rightarrow R(\omega)\big|_{\omega=\omega_r} = \alpha$$

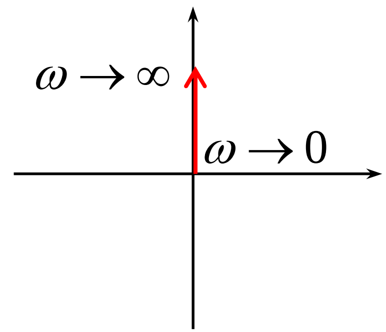
- *To Determine the the asymptotic line*

$$\lim_{\omega\to 0} T(j\omega) = \lim_{\omega\to 0}\left[R(\omega) + jX(\omega)\right] = a + jb$$

★ *Case 1: Integral and derivative factor* $s^{\pm 1}$

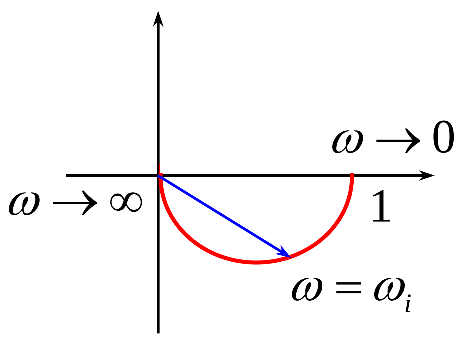


$$A).\ T(j\omega) = \frac{1}{j\omega}$$

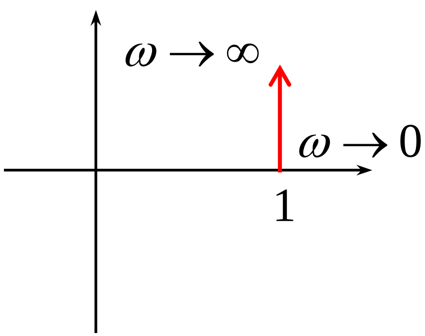


$$B).\ T(j\omega) = j\omega$$

★ *Case 2: First-order factor* $T(s) = (1+\tau s)^{\pm 1}$



$$A).\ T(j\omega) = \frac{1}{1+j\omega\tau}$$



$$B).\ T(j\omega) = 1 + j\omega\tau$$

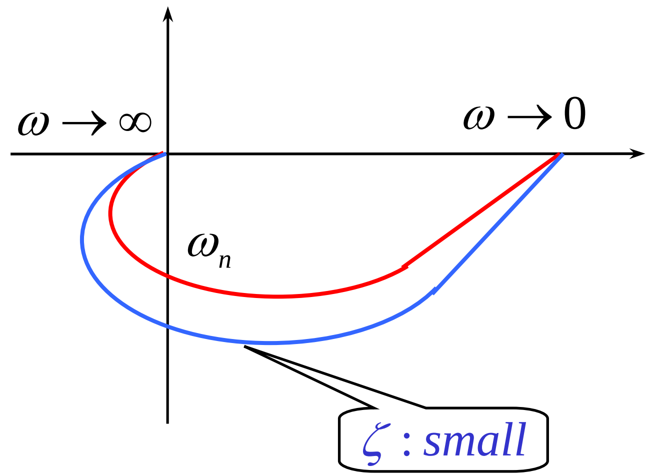
★ *Case 3: Quadratic factor*

$$A).\ T(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

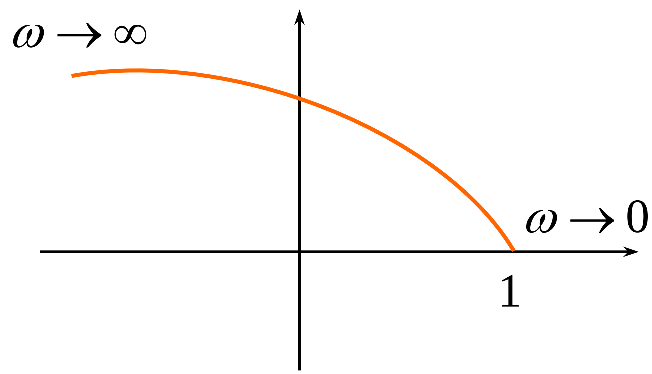
$$=\frac{1}{\frac{1}{\omega_n^2}s^2+2\zeta\frac{1}{\omega_n}s+1}$$

$$T(j\omega)=\frac{1}{1+2\zeta\left(\frac{j\omega}{\omega_n}\right)+\left(\frac{j\omega}{\omega_n}\right)^2}$$

$$=\frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2+j2\zeta\left(\frac{\omega}{\omega_n}\right)}$$



$$B).\ T(s)=\frac{s^2+2\zeta\omega_n s+\omega_n^2}{\omega_n^2}$$



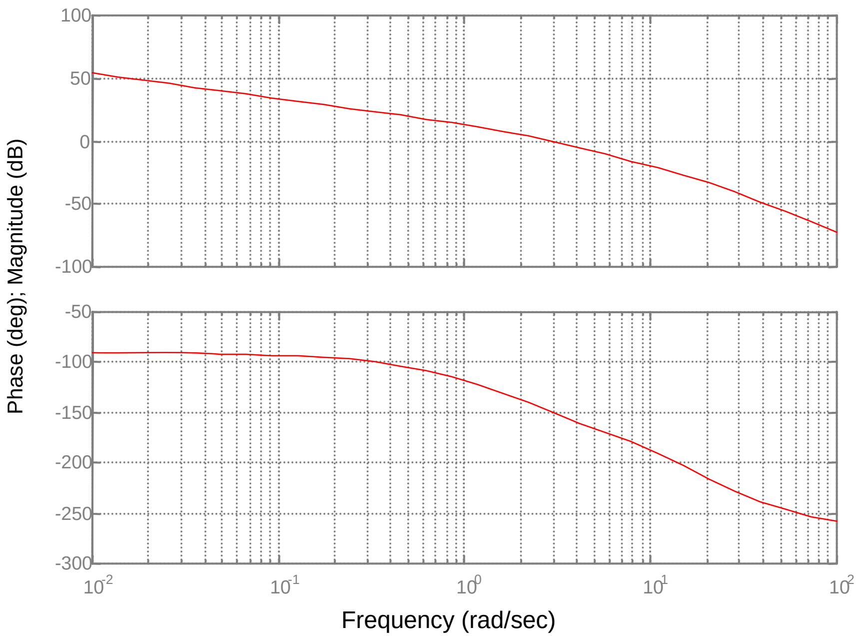
Bode plot

The Bode plot of the function $T(j\omega)$ is composed of two plots:

- *The amplitude of $T(j\omega)$ in decibels (**dB**) versus ω or $\log_{10}\omega$.*
- *The phase of $T(j\omega)$ in degree versus ω or $\log_{10}\omega$.*

$$T(j\omega) = |T(j\omega)| e^{j\angle T(j\omega)}$$

$$Logarithmic\ gain = 20 \cdot \log_{10} |T(j\omega)|$$

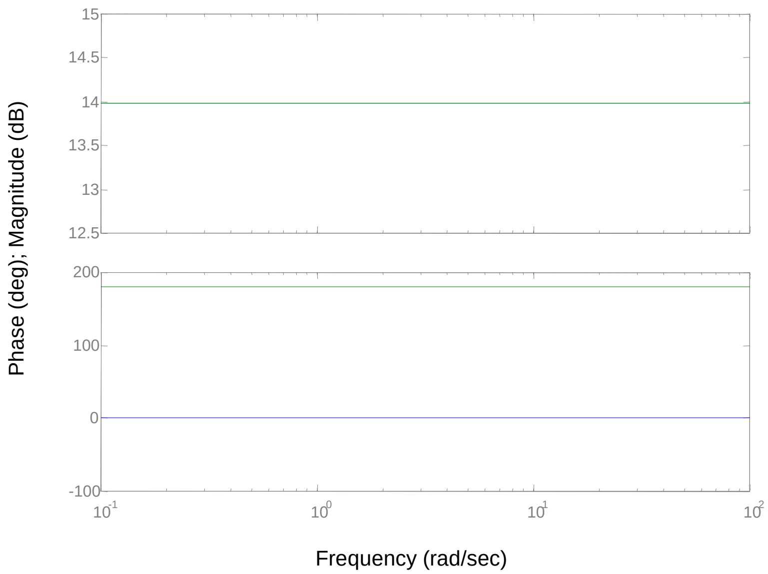


- *Constant factor: K*

$$K\big|_{dB} = 20 \cdot \log|K|$$

and

$$\angle K = \begin{cases} 0^\circ & for \quad K > 0 \\ \pm 180^\circ & for \quad K < 0 \end{cases}$$

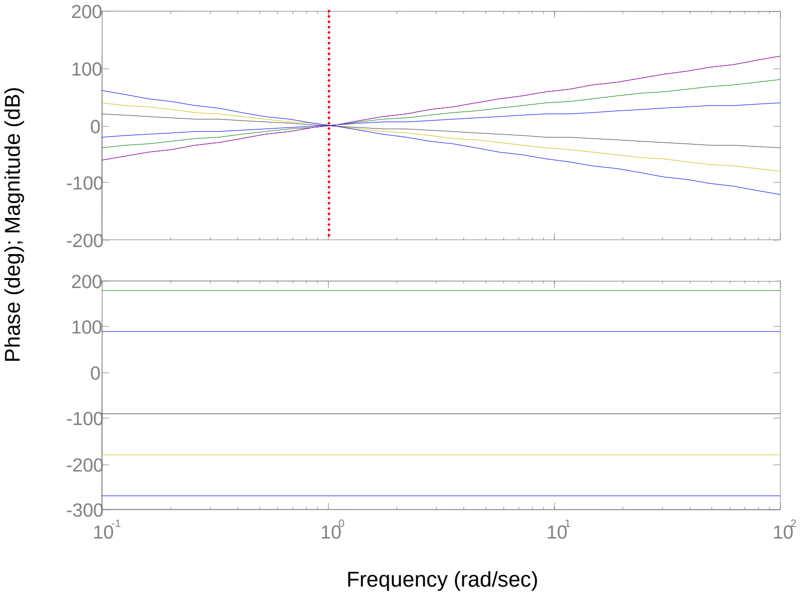


$$|T(j\omega)|_{dB} = 20 \cdot \log\left|(j\omega)^{\pm p}\right| = \pm 20p \cdot \log(\omega)$$

$$\frac{d}{d(\log\omega)}[\pm 20p \cdot \log(\omega)] = \pm 20p \quad dB/decade$$

and

$$\angle(j\omega)^{\pm p} = \pm p \times 90°$$



- *Simple zero:* $1+s\tau$

$$|T(j\omega)|_{dB} = 20\cdot\log|T(j\omega)| = 20\cdot\log\sqrt{1+(\omega\tau)^2}$$

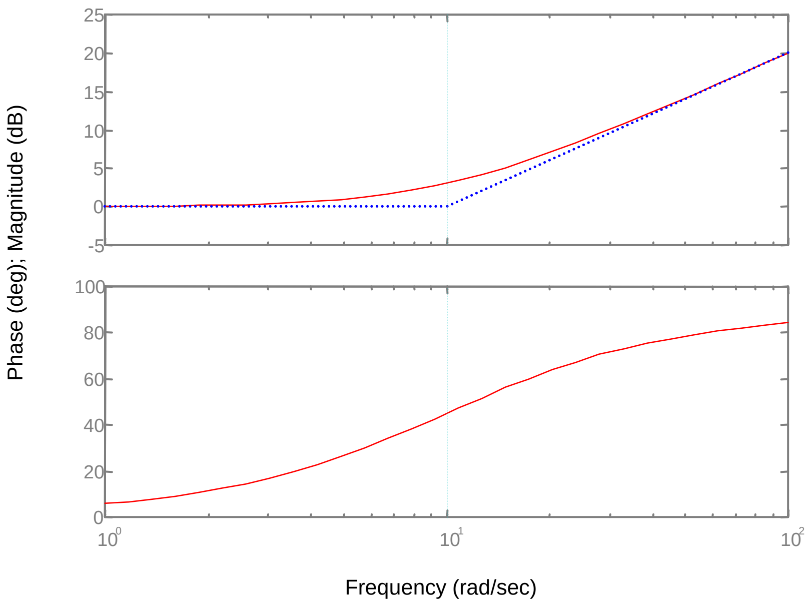
$$\omega\tau << 1 \Rightarrow |T(j\omega)|_{dB} \cong 20\cdot\log(1) = 0$$

$$\omega\tau >> 1 \Rightarrow |T(j\omega)|_{dB} \cong 20\cdot\log\sqrt{(\omega\tau)^2} = 20\cdot\log(\omega\tau)$$

and

$$\angle T(j\omega) = \tan^{-1}(\omega\tau)$$

$$\omega = \frac{1}{\tau} \Rightarrow \begin{cases} |T(j\omega)|_{dB} = 20\cdot\log\sqrt{2} \cong 3 \\ \angle T(j\omega) = \tan^{-1}(1) = 45° \end{cases}$$



- *Simple pole:* $\dfrac{1}{1+s\tau}$

$$\left|T(j\omega)\right|_{dB} = 20\cdot\log\left|T(j\omega)\right| = 20\cdot\log \frac{1}{\sqrt{1+(\omega\tau)^2}}$$

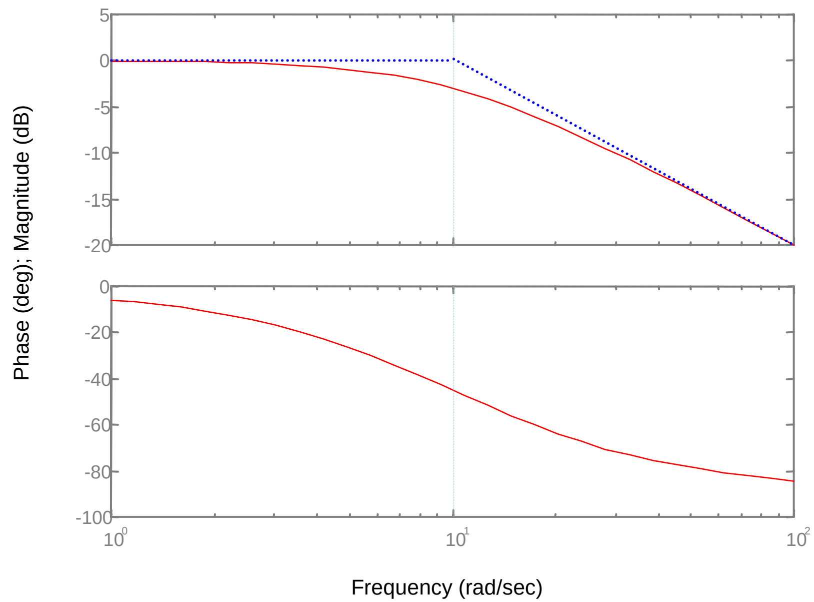
$$\omega\tau << 1 \Rightarrow \left|T(j\omega)\right|_{dB} \cong 20\cdot\log(1) = 0$$

$$\omega\tau >> 1 \Rightarrow \left|T(j\omega)\right|_{dB} \cong -20\cdot\log(\omega\tau)$$

and

$$\angle T(j\omega) = -\tan^{-1}(\omega\tau)$$

$$\omega = \frac{1}{\tau} \Rightarrow \begin{cases} \left|T(j\omega)\right|_{dB} = -20\cdot\log\sqrt{2} \cong -3 \\ \angle T(j\omega) = -\tan^{-1}(1) = -45^\circ \end{cases}$$



- *Quadratic poles:* $\dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$$\left|T(j\omega)\right|_{dB} = 20 \cdot \log \frac{1}{\sqrt{\left[1-(\frac{\omega}{\omega_n})^2\right]^2 + \left[2\zeta(\frac{\omega}{\omega_n})\right]^2}}$$

$$\frac{\omega}{\omega_n} << 1 \Rightarrow \left|T(j\omega)\right|_{dB} \cong 20 \cdot \log(1) = 0$$

$$\frac{\omega}{\omega_n} >> 1 \Rightarrow \left|T(j\omega)\right|_{dB} \cong -20 \cdot \log\left(\frac{\omega}{\omega_n}\right)^2 = -40 \cdot \log\left(\frac{\omega}{\omega_n}\right)$$

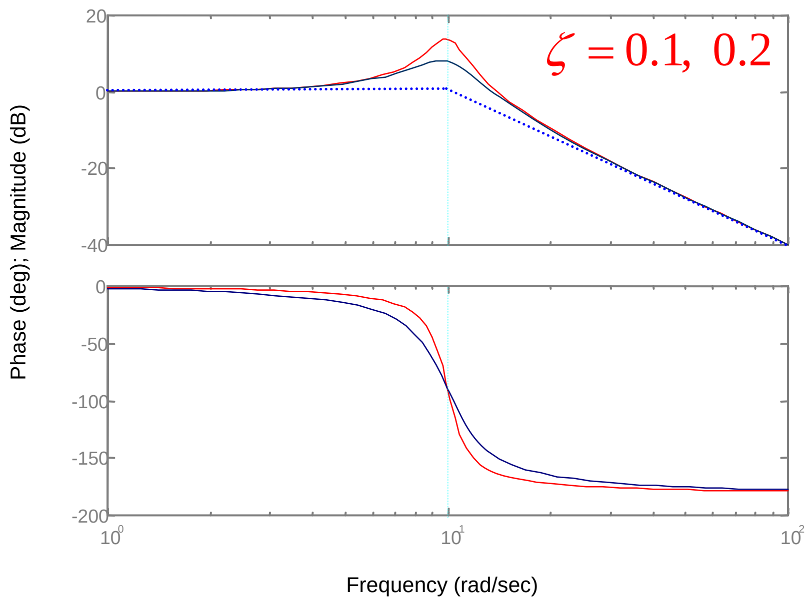
and

$$\angle T(j\omega) = -\tan^{-1}\left[\frac{2\zeta(\frac{\omega}{\omega_n})}{1-(\frac{\omega}{\omega_n})^2}\right]$$

$$\frac{\omega}{\omega_n} << 1 \Rightarrow \angle T(j\omega) = 0^\circ$$

$$\frac{\omega}{\omega_n} >> 1 \Rightarrow \angle T(j\omega) = -180^\circ$$

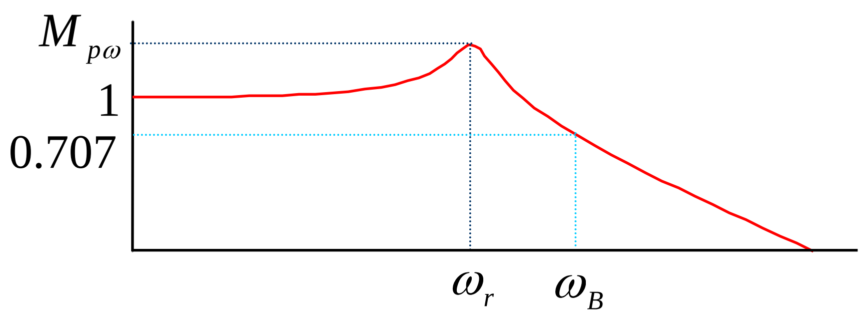
$$\omega = \omega_n \Rightarrow \left|T(j\omega)\right|_{dB} = -20 \cdot \log(2\zeta)$$
$$\omega = \omega_n \Rightarrow \angle T(j\omega) = -90^\circ$$



- ***Log Magnitude and Phase Plot***

☑Performance Specifications in The Frequency Domain

- ***Resonant peak: $M_{p\omega}$***
  The resonant peak is the maximum value of $|T(j\omega)|$.
- ***Resonant frequency: ω_r***
  The resonant frequency is the frequency at which the peak resonant occur.
- ***Bandwidth: BW (ω_B)***
  The bandwidth is the frequency at which $T(j\omega)$ drops to 70.7% of, or 3 dB down from, its zero-frequency value.
- ***Cutoff rate***
  The cutoff rate is the slop of log-magnitude curve near the cutoff frequency.



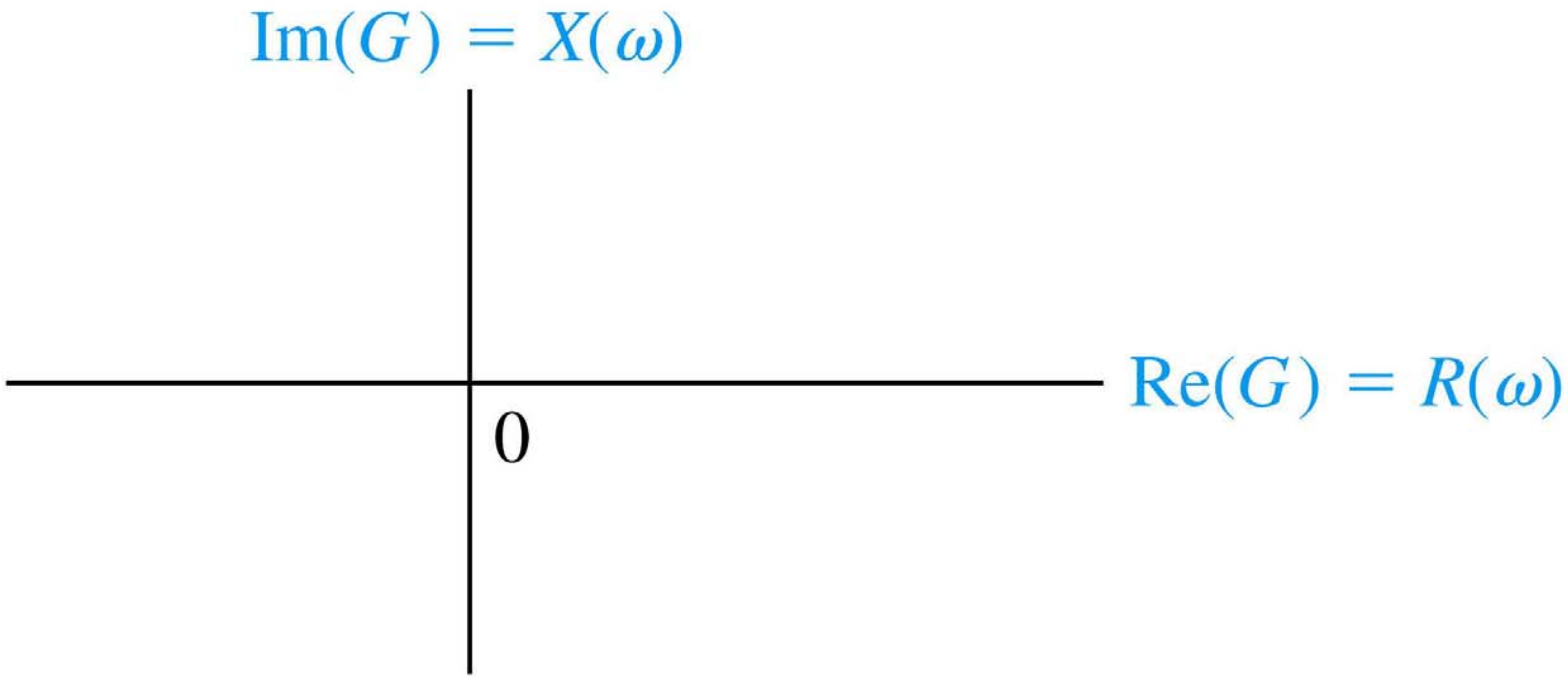


FIGURE 8.1

The polar plane.

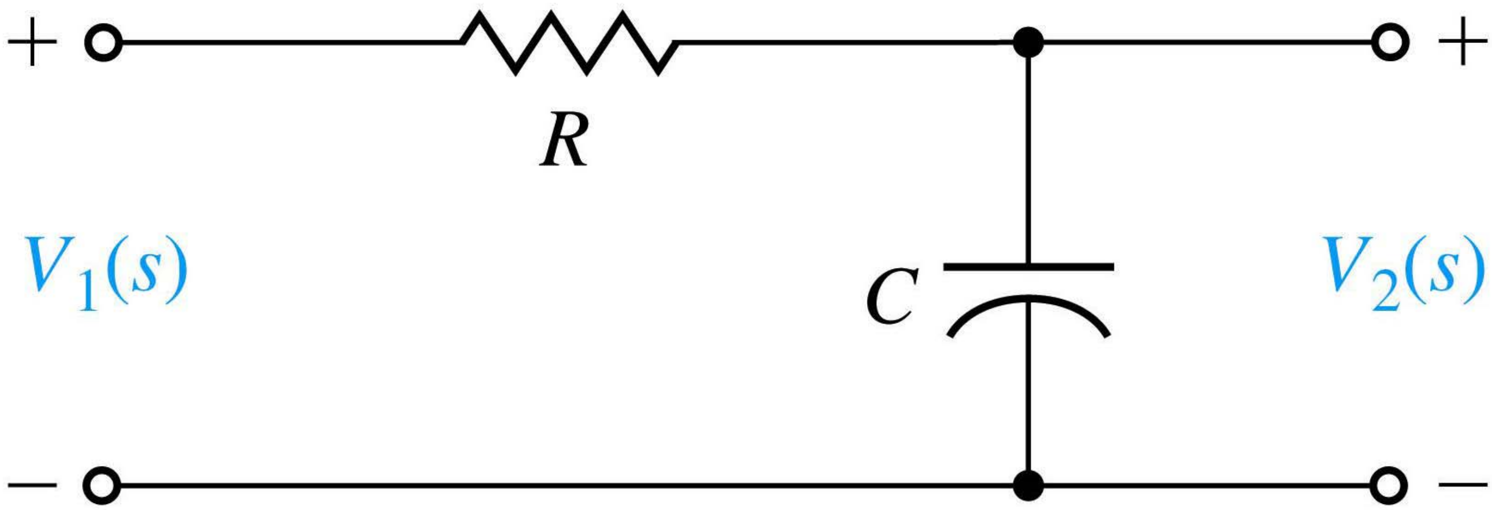


FIGURE 8.2

An *RC* filter.

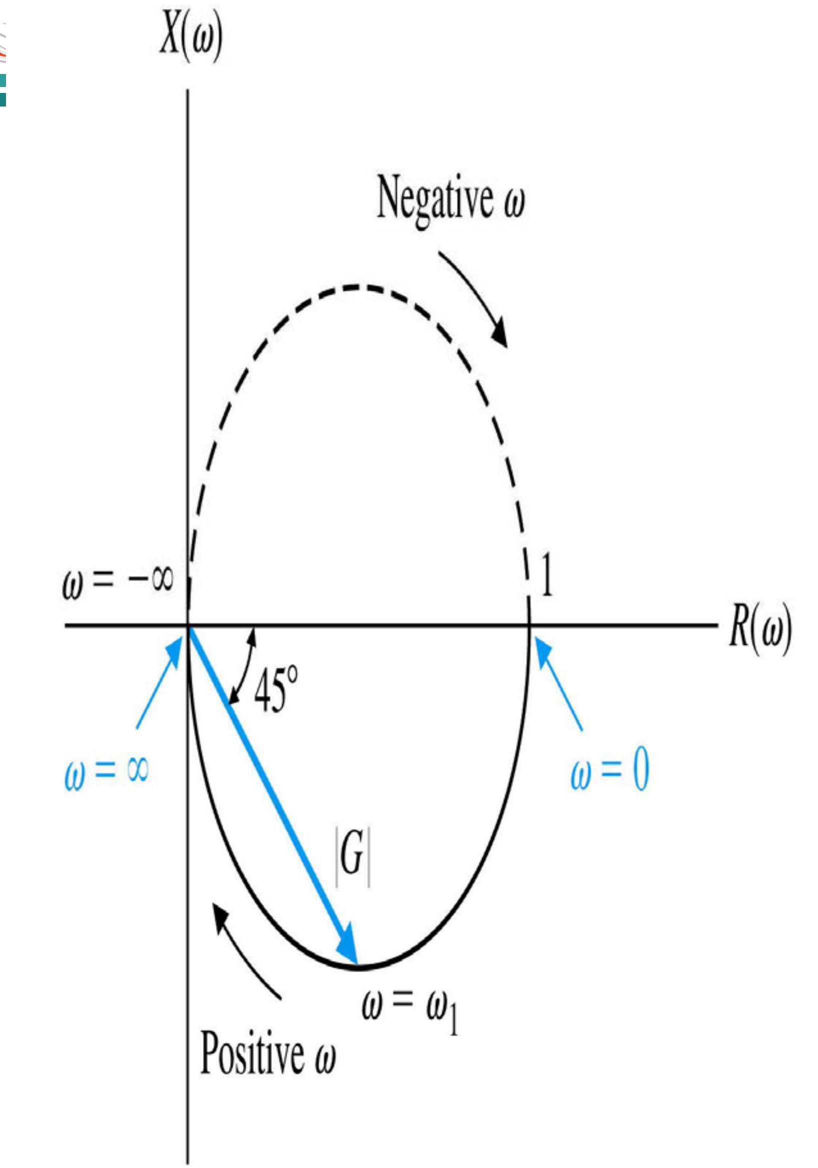


FIGURE 8.3

Polar plot for *RC* filter.

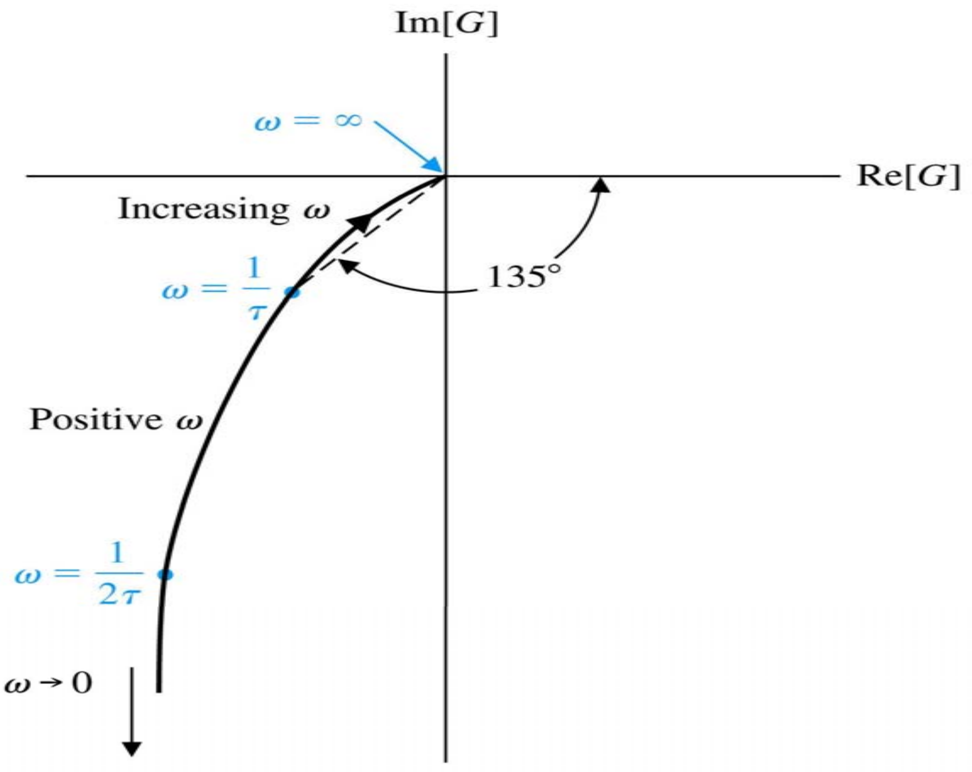


FIGURE 8.4

Polar plot for $G(j\omega) = K/j\omega(j\omega\tau + 1)$. Note that $\omega = \infty$ at the origin.

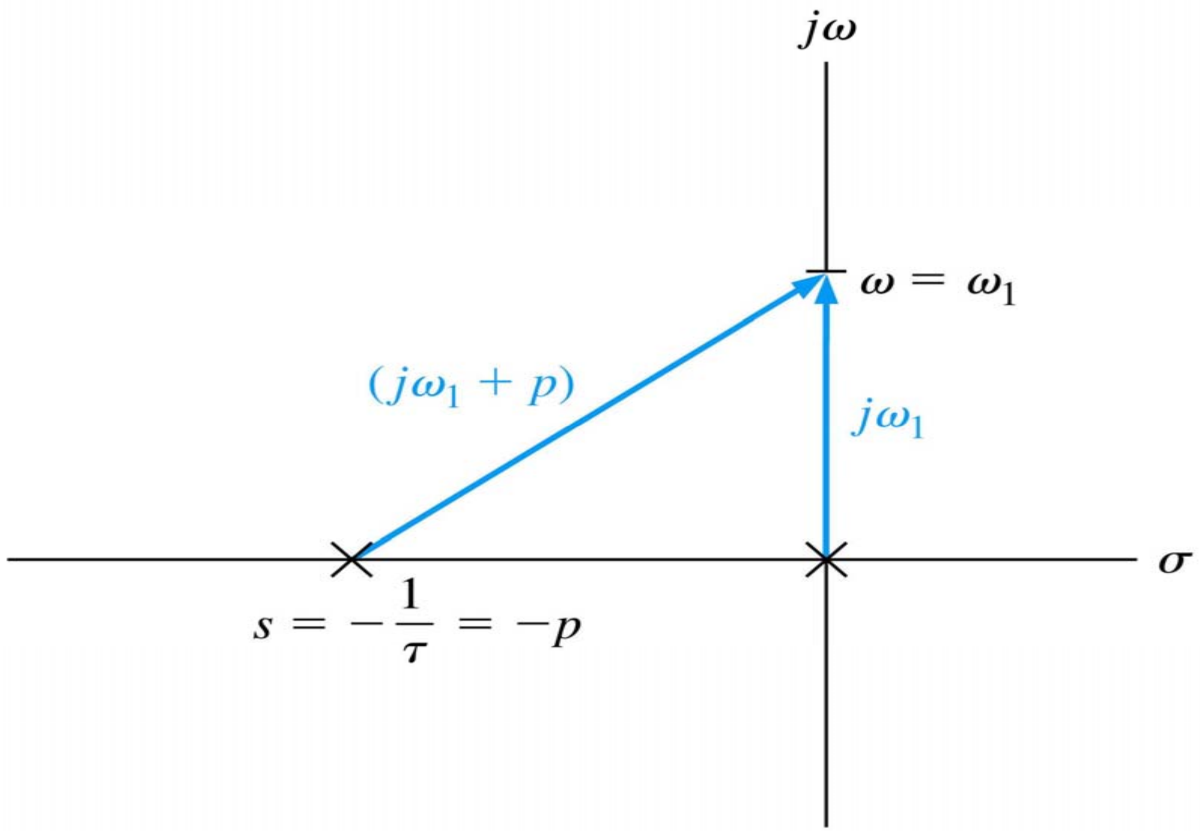


FIGURE 8.5

Two vectors on the s-plane to evaluate $G(j\omega_1)$.

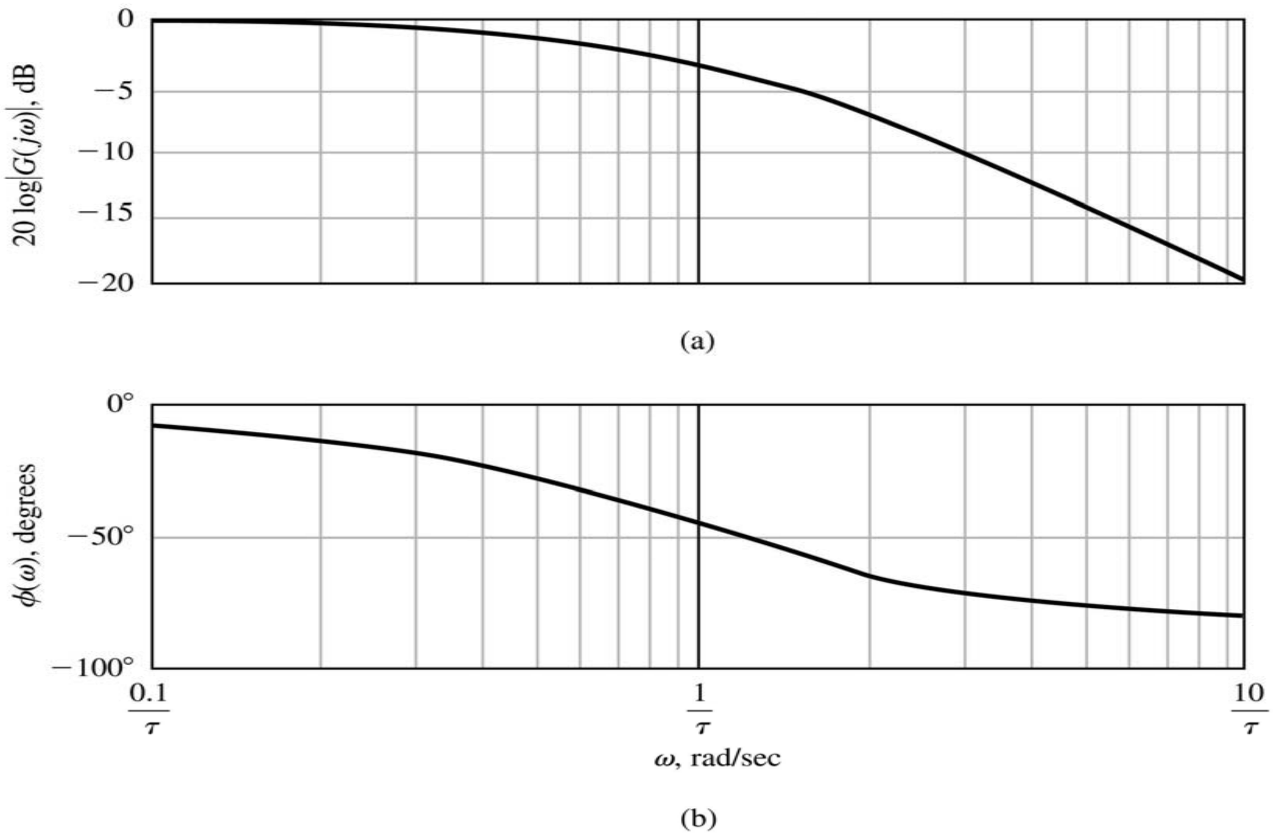


FIGURE 8.6

Bode diagram for $G(j\omega) = 1/(j\omega\tau + 1)$: (a) magnitude plot and (b) phase plot.

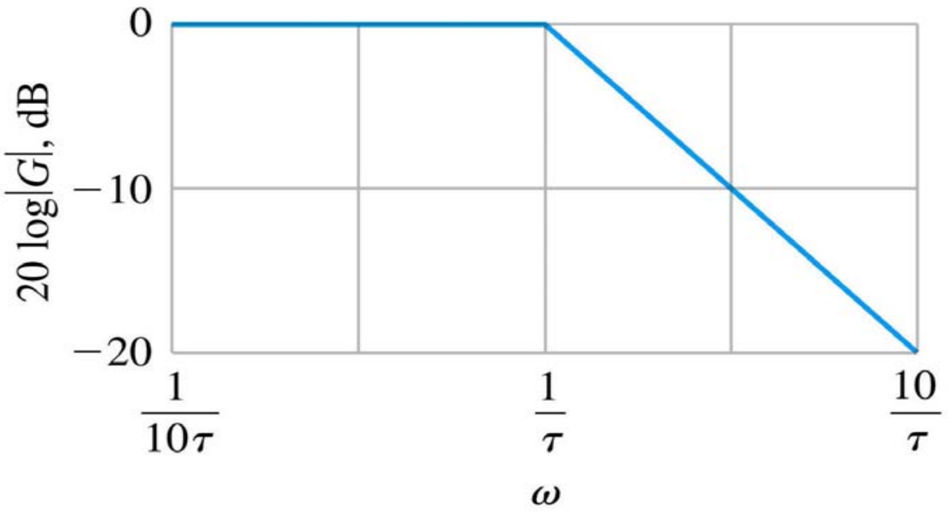


FIGURE 8.7

Asymptotic curve for $(j\omega\tau + 1)^{-1}$.

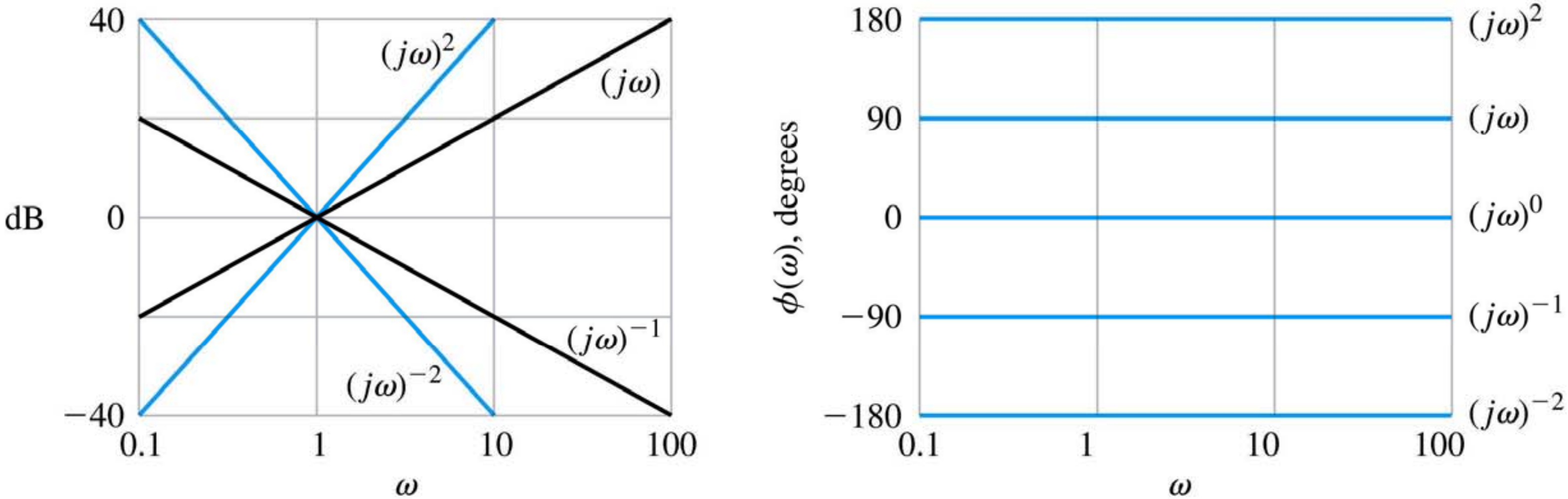


FIGURE 8.8

Bode diagram for $(j\omega)^{\pm N}$.

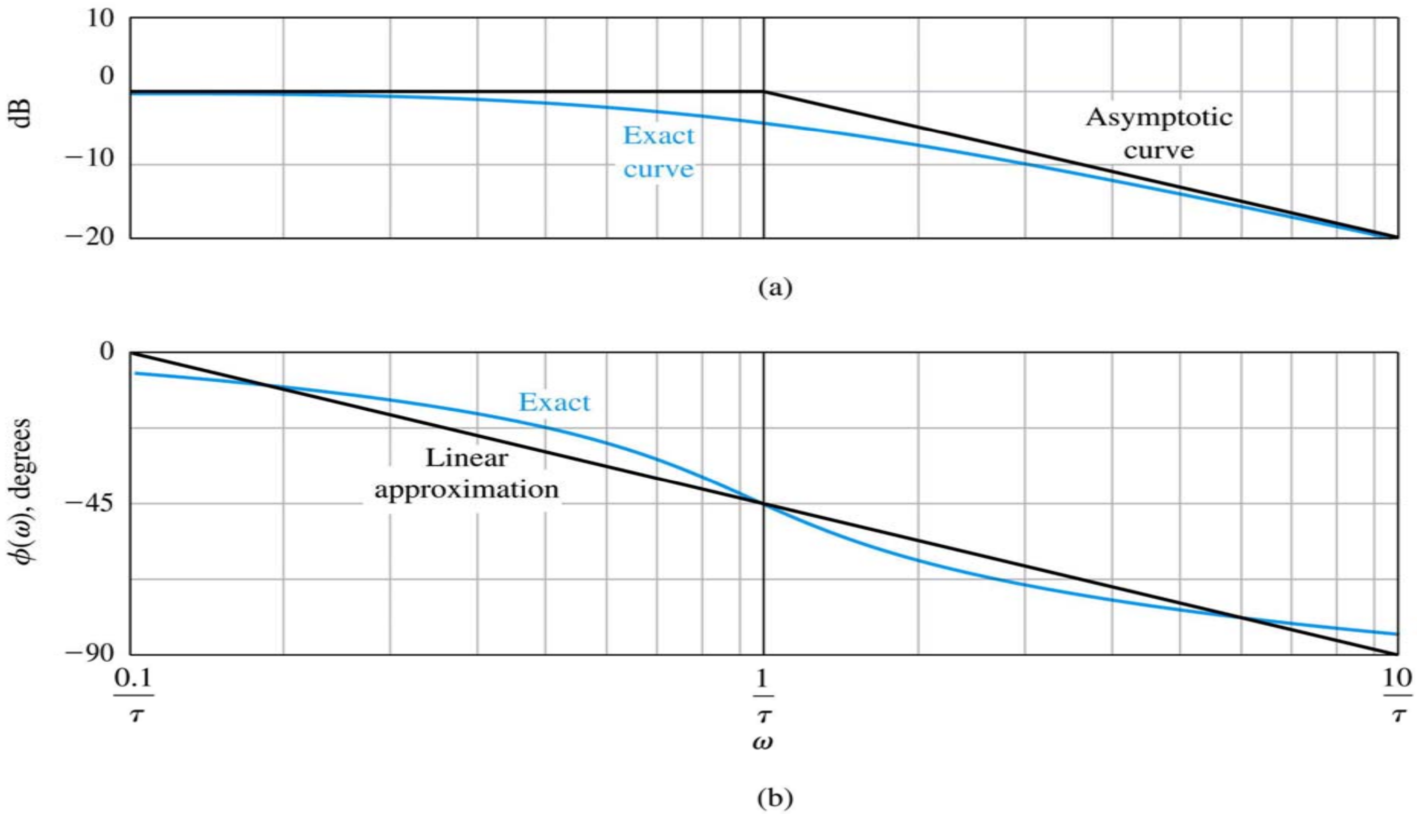


FIGURE 8.9

Bode diagram for $(1 + j\omega\tau)^{-1}$.

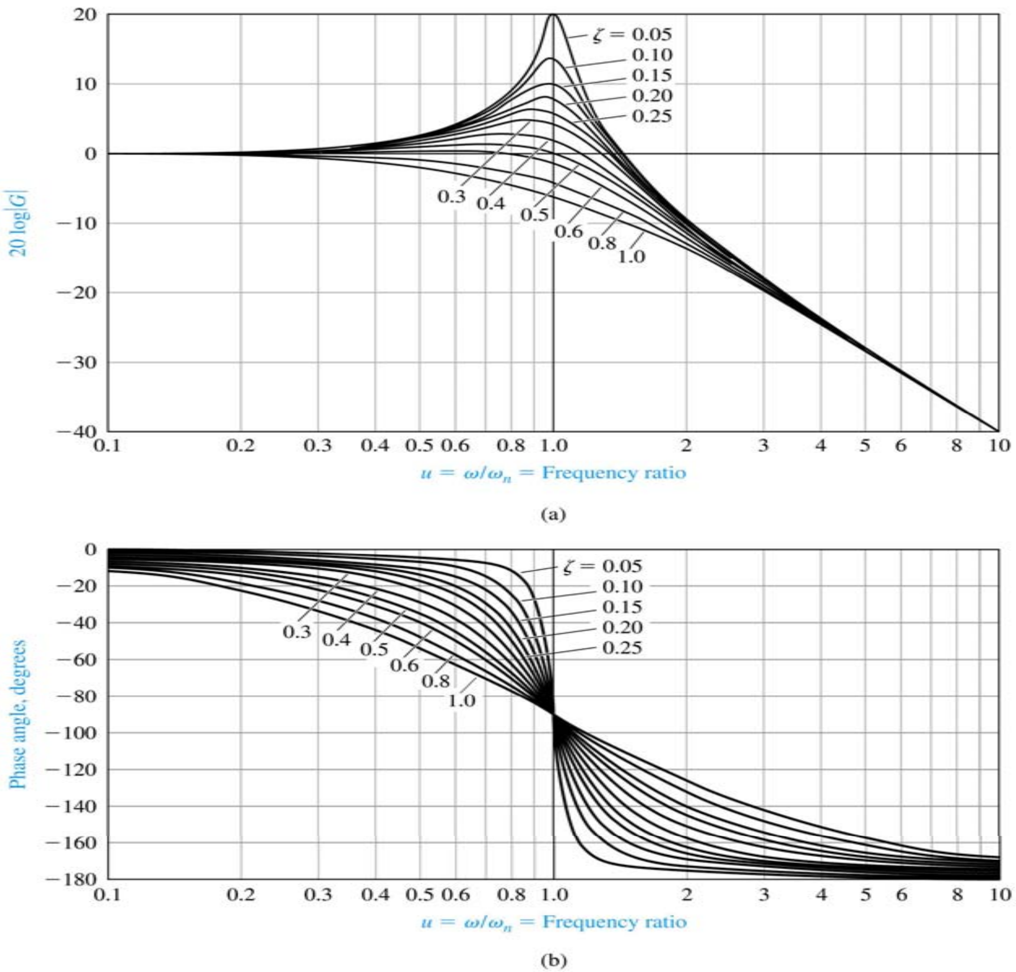


FIGURE 8.10

Bode diagram for $G(j\omega) = [1 + (2\zeta/\omega_n)j\omega + (j\omega/\omega_n)^2]^{-1}$.

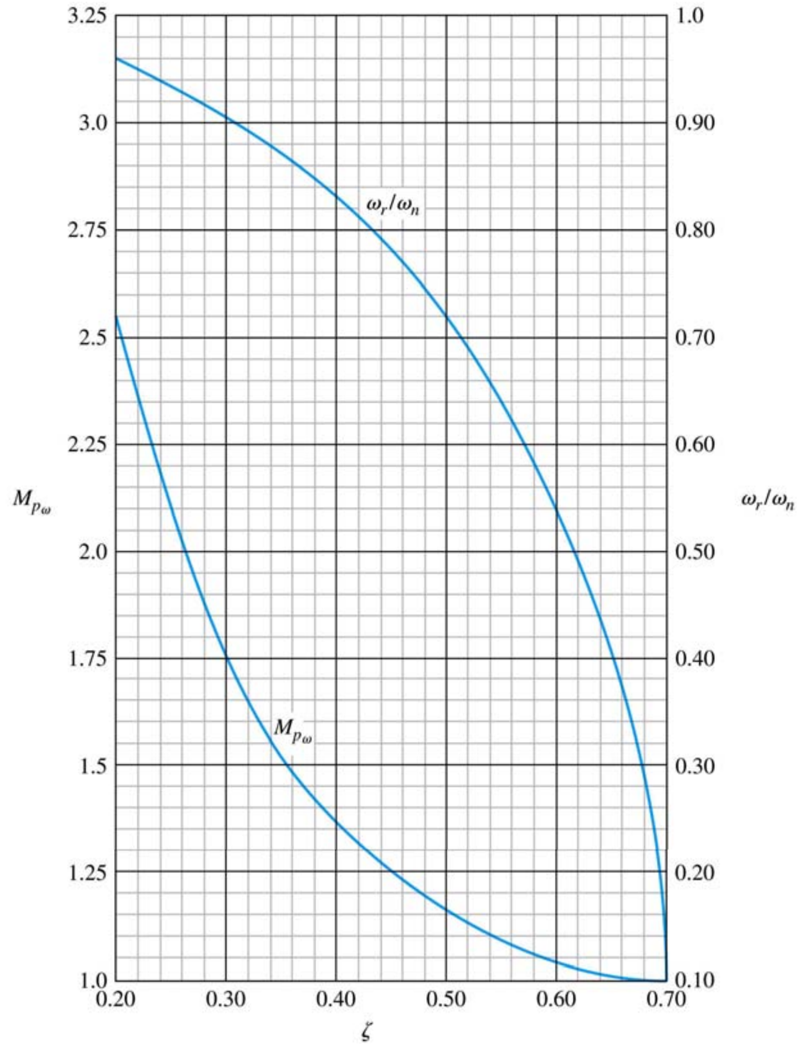


FIGURE 8.11

The maximum of the frequency response $M_{p\omega}$ and the resonant

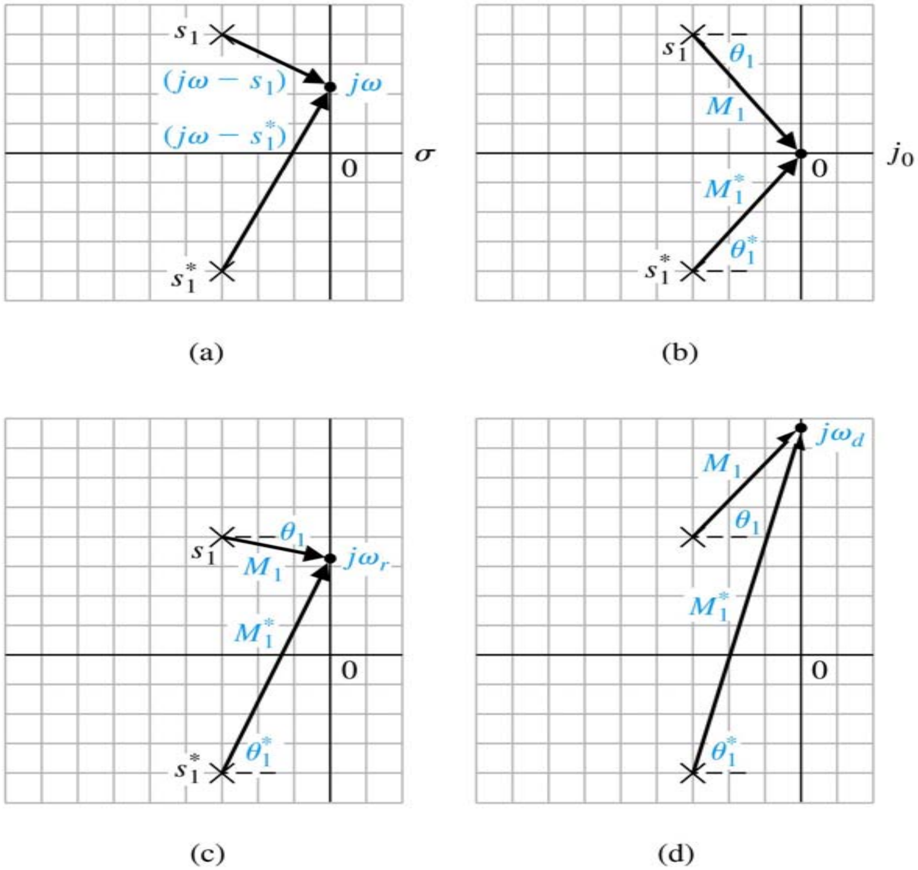


FIGURE 8.12

Vector evaluation of the frequency response for selected values of ω.

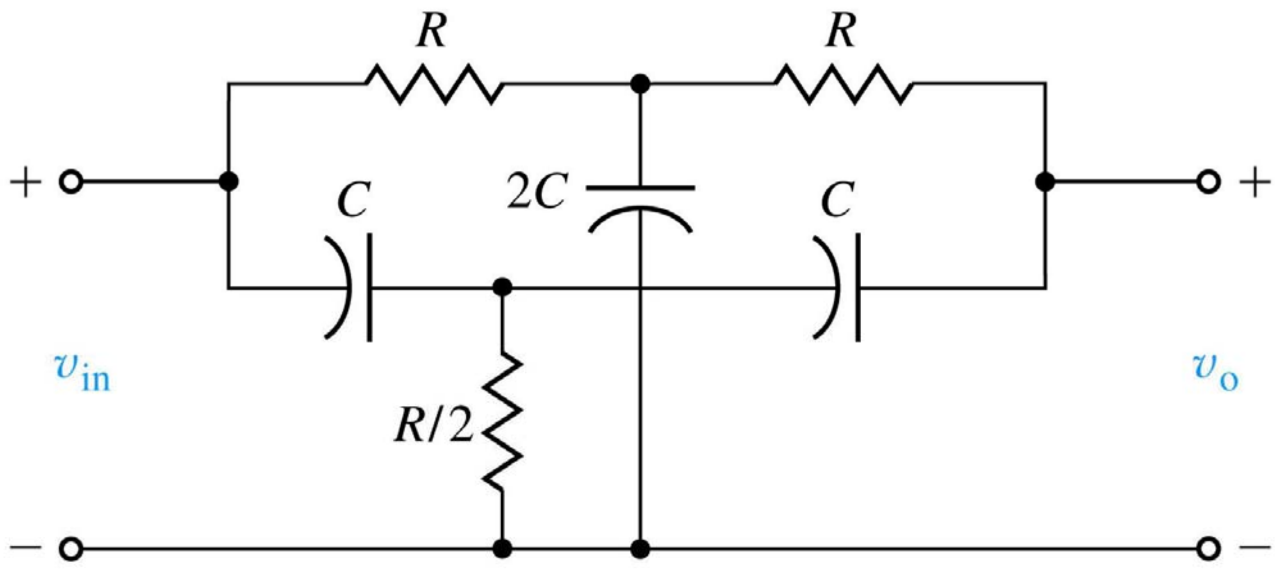


FIGURE 8.14

Twin-T network.

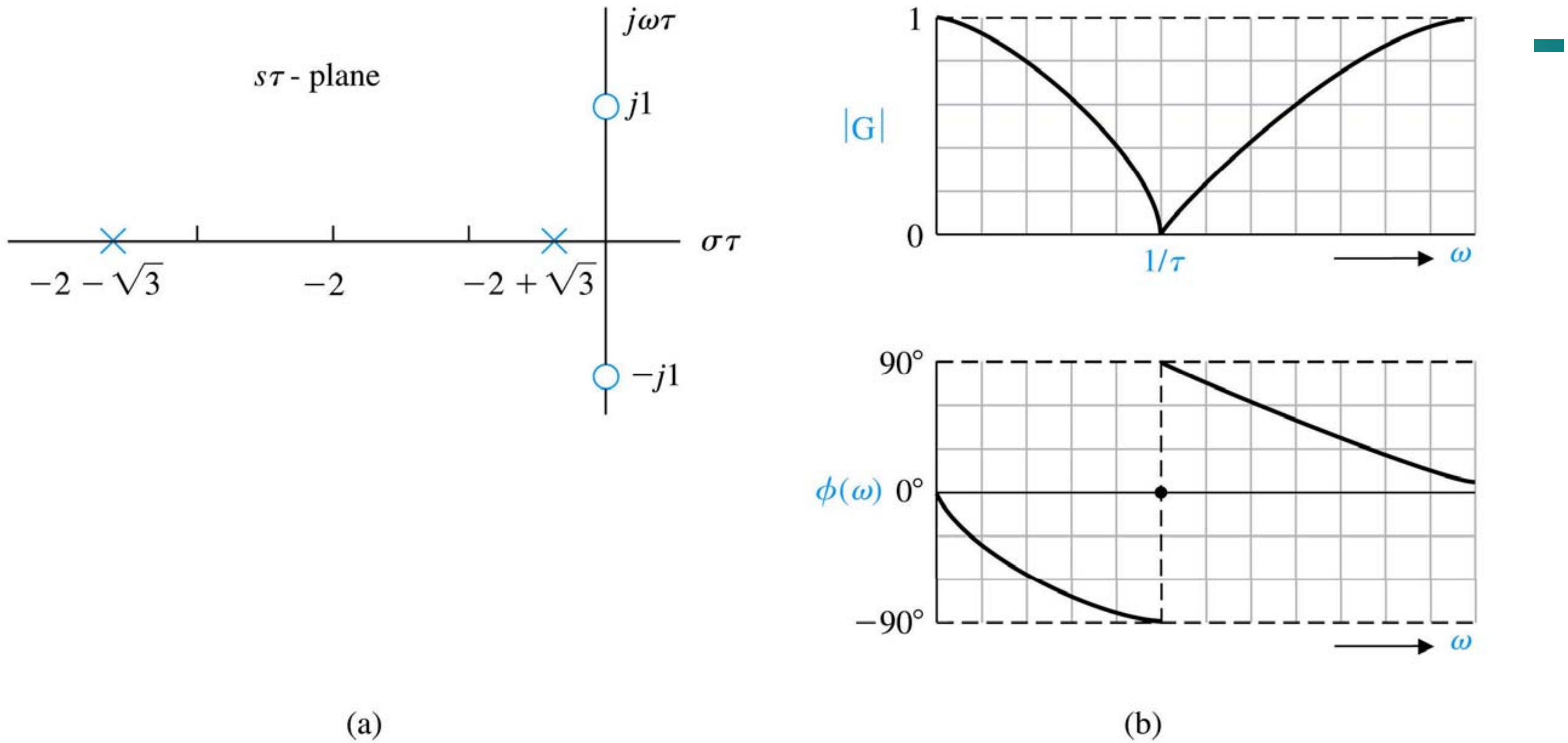


FIGURE 8.15

Twin-T network.

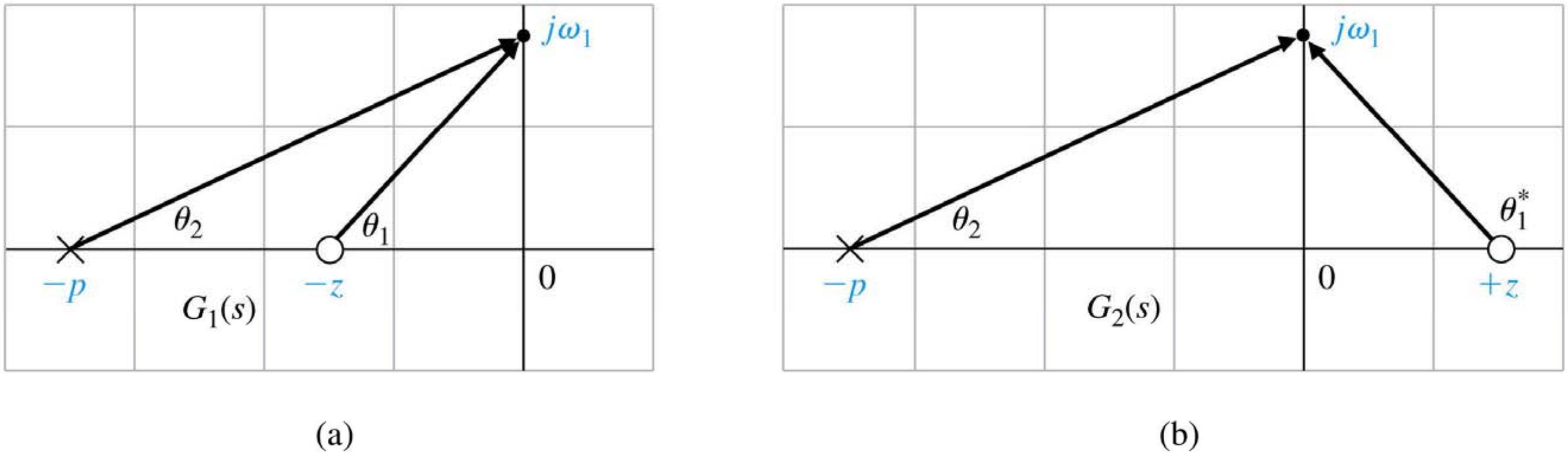


FIGURE 8.16

Pole–zero patterns giving the same amplitude response and different phase characteristics.

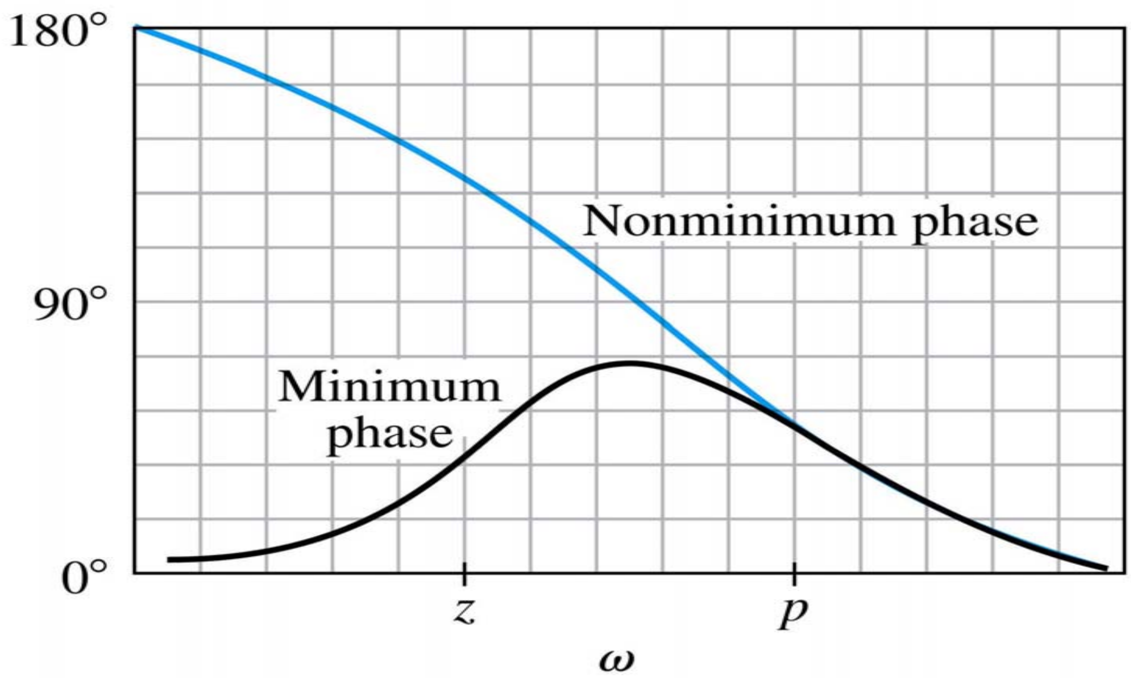


FIGURE 8.17

The phase characteristics for the minimum phase and nonminimum phase transfer function.

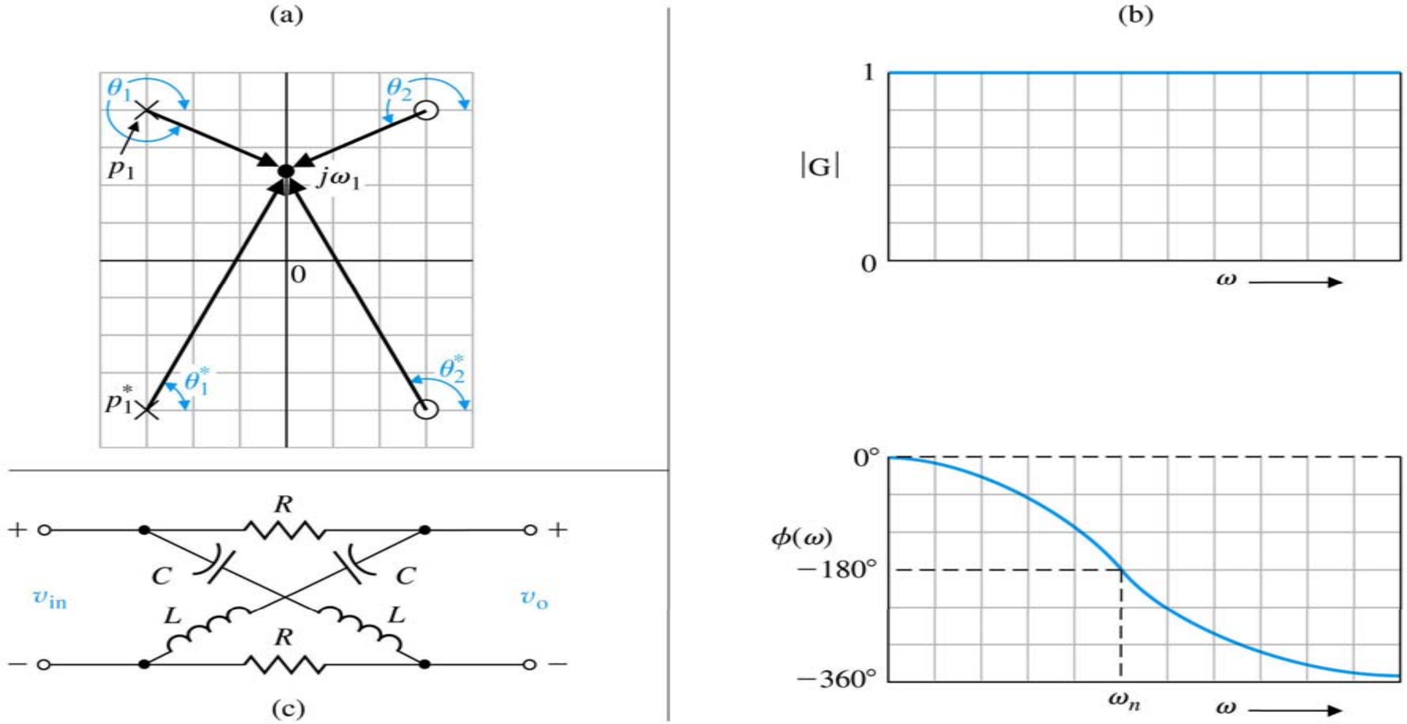


FIGURE 8.18

The all-pass network (a) pole–zero pattern, and (b) frequency response, and (c) a lattice network.

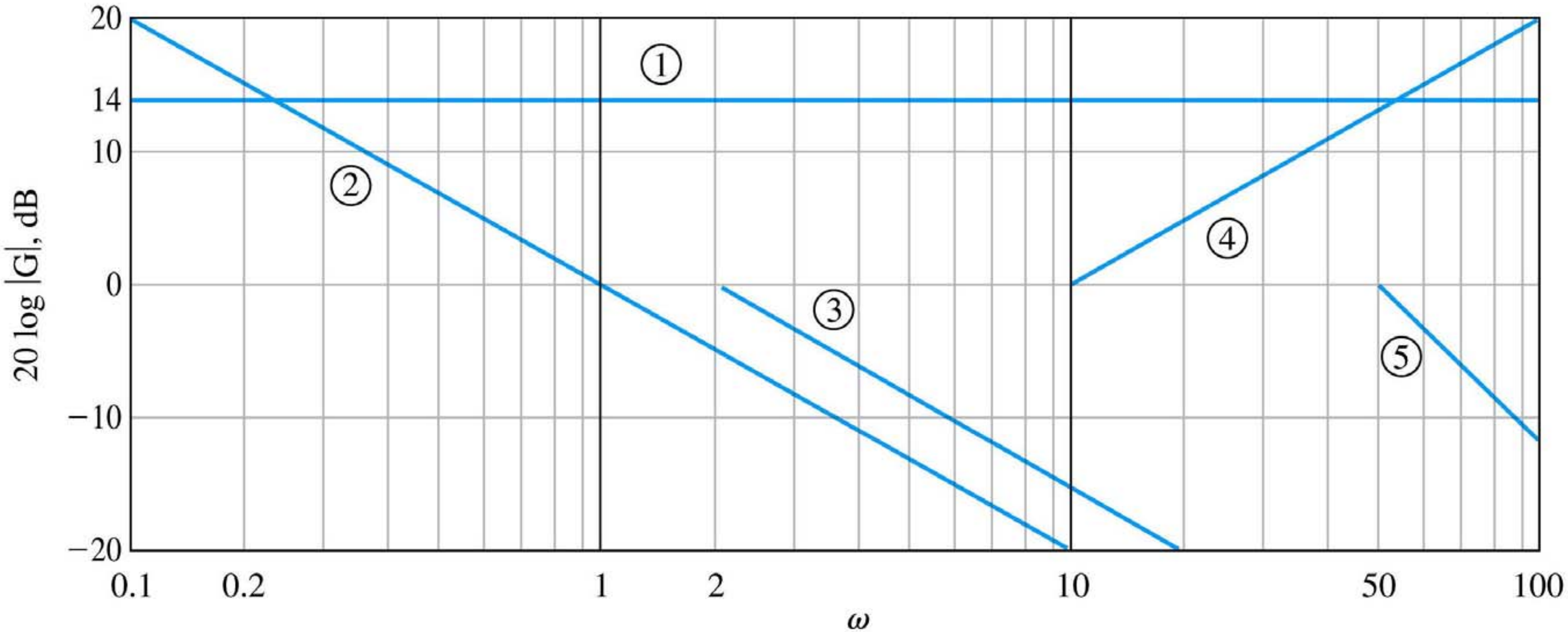


FIGURE 8.19

Magnitude asymptotes of poles and zeros used in the example.

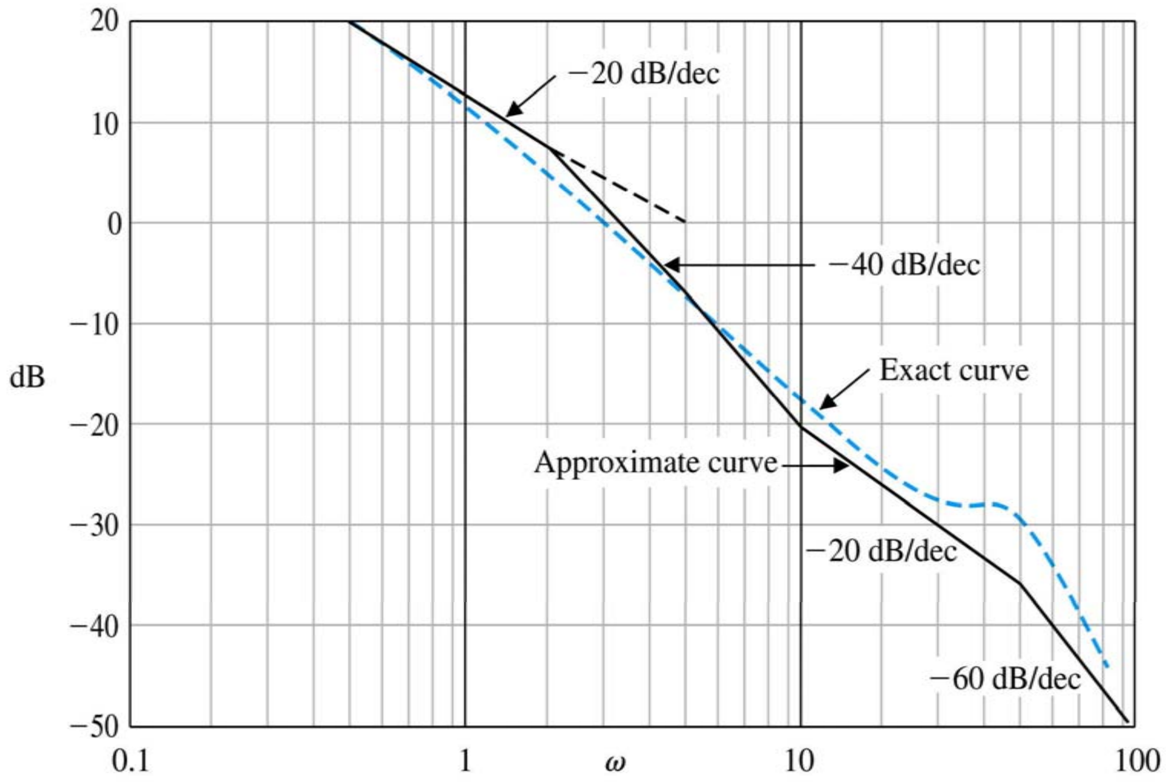


FIGURE 8.20

Magnitude characteristic.

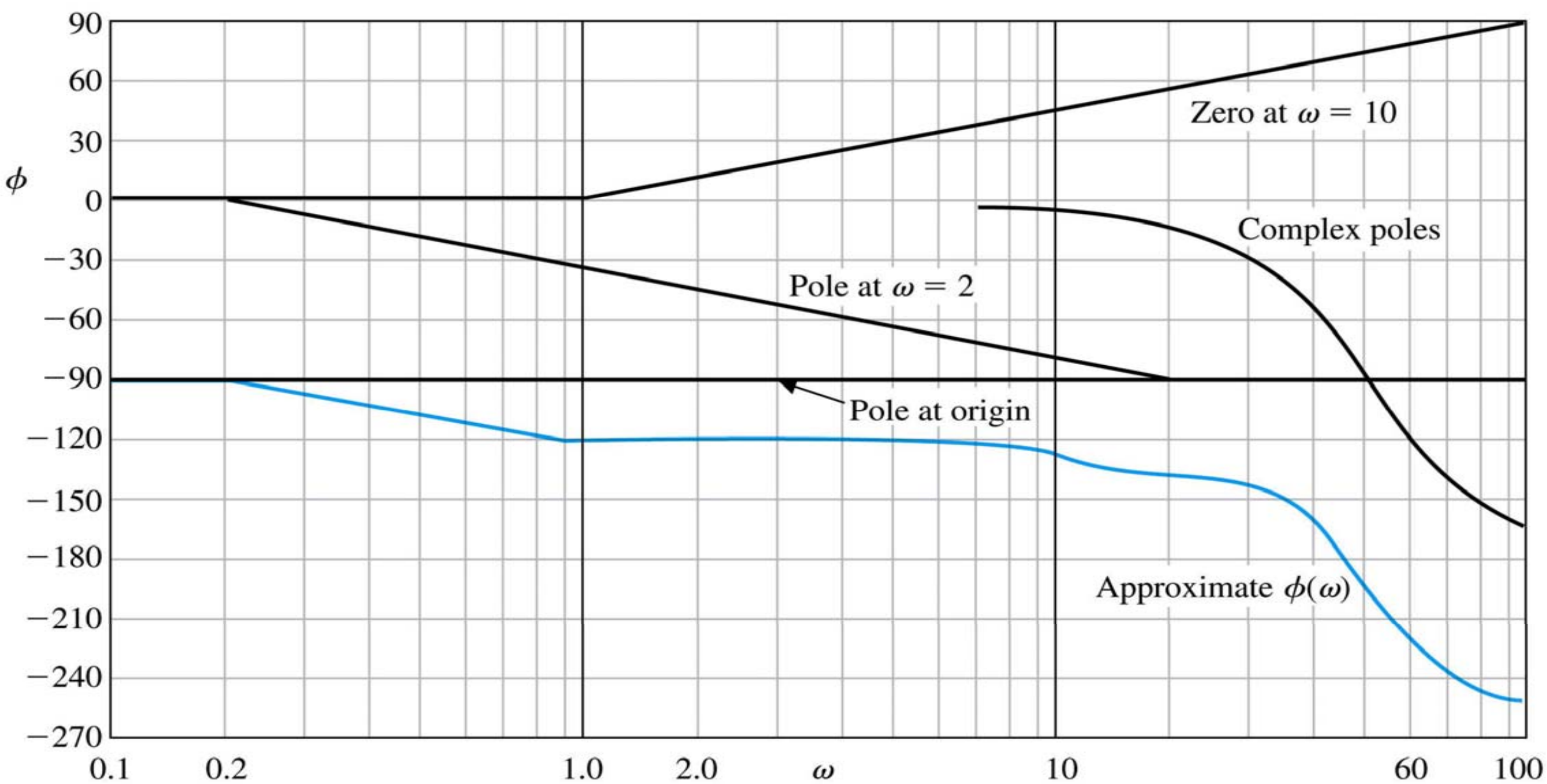


FIGURE 8.21

Phase characteristic.

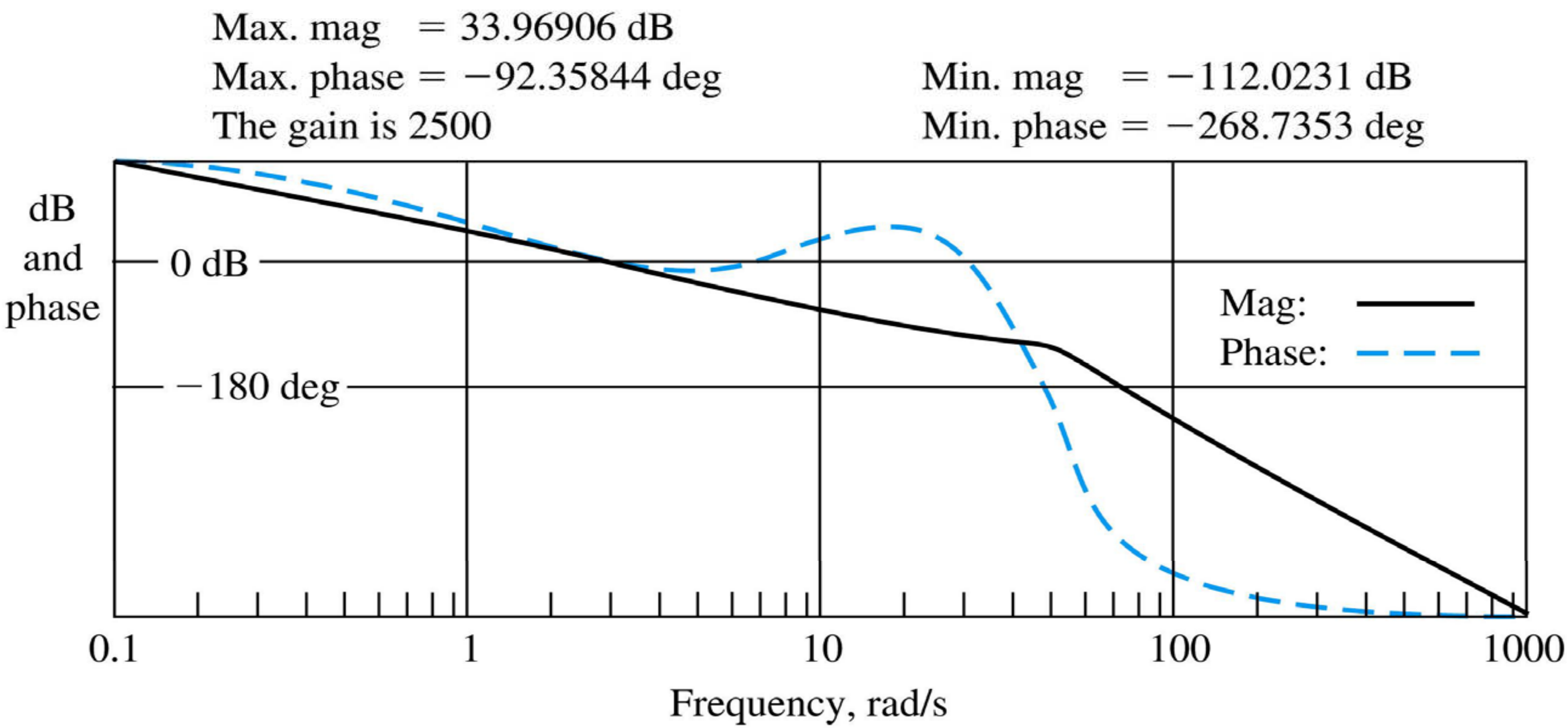


FIGURE 8.22

The Bode plot of the $G(j\omega)$ of Eq. (8.42).

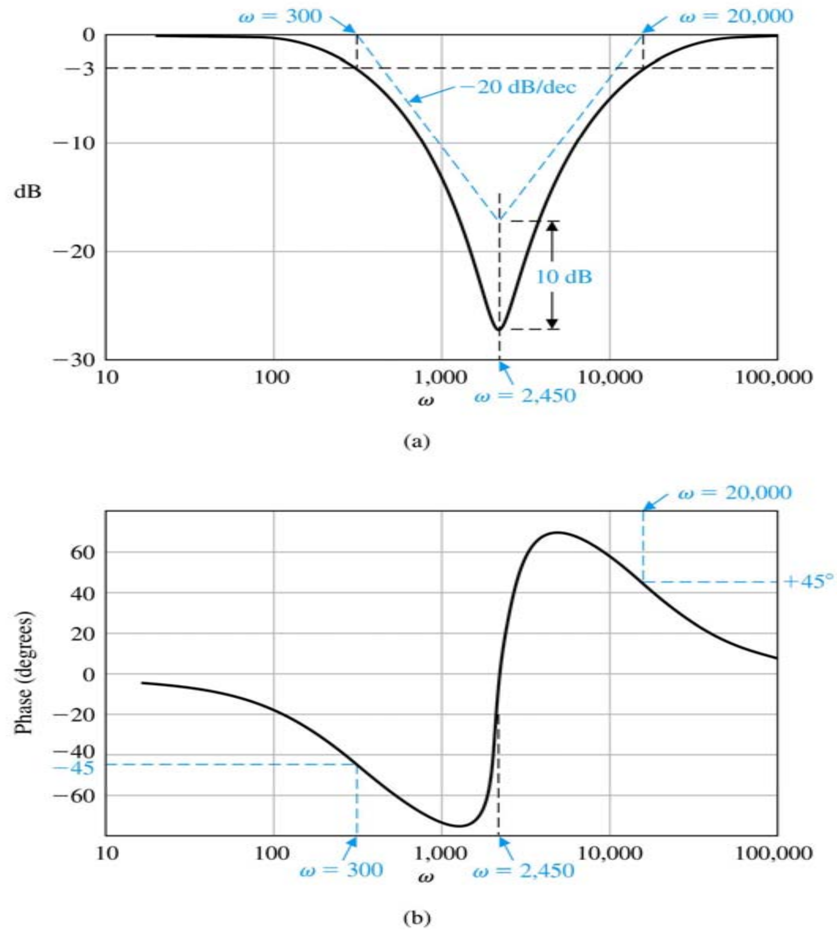


FIGURE 8.23

A Bode diagram for a system with an unidentified transfer function.

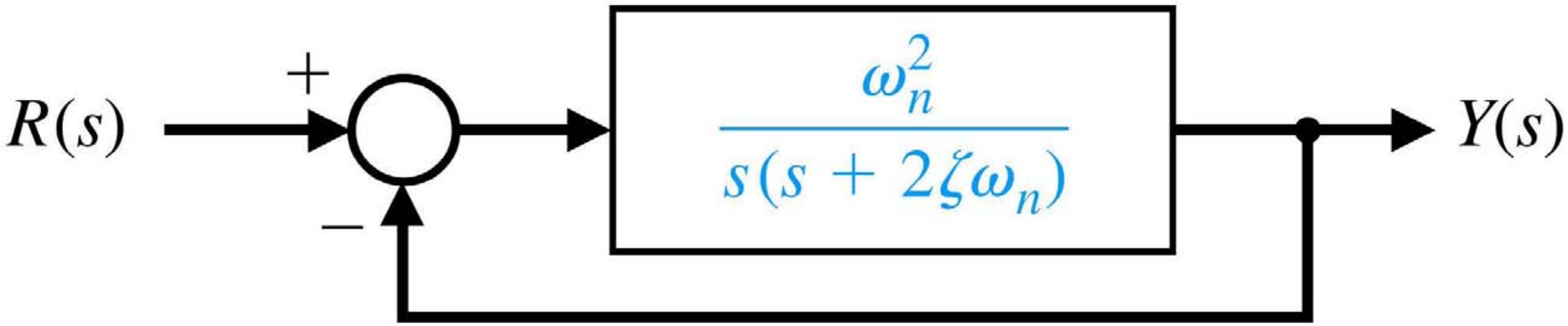


FIGURE 8.24

A second-order closed-loop system.

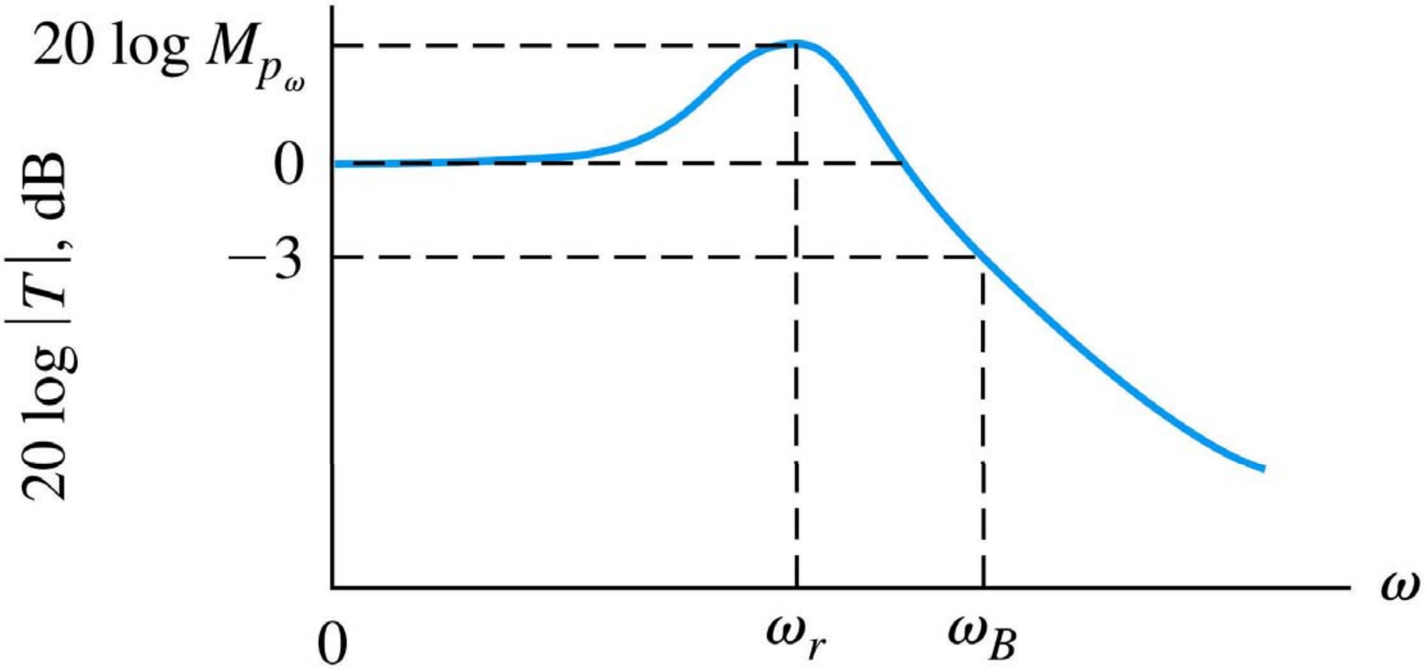


FIGURE 8.25

Magnitude characteristic of the second-order system.

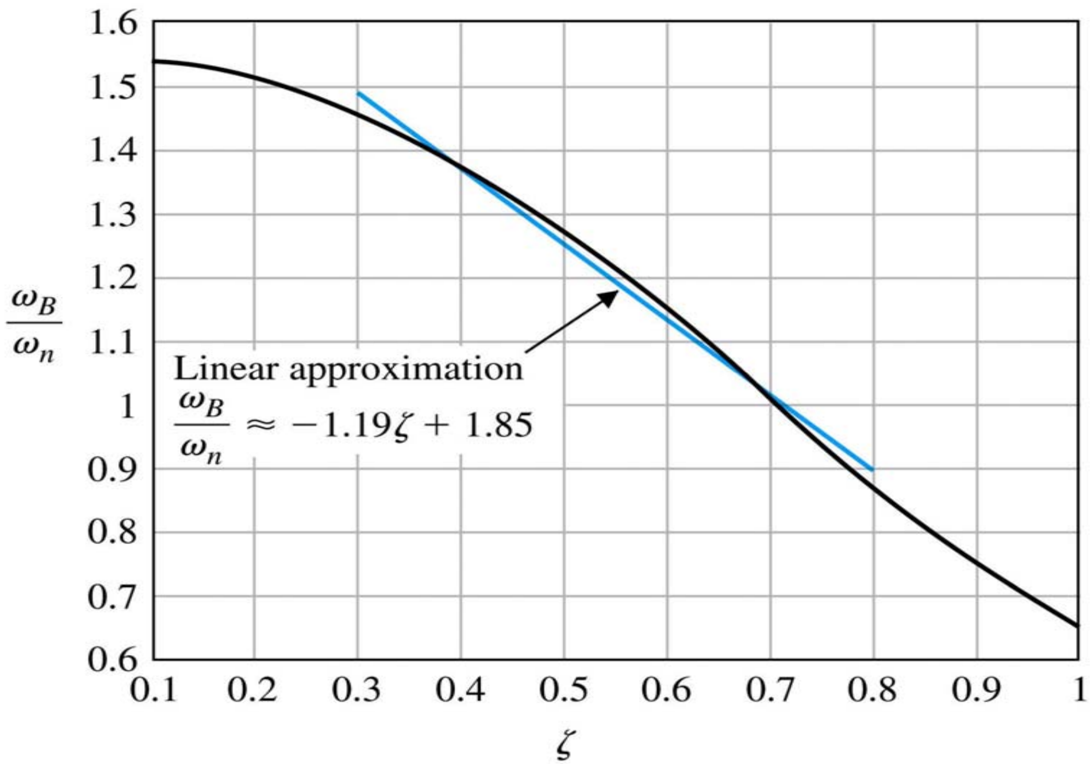


FIGURE 8.26

Normalized bandwidth, ω_B/ω_n, versus ζ for a second-order system (Eq. 8.46). The linear approximation $\omega_B/\omega_n = -1.19\zeta + 1.85$ is accurate for $0.3 \leq \zeta \leq 0.8$.

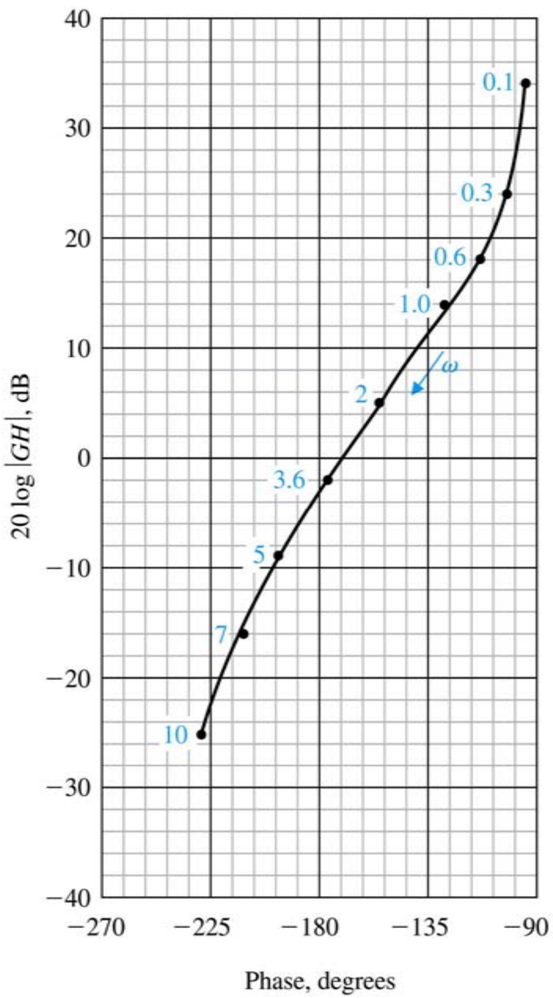


FIGURE 8.27

Log-magnitude–phase curve for $GH_1(j\omega)$.

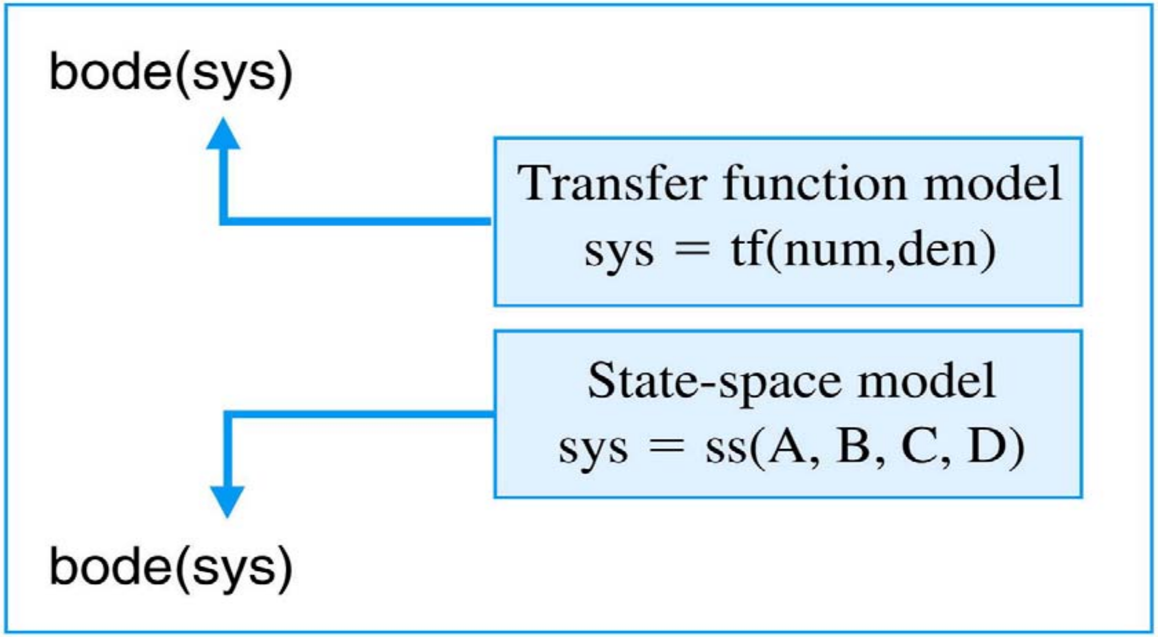
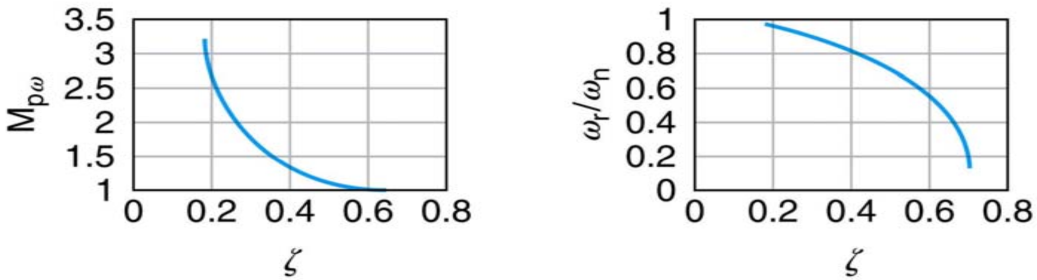


FIGURE 8.36

The bode function with a state variable model.



(a)

```
zeta=[0.15:0.01:0.7];
wr_over_wn=sqrt(1-2*zeta.^2);
Mp=(2*zeta .* sqrt(1-zeta.^2)).^(-1);
%
subplot(211),plot(zeta,Mp),grid
xlabel('\zeta'), ylabel('M_{p\omega}')
subplot(212),plot(zeta,wr_over_wn),grid
xlabel('\zeta'), ylabel('\omega_r/\omega_n')
```

zeta ranges from 0.15 to 0.70

Generate plots

(b)

FIGURE 8.37

(a) The relationship between (M_{p_ω}, ω_r) and (ζ, ω_n) for a second-order system. (b) MATLAB script.

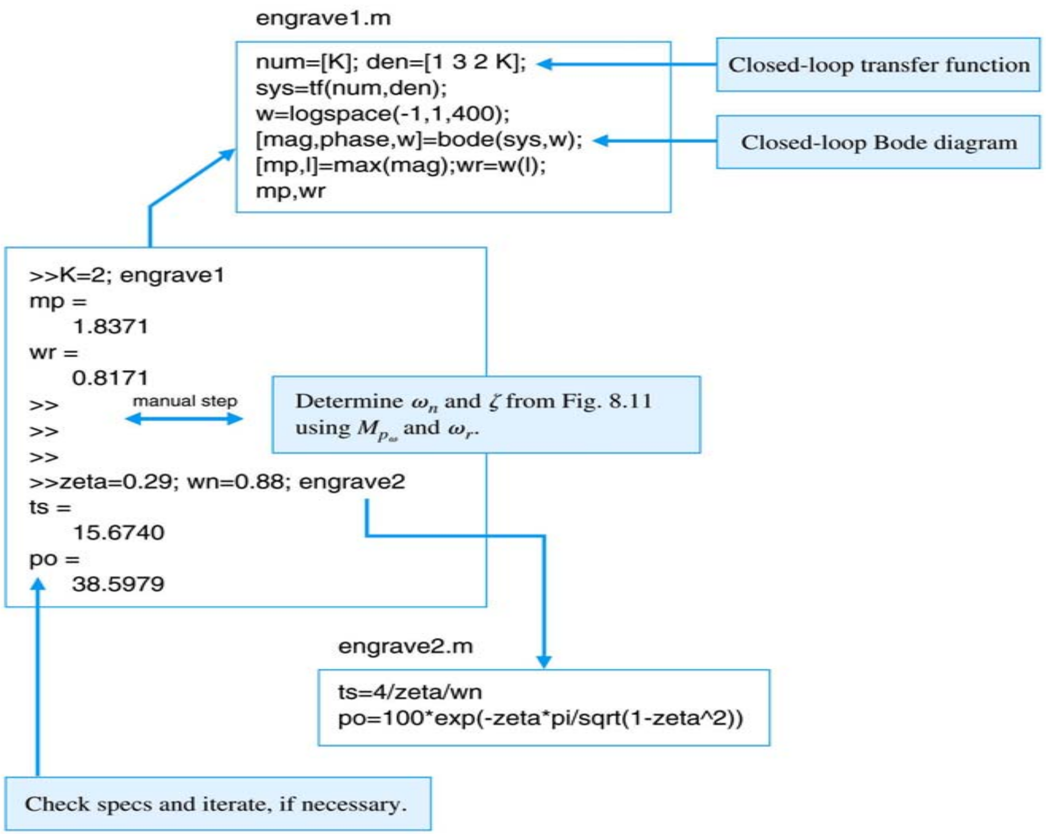


FIGURE 8.39

Script for the design of an engraving machine.

CHAPTER 9
Stability in the Frequency Domain

- **Introduction**
- **Mapping Contours in the s-Plane**
- **The Nyquist Criterion**
- **Relative Stability and the Nyquist Criterion**
- ***Time-Domain performance Criteria Specified in the Frequency Domain***

☑Introduction

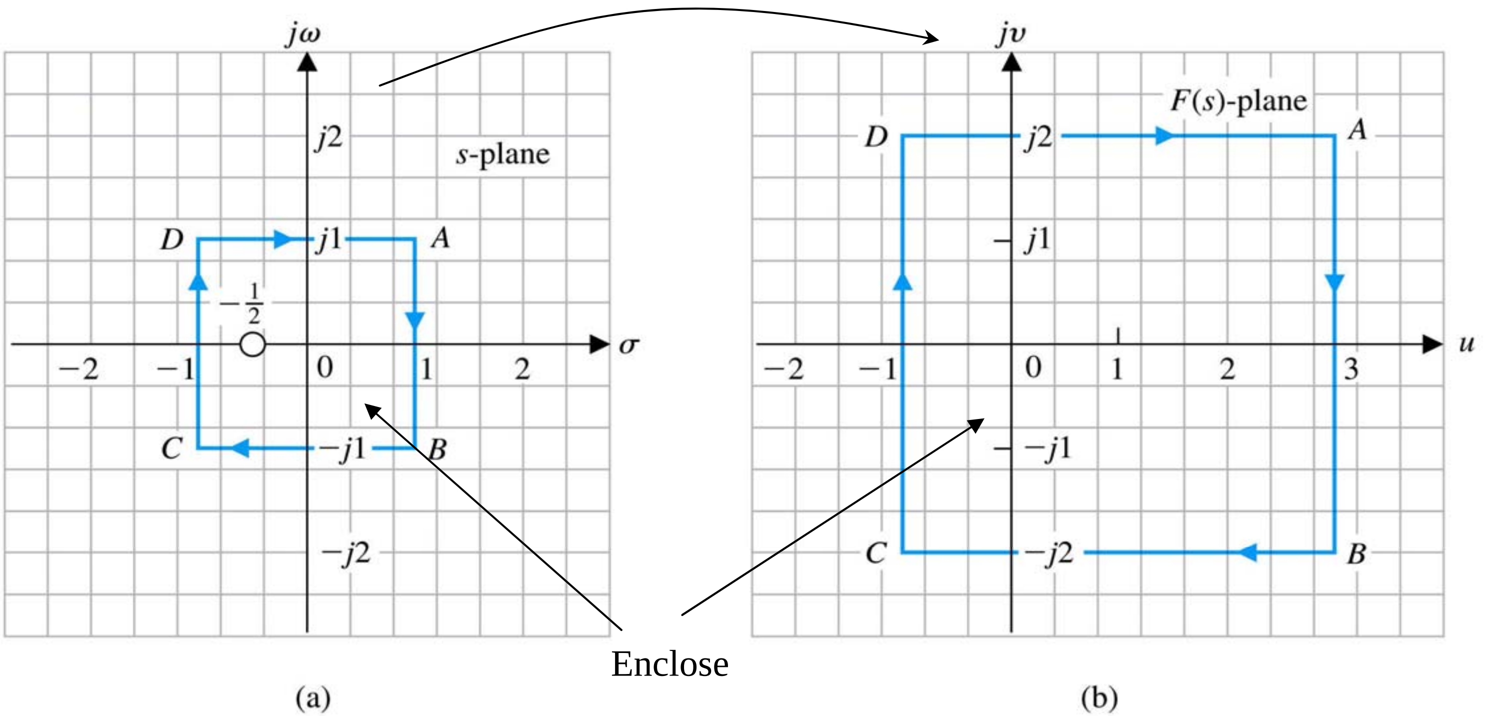
- ***Time-domain***
  - Routh-Hurwitz criterion
  - Root-Locus

  by locating the roots of characteristic equation in the s-plane.

- ***Frequency domain***
  - Nyquist stability criterion

  based on the theorem of complex variables due to Cauchy,
  commonly known as principle of argument.
  H. Nyquist 1932

$$F(s) = 2s + 1$$



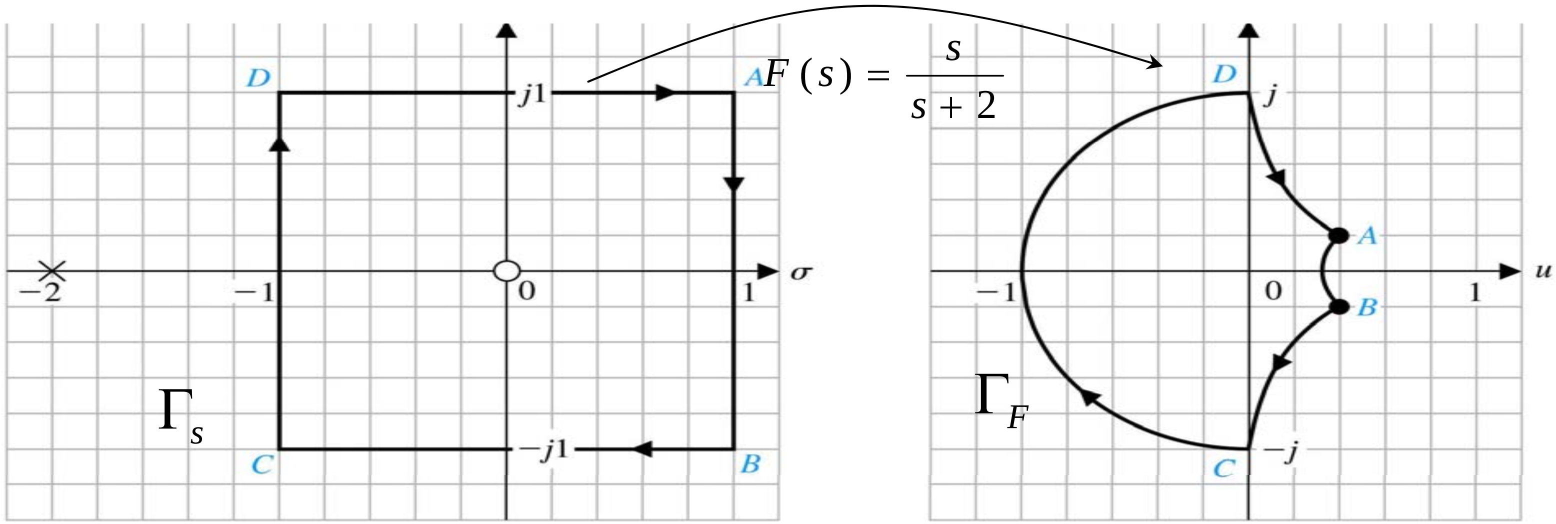
Clockwise and eyes right

$$A: F(s)|_{s=1+j} = 2s + 1 = 2 + 2j + 1 = 3 + 2j$$

$$B: F(s)|_{s=1-j} = 2s + 1 = 3 - 2j$$

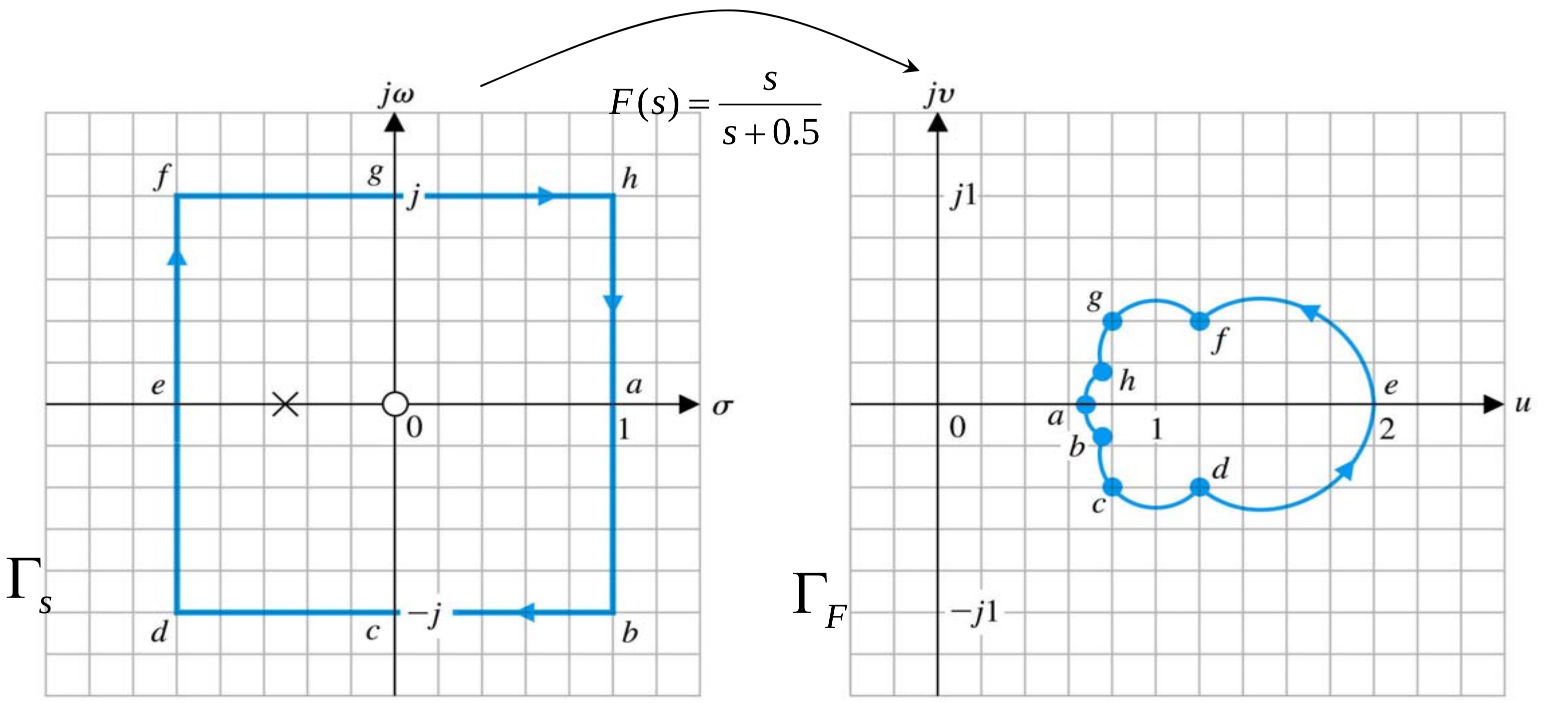
Table 9.1 Values of $F(s)$

	Point A		Point B		Point C		Point D	
$s=\sigma+j\omega$	$1+j1$	1	$1-j1$	$-j1$	$-1-j1$	-1	$-1+j1$	$j1$
$F(s)=u+jv$	$\frac{4+2j}{10}$	$\frac{1}{3}$	$\frac{4-2j}{10}$	$\frac{1-2j}{5}$	$-j$	-1	$+j$	$\frac{1+2j}{5}$



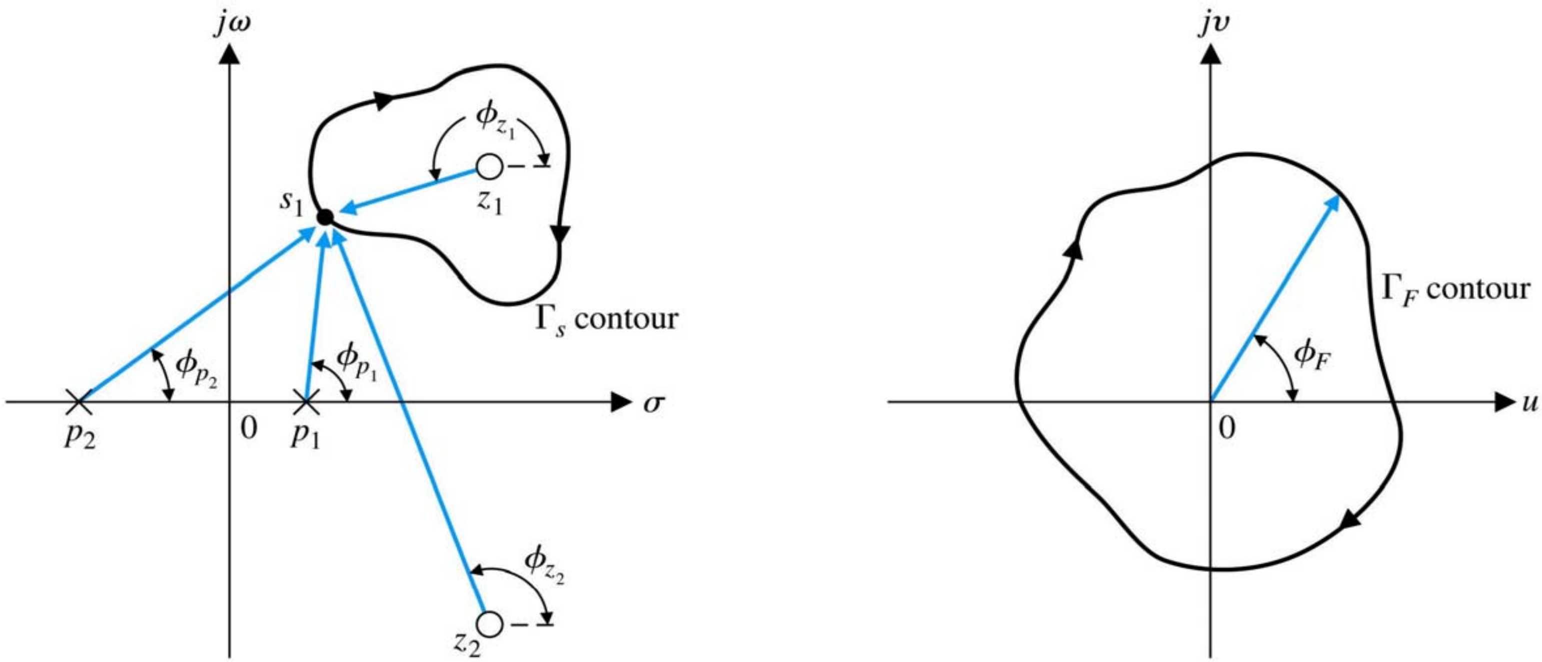
(a) (b)

Cauchy's Theorem: If a contour Γ_s in the s-plane encircles Z zeros and P poles of F(s) and does not pass through any poles or zeros of F(s) and the traversal is in the clockwise direction along the contour, the corresponding contour Γ_F in the F(s)-plane encircles the origin of the F(s)-plane N=Z-P time in the clockwise direction


$F(s) = \frac{s}{s+0.5}$
$j\omega$
σ
jv
u
Γ_s
Γ_F
j
$-j$
$j1$
$-j1$
0
1
2
a
b
c
d
e
f
g
h

(a)

(b)

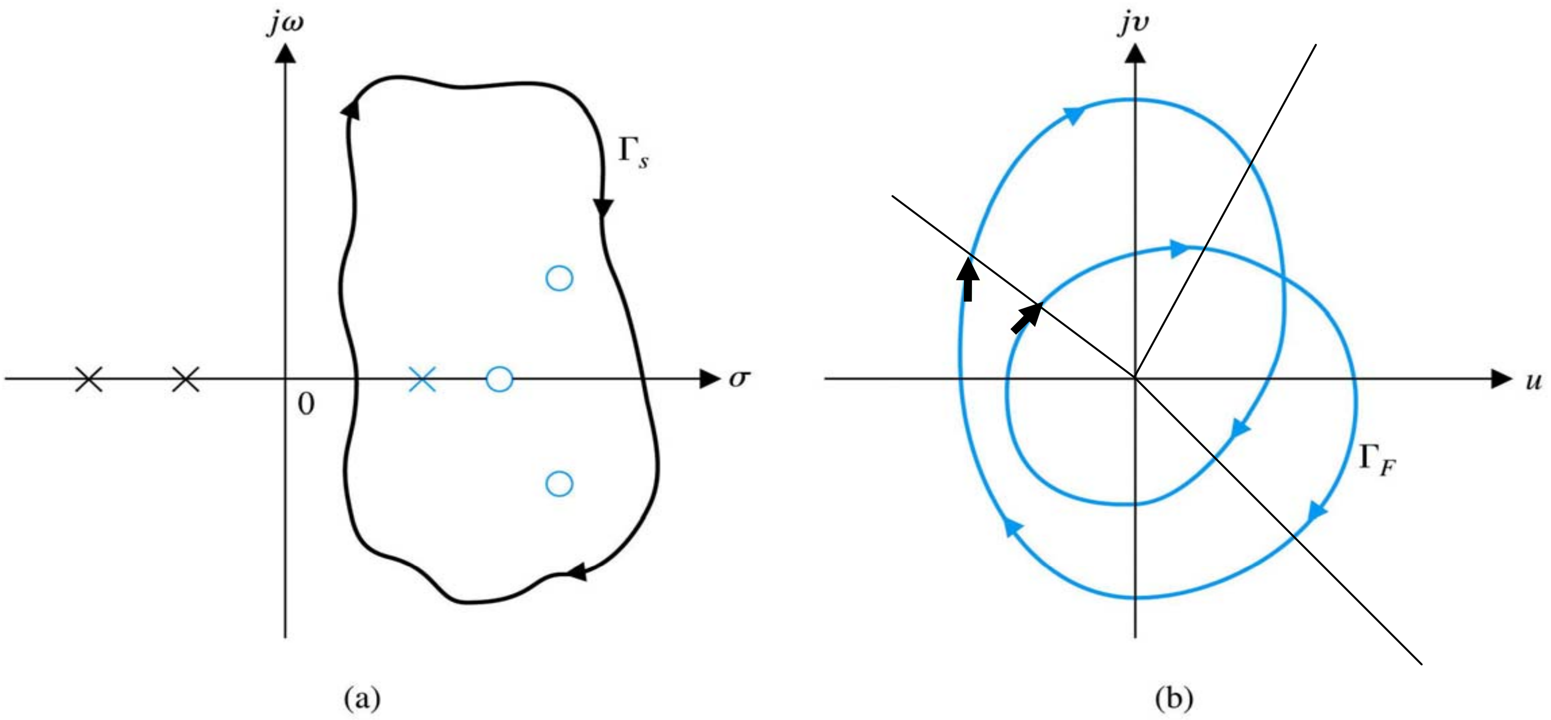


(a) (b)

$$F(s) = |F(s)| \angle F(s) = \frac{|s+z_1||s+z_2|}{|s+p_1||s+p_2|}(\angle s+z_1 + \angle s+z_2 - \angle s+p_1 - \angle s+p_2)$$

$$= |F(s)| \angle(\phi_{z_1} + \phi_{z_2} - \phi_{p_1} - \phi_{p_2}) = |F(s)| \angle \phi_F$$

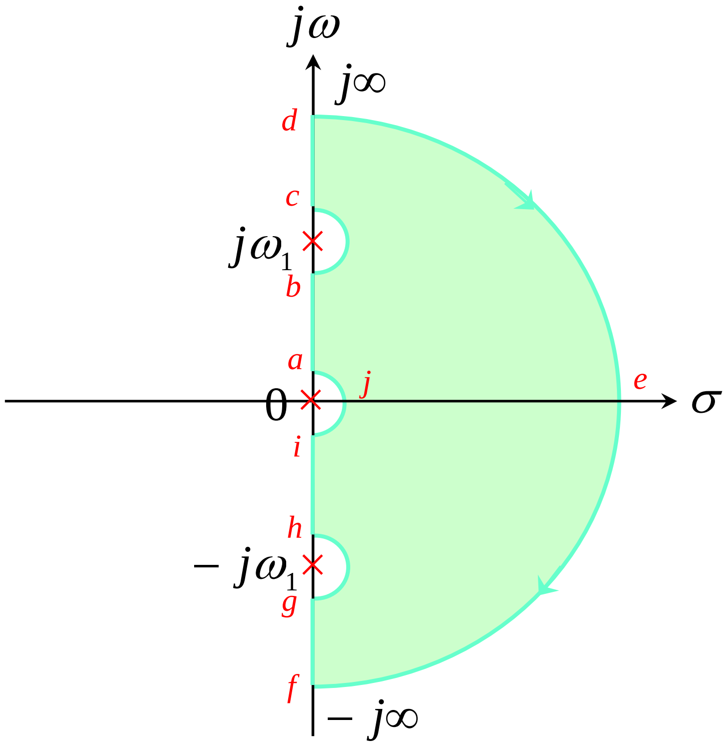
$$\phi_F = \phi_z - \phi_p$$

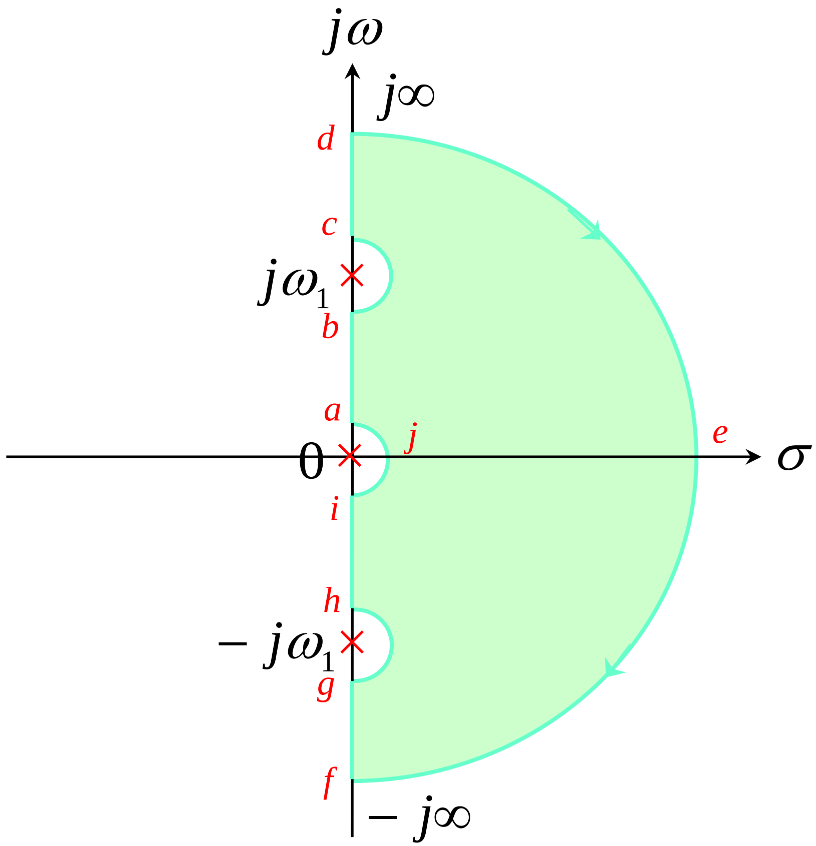


$$P = 1, Z = 3, \qquad N = Z - P = 3 - 1 = 2$$

◆ ***Nyquist contour (Nyquist path)***

Since the Nyquist contour must not pass through any poles and zeros of $F(s)$, the small semicircles shown along the $j\omega$-axis are used to indicate that the path should go around these poles and zeros if they fall on the $j\omega$-axis.





- *Path ab*:

  $s=j\omega\ ;\quad 0<\omega<\omega_1$
- *Path bc*:

  $s=j\omega_1+\varepsilon\cdot e^{j\theta};\quad \varepsilon\to 0$ and $-90^o<\theta<90^o$
- *Path cd*:

  $s=j\omega\ ;\quad \omega_1<\omega<\infty$
- *Path def*:

  $s=R\cdot e^{j\theta};\quad R\to\infty$ and $90^o<\theta<-90^o$
- *Path fg*:

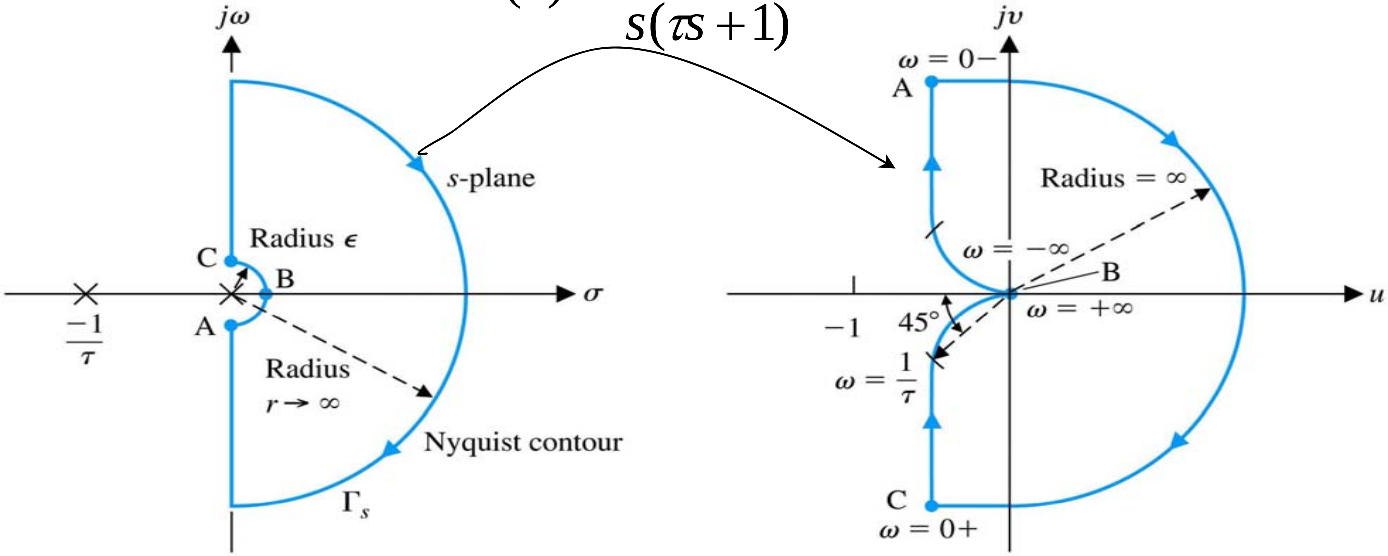
  $s=j\omega\ ;\quad -\infty<\omega<-\omega_1$
- *Path gh*:

  $s=-j\omega_1+\varepsilon\cdot e^{j\theta};\quad \varepsilon\to 0$ and $-90^o<\theta<90^o$
- *Path hi*:

  $s=j\omega\ ;\quad -\omega_1<\omega<0$
- *Path ija*:

  $s=\varepsilon\cdot e^{j\theta};\quad \varepsilon\to 0$ and $-90^o<\theta<90^o$

$$GH(s) = \frac{K}{s(\tau s+1)}$$



1. $\omega: 0^- \to 0^+, \phi: -90 \sim 90, s = \varepsilon e^{j\phi}$

$$G(s) = \lim_{\varepsilon \to 0} \frac{K}{\varepsilon e^{j\phi}(\tau \varepsilon e^{j\phi}+1)} = \infty e^{-j\phi}, 90^0 \sim -90^0$$

2. $\omega: 0^+ \to \infty, s = j\omega$

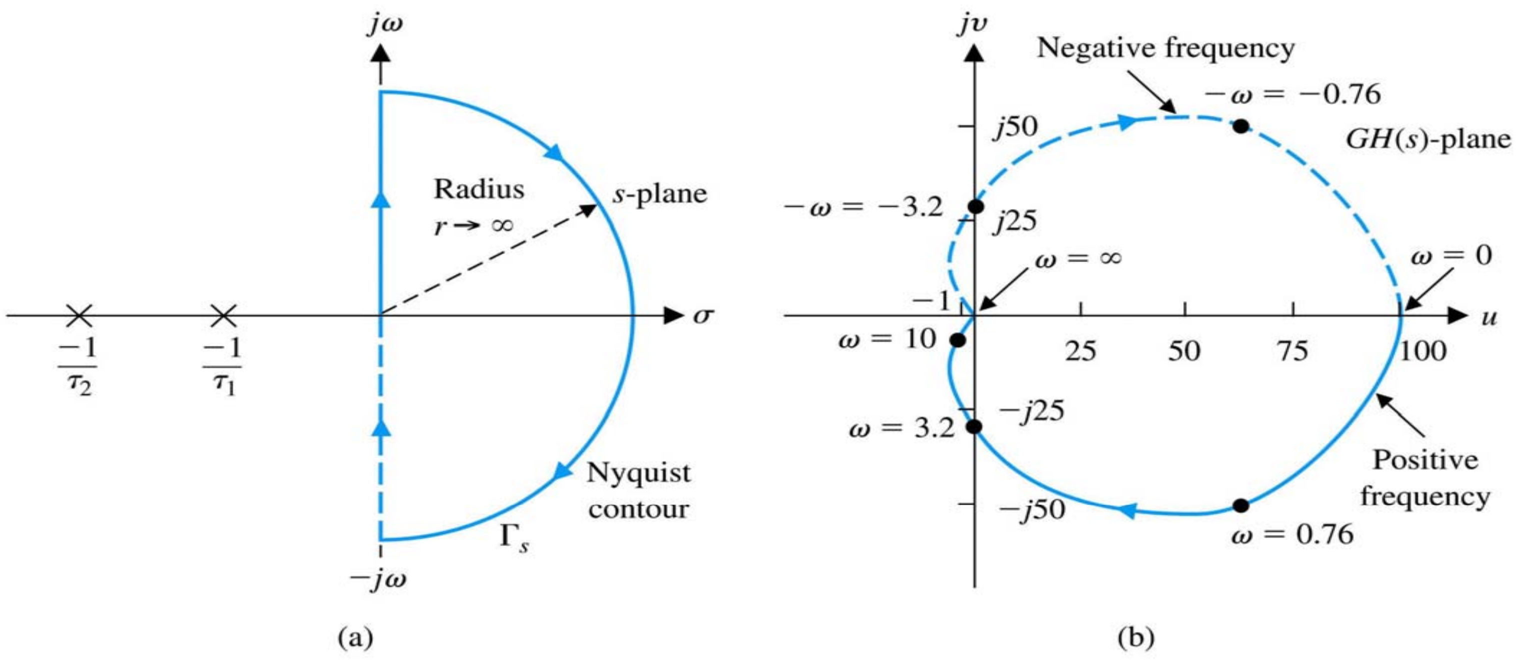
$$GH(s) = \frac{K}{j\omega(j\omega\tau+1)} = \frac{K}{-\omega^2\tau + j\varpi}, \omega \to \infty, |GH(j\omega)| \approx 0, \phi \approx -180^0$$

3. $\omega: \infty \sim -\infty, s = \mathrm{Re}^{j\phi}, \quad R \to \infty, \phi: 90 \sim -90$

$$GH(s)|_{s=\mathrm{Re}^{j\phi}} = \lim_{R\to\infty} \frac{K}{\mathrm{Re}^{j\phi}(\tau\,\mathrm{Re}^{j\phi}+1)} = \frac{K}{\tau R^2} e^{-j2\phi} \approx 0e^{-j2\phi}, -180 \sim +180$$

4. $\omega: -\infty \to 0^-$

The portion of the polar plot from $\omega = -\infty$ to $\omega = 0^-$ is symmetrical to the polar plot from $\omega = 0^+$ to $\omega = +\infty$

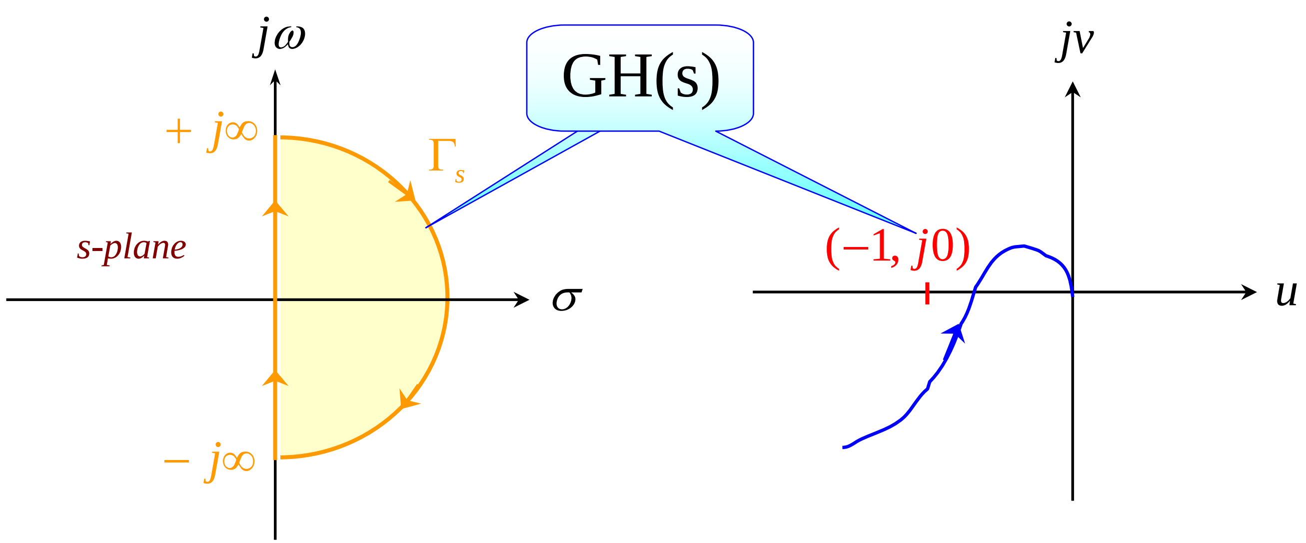


$$G(s) = \frac{100}{(s+1)(s/10+1)}$$

◆ *Nyquist criterion and the GH(s) plot*

Since the open-loop transfer function GH(s) is generally known, it would be simpler to construct the GH(s) plot that corresponds to the Nyquist path, and the same stability conditions of the closed-loop system can be obtained by observing the number of encirclements of the (-1,*j*0) point in the GH(s)-plane.

$$F(s) = 1 + GH(s) \Leftrightarrow GH(s) = 1 - F(s)$$



Stability requirements

- For the closed-loop system to be stable, there should be no zeros of $F(s)$ in right half s-plane; *i.e.*,

  Z=0

  this condition is met if

  N=Z–P=0 – P= – P
- The special case of P=0, means open-loop stable system, the closed-loop system is stable if

  N=Z–P=0

$$Open-loop: GH(s)=\frac{N(s)}{D(s)}$$

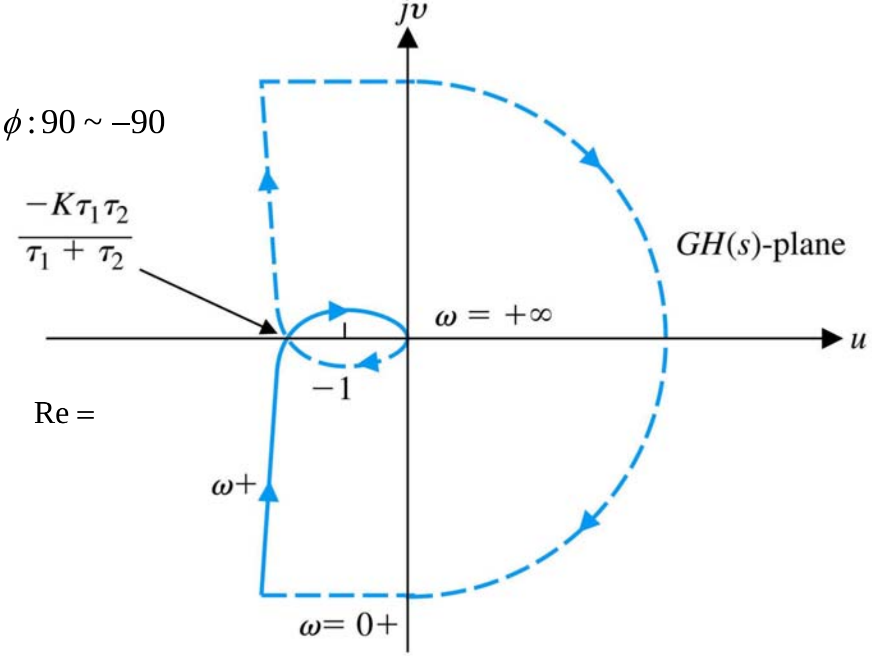
$$closed-loop: F(s)=1+GH(s)=1+\frac{N(s)}{D(s)}=\frac{D(s)+N(s)}{D(s)}$$

$$\boxed{\begin{array}{l} N_0 = Z_0 - P_0 \\ P_{-1} = P_0 \Leftarrow \text{poles of } L(s)=F(s) \\ N_{-1} = Z_{-1} - P_{-1} \\ Z_{-1} = 0 \Leftarrow \text{stable for closed-loop} \\ N_{-1} = 0 - P_{-1} = -P_{-1} \end{array}}$$

$$GH(s) = \frac{K}{s(\tau_1 s + 1)(\tau_2 s + 1)}$$

$1. \omega : 0^- \to 0^+, s = \varepsilon e^{j\phi}, \varepsilon \to 0, \phi : -90 \sim 90$

$$GH(s) = \lim_{\varepsilon \to 0} \frac{K}{\varepsilon e^{j\phi}(\tau_1 \varepsilon e^{j\phi} + 1)(\tau_2 \varepsilon e^{j\phi} + 1)} = \infty e^{-j\phi}, |GH(S)| = \infty, \phi : 90 \sim -90$$



$2. \ \omega : 0^+ \to \infty, s = j\omega,$

$$GH(s) = \frac{K}{j\omega(j\tau_1\omega + 1)(j\tau_2\omega + 1)}$$

$$= \frac{-K(\tau_1 + \tau_2) - jK(1/\omega)(1 - \omega^2\tau_1\tau_2)}{1 + \omega^2(\tau_1^2 + \tau_2^2) + \omega^4\tau_1^2\tau_2^2} = \text{Re} + J\,\text{Im}$$

Re =

Let Im=0 $\omega = \frac{1}{\sqrt{\tau_1\tau_2}}$

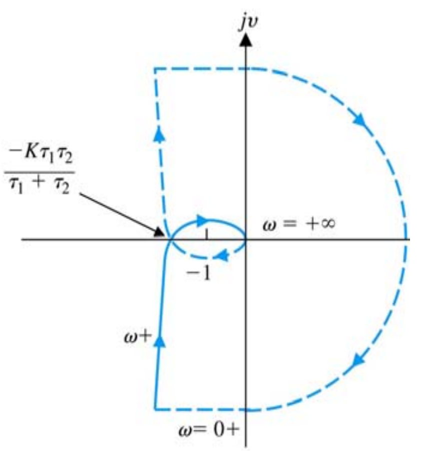
The magnitude of the real part,Re of the GH(jw) at this frequency is

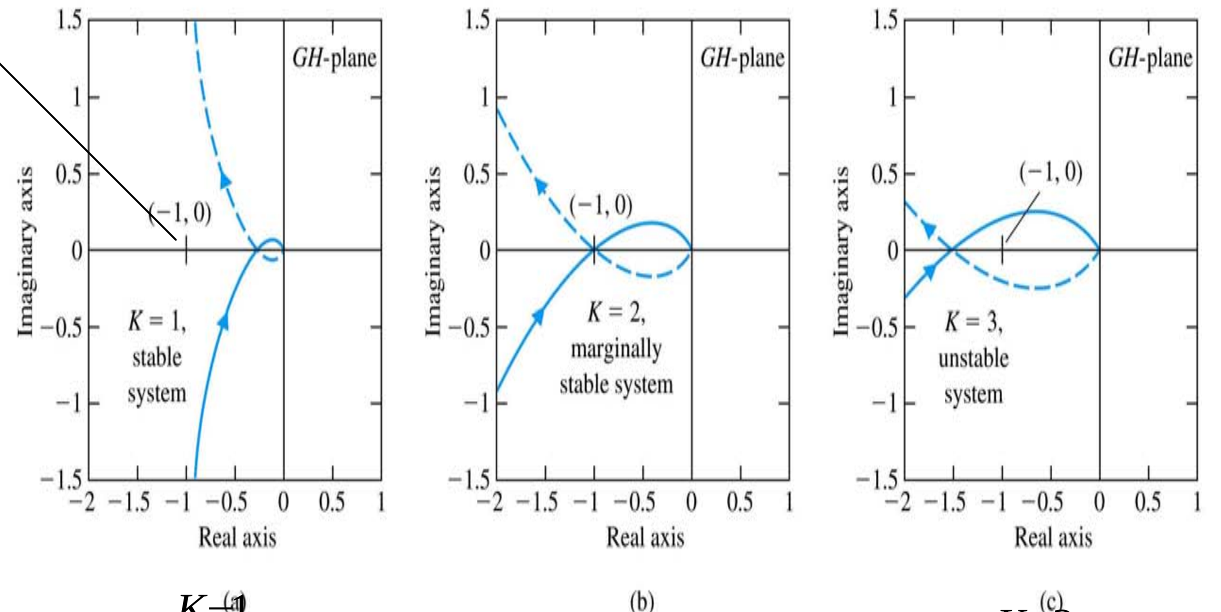
$$\text{Re} = \frac{-K(\tau_1 + \tau_2)}{1 + \omega^2(\tau_1^2 + \tau_2^2) + \omega^4\tau_1^2\tau_2^2}, \omega = \frac{1}{\sqrt{\tau_1\tau_2}}$$

$$= \frac{-K\tau_1\tau_2}{\tau_1 + \tau_2}$$

$3. \ \omega : +\infty \to -\infty, \ s = re^{j\phi}, r \to \infty, \phi : 90 \sim -90$

$$GH(s) = \lim_{r \to \infty} \frac{K}{re^{j\phi}(r\tau_1 e^{j\phi} + 1)(r\tau_2 e^{j\phi} + 1)} \approx 0e^{-j3\phi}$$





$$GH(s)=\frac{K}{s(s+1)(s+1)}$$

K=1

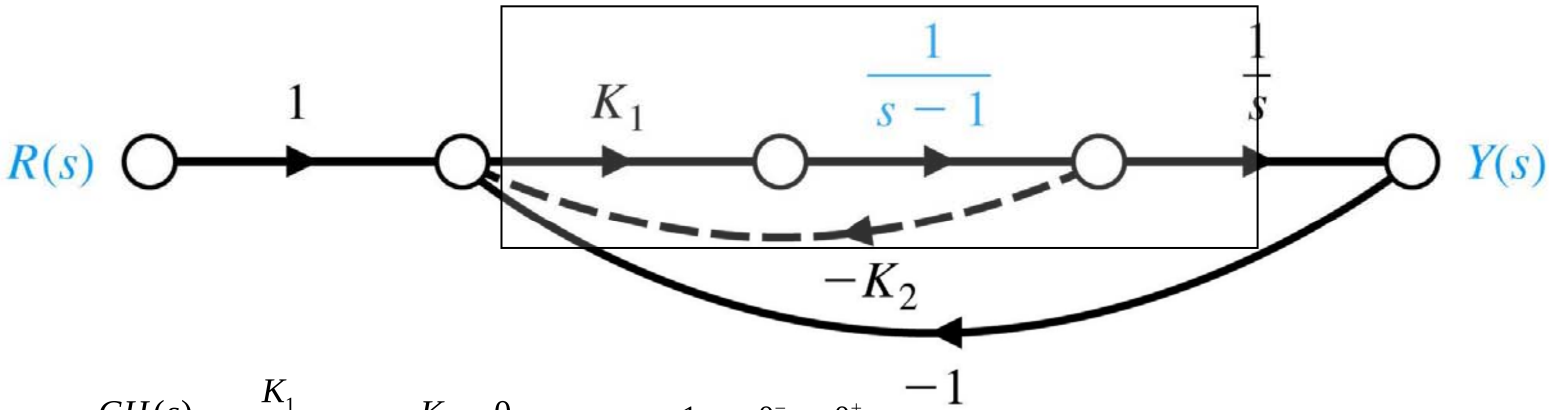
stabl

K=2

K=3

unstab

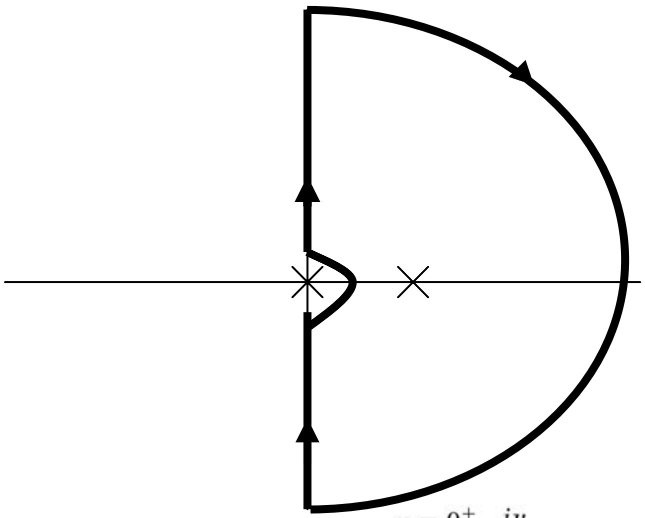
System with a pole in the right-hand s-plane



$GH(s)=\dfrac{K_1}{s(s-1)}$, $K_2=0$ $1.\omega=0^- \rightarrow 0^+$

$1. w: 0^- \to 0^+, s = \varepsilon e^{j\phi}, \phi: -90 \sim 90$

$$GH(s) = \lim_{\varepsilon \to 0} \frac{K}{\varepsilon e^{j\phi}(\varepsilon e^{j\phi} - 1)} = -\infty e^{-j\phi} = \infty \angle -180 - \phi$$



$2. w: 0^+ \to \infty, s = jw$

$$GH(jw) = \frac{K}{jw(jw-1)} = \frac{K}{(w^4 + w^2)^{\frac{1}{2}}} \angle -90^0 - \tan^{-1}(-w)$$

$$= \frac{K}{(w^4 + w^2)^{\frac{1}{2}}} \angle 90 + \tan^{-1}(w)$$

$3. w: \infty \to -\infty, s = re^{j\phi}, \phi: 90 \sim -90$

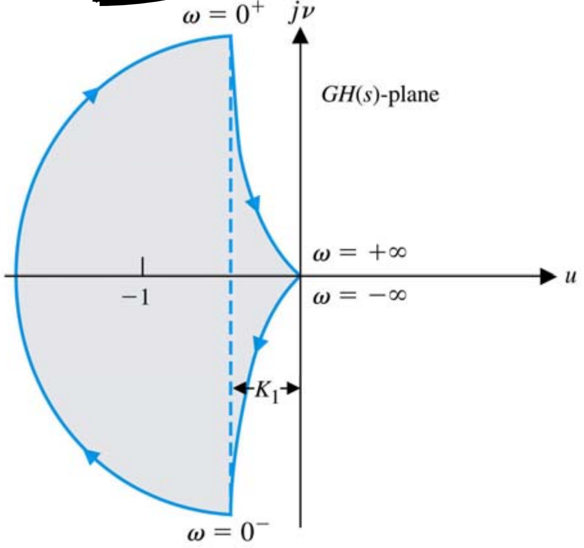
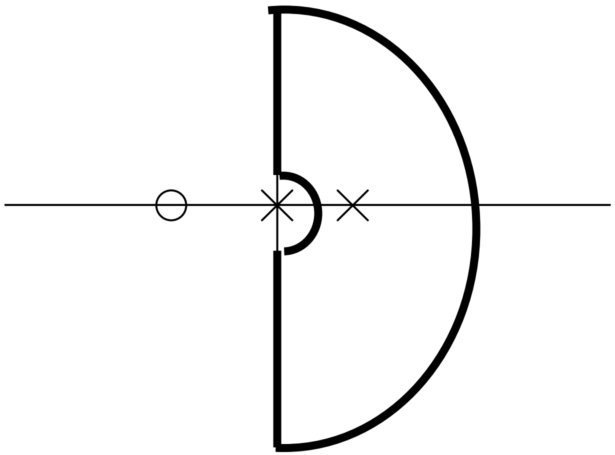
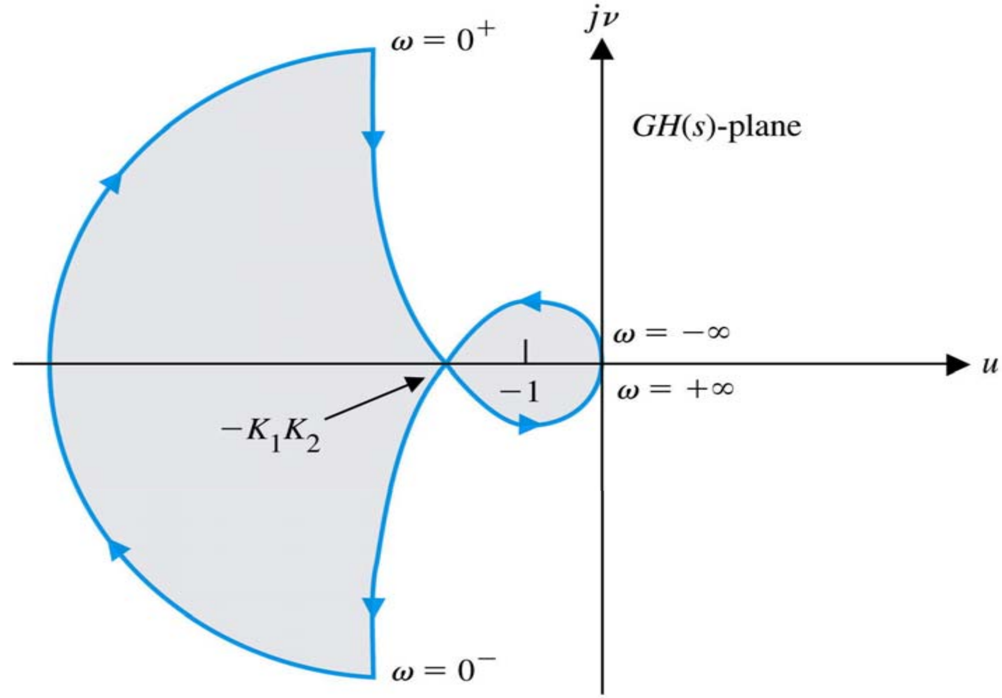


Table 9.3 Values of *GH*(s)

s	$j0^-$	$j0^+$	$j1$	$+j\infty$	$-j\infty$
$\lvert GH \rvert / K_1$	∞	∞	$1/\sqrt{2}$	0	0
$\angle GH$	$-90°$	$+90°$	$+135°$	$+180°$	$-180°$





closed − loop

$$\frac{Y(s)}{R(s)} = \frac{K_1}{s^2 + (K_1K_2 - 1)s + K_1}, F(s) = s^2 + K_1K_2 s - s + K_1 = 0$$

$$F(s) = 1 + \frac{K_1(1 + K_2 s)}{s(s-1)} = 0$$

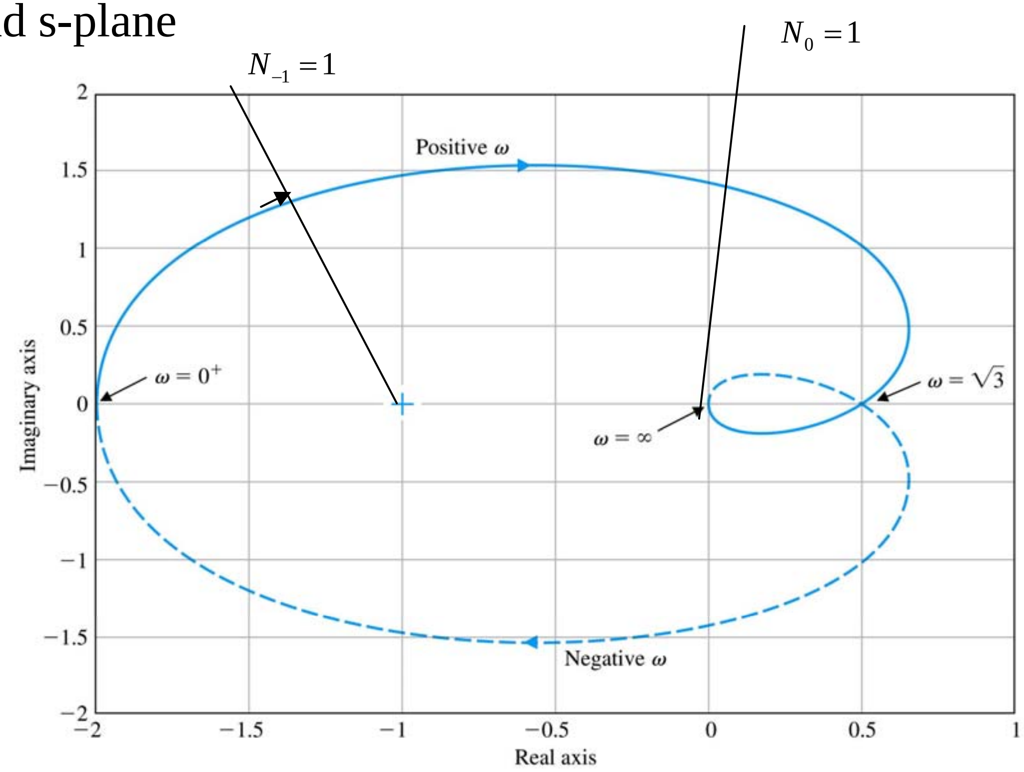
System with a zero in the right-hand s-plane

$$GH(s)=\frac{K(s-2)}{(s+1)^2}$$

$Z_0=1, N_0=1 \Rightarrow P_0=0$

$P_{-1}=P_0=1, N_{-1}=1, \Rightarrow Z_{-1}=2$

$\therefore$ Closed-loop system is unstable



☑Relative Stability and the Nyquist Criterion

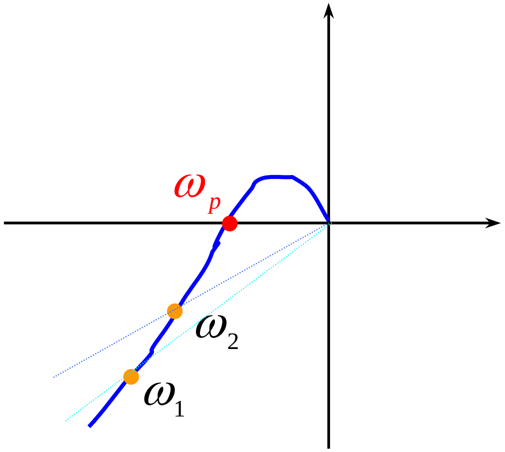
The Nyquist stability criterion is defined in term of (-1,*j*0) *points on the polar plot or the 0 dB, -180 º point on the Bode diagram or Log-magnitude-phase diagram. Clearly, the proximity of* $L(j\omega)$ *locus to this stability point is a measure of the relative stability of the system.*

- ***Definition***
  - Phase Crossover

    A phase crossover on the $L(j\omega)$ *plot is a point at which the plot intersects the negative real axis.*
  - Phase Crossover Frequency: ω_p

    The phase crossover frequency is the frequency at the phase crossover, or where

$$\angle L(j\omega_p)=\pm 180^o$$



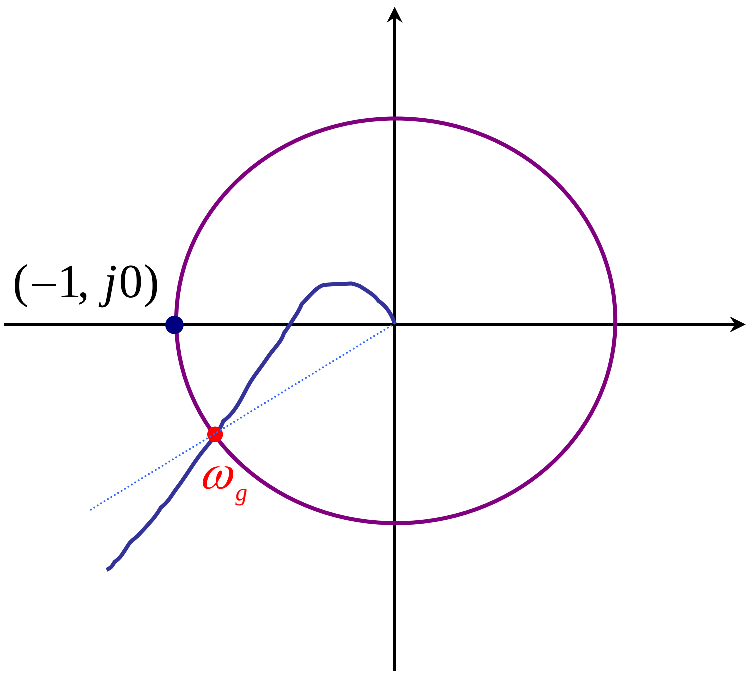
- Gain Crossover

  The gain crossover is a point on $L(j\omega)$ plot at which the magnitude of $L(j\omega)$ is equal to 1.

- Gain Crossover Frequency: ω_g

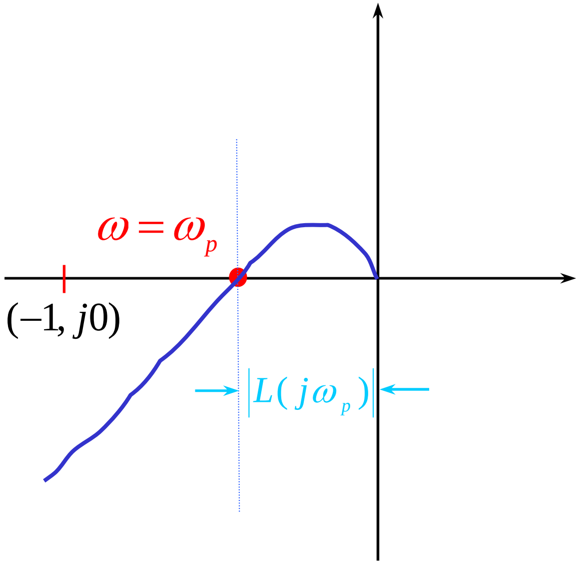
  The gain crossover frequency is the frequency of $L(j\omega)$ at the gain crossover, or where

  $|L(j\omega_g)|=1$

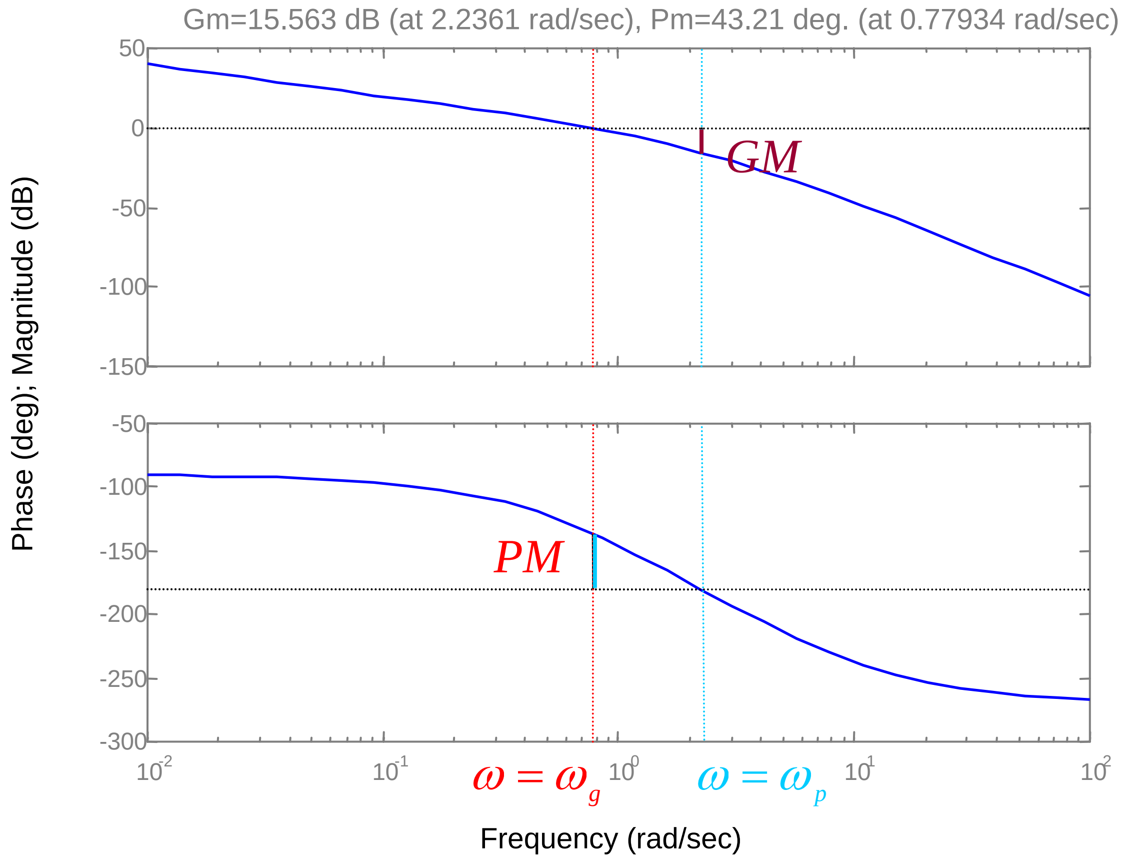


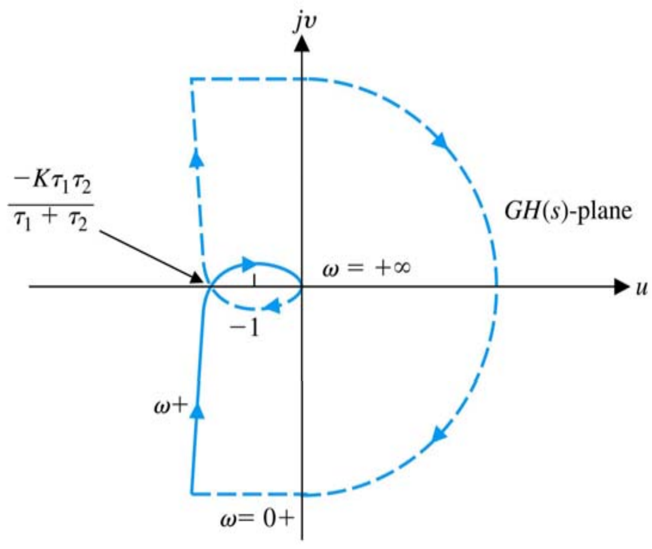
- ***Gain Margin*: (*GM*)**
  - The gain margin is the increase in the system gain when phase = - *180* ^{o} that will result a marginally stable system with intersection of (-1,*j*0) point on the Nyquist plot.
  - Gain margin is the amount of gain in decibels (dB) that can be added to the loop before the closed-loop system become unstable.

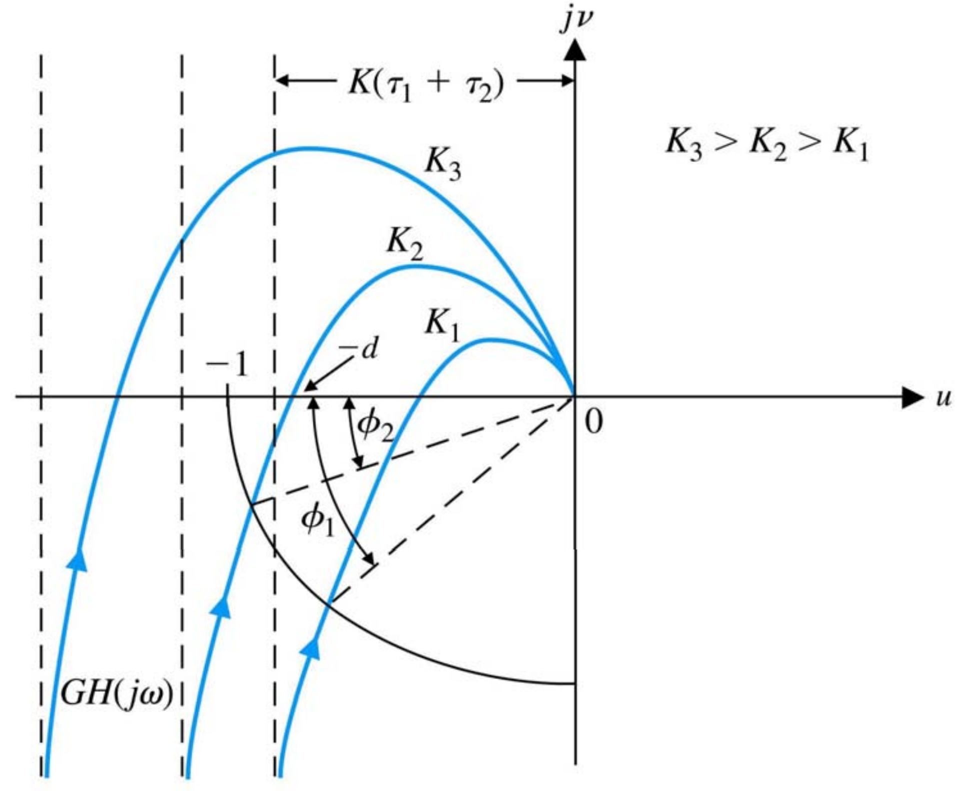
$$GM = 20\log\{1 - |L(j\omega_p)|\}$$
$$= -20\log\{|L(j\omega_p)|\}$$
$$\underline{\underline{or}}\, 20\log\{\left|\frac{1}{L(j\omega_p)}\right|\}$$



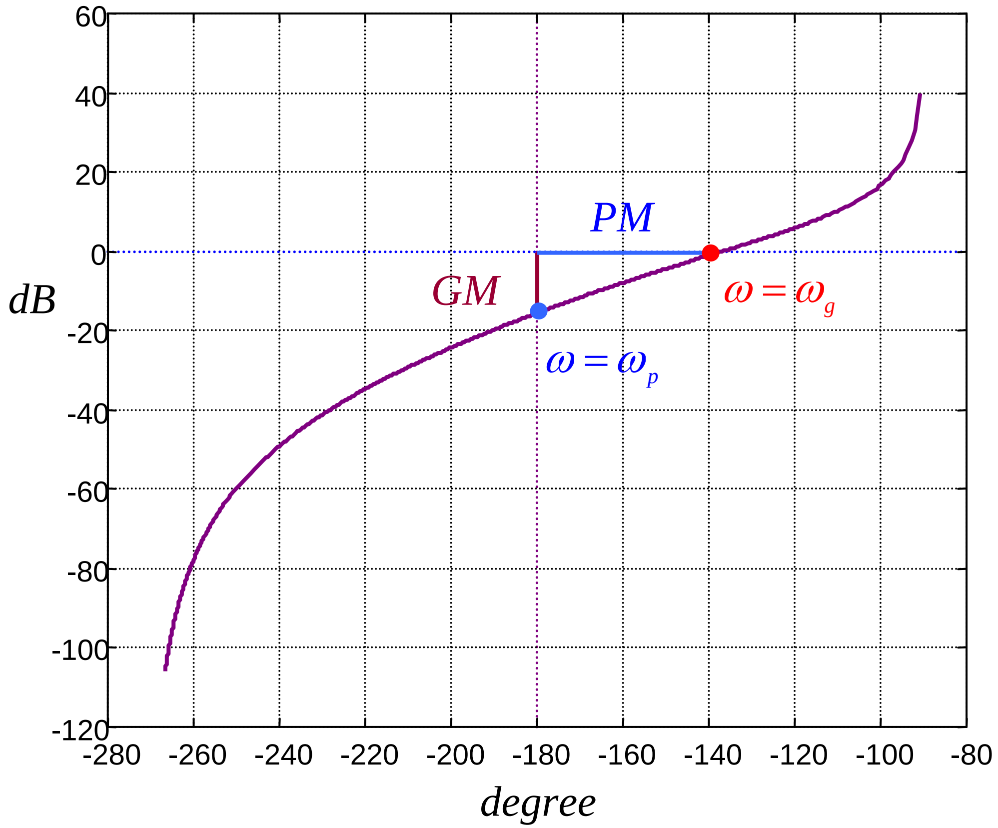
- ***Bode plot***


Gm=15.563 dB (at 2.2361 rad/sec), Pm=43.21 deg. (at 0.77934 rad/sec)
Phase (deg); Magnitude (dB)
50
0
-50
-100
-150
GM
-50
-100
-150
-200
-250
-300
PM
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
$\omega = \omega_g$
$\omega = \omega_p$
Frequency (rad/sec)





$$GH(s) = \frac{K}{s(\tau_1 s + 1)(\tau_2 s + 1)}$$



Log-magnitude-phase curve

$$\zeta = 0.01\phi_{PM}$$

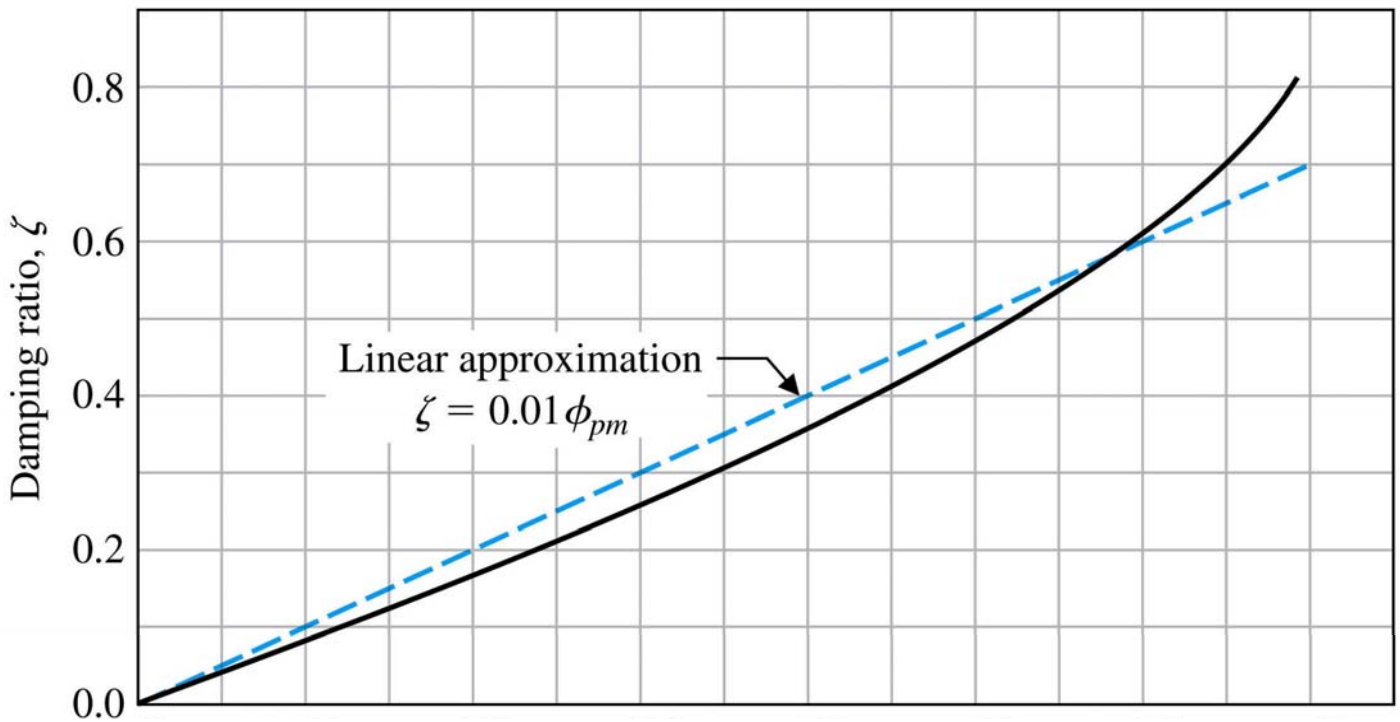


FIGURE 9.21

Damping ratio versus phase margin for a second-order system.

$$\frac{Y(jw)}{R(jw)} = T(jw) = \frac{G(jw)}{1+GH(jw)} = M(w)e^{j\phi(w)}$$

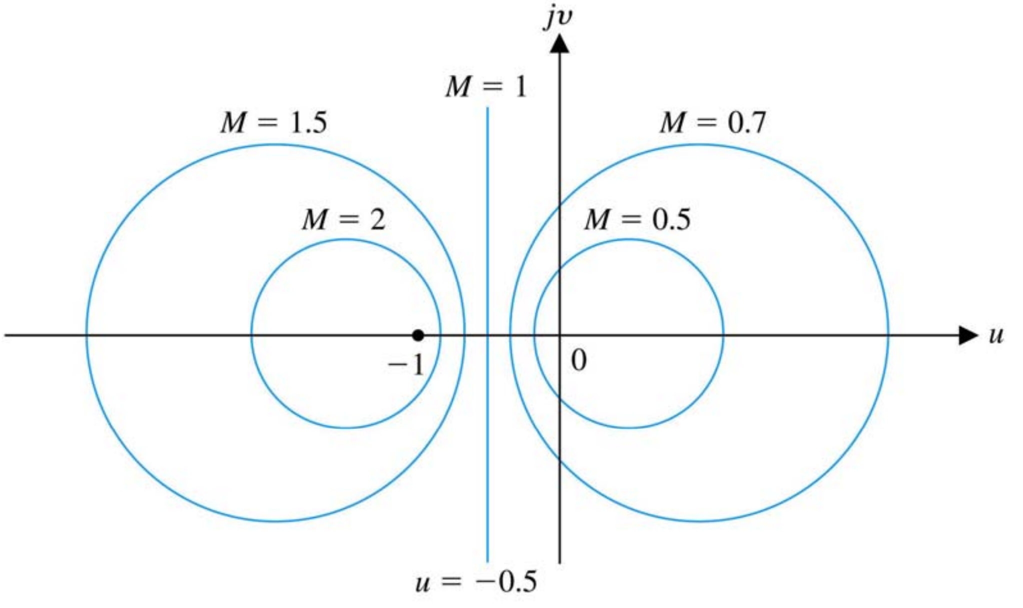
$$G(jw) = u + jv$$

$$M = \left|\frac{G(jw)}{1+G(jw)}\right| = \left|\frac{u+jv}{1+u+jv}\right| = \left|\frac{u^2+v^2}{(1+u)^2+v^2}\right|^{1/2}$$

$$(1-M^2)u^2 + (1-M^2)v^2 - 2M^2u = M^2$$

$$u^2+v^2-\frac{2M^2u}{1-M^2}+\left(\frac{M^2}{1-M^2}\right) = \left(\frac{M^2}{1-M^2}\right)+\left(\frac{M^2}{1-M^2}\right)^2$$

$$\left(u-\frac{M^2}{1-M^2}\right)^2+v^2 = \left(\frac{M}{1-M^2}\right)^2$$

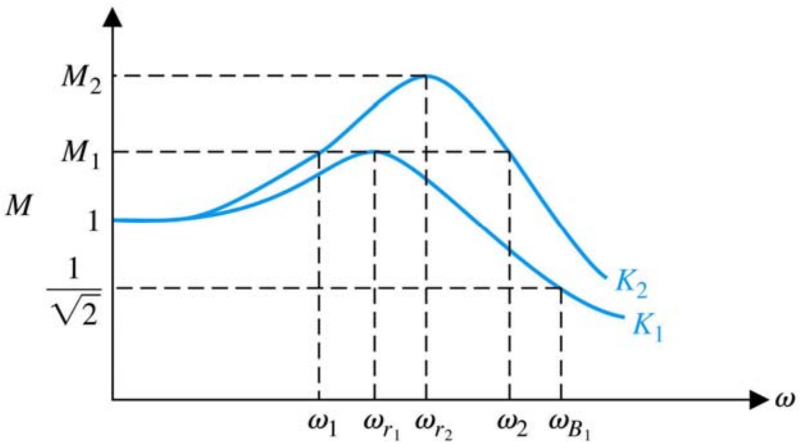


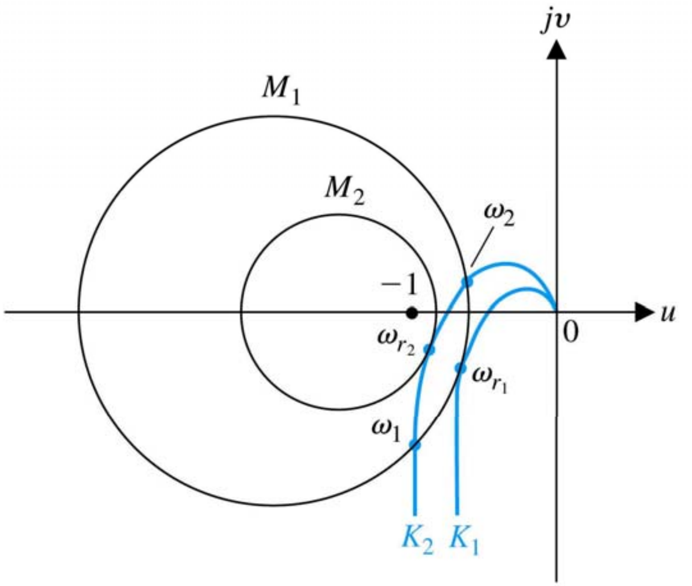
$$\phi = \angle T(jw) = \angle(u+jv)/(1+u+jv) = \tan^{-1}\left(\frac{v}{u}\right) - \tan^{-1}\left(\frac{v}{1+u}\right)$$

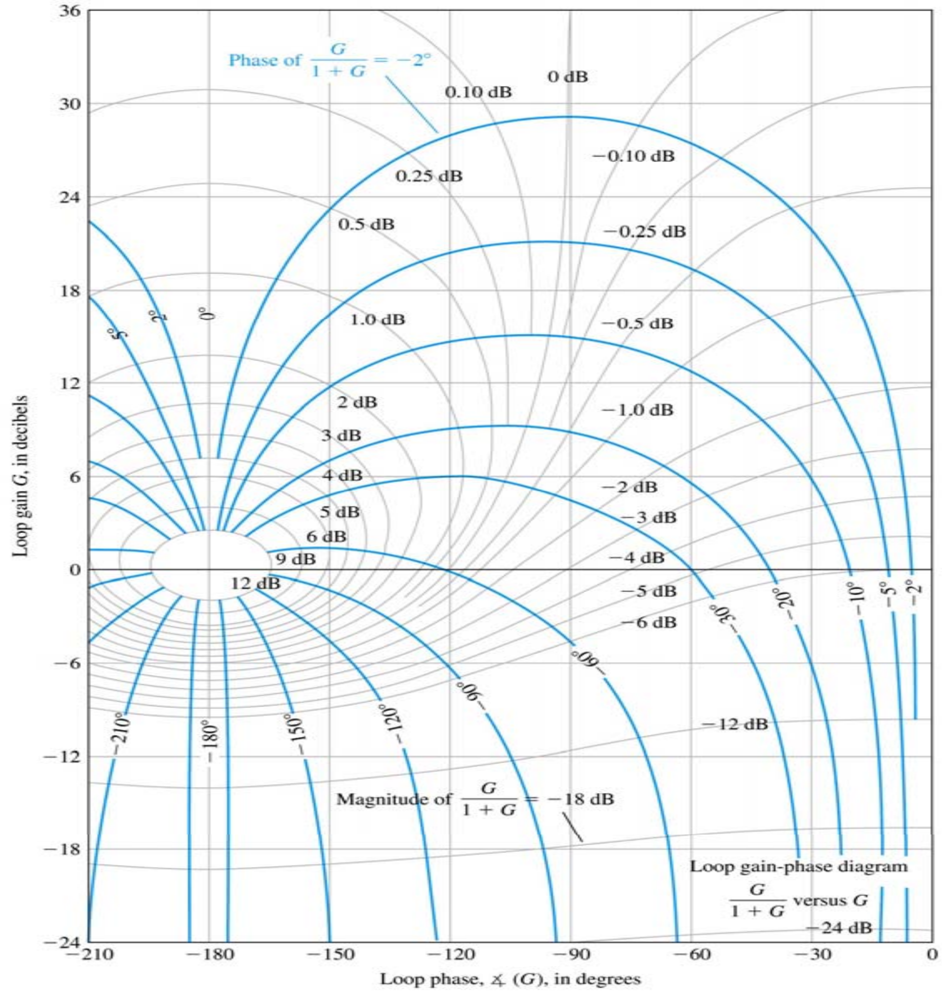
$$let \;\; N = \tan\phi$$

$$u^2+v^2+u-\frac{v}{N} = 0$$

$$(u+0.5)^2+(v-\frac{1}{2N})^2 = \frac{1}{4}\left(1+\frac{1}{N^2}\right)$$


M_2
M_1
M
1
$\frac{1}{\sqrt{2}}$
K_2
K_1
ω
ω_1
ω_{r_1}
ω_{r_2}
ω_2
ω_{B_1}


jv
M_1
M_2
ω_2
-1
u
0
ω_{r_2}
ω_{r_1}
ω_1
K_2
K_1


Phase of $\frac{G}{1+G} = -2°$
0 dB
0.10 dB
0.25 dB
0.5 dB
1.0 dB
2 dB
3 dB
4 dB
5 dB
6 dB
9 dB
12 dB
−0.10 dB
−0.25 dB
−0.5 dB
−1.0 dB
−2 dB
−3 dB
−4 dB
−5 dB
−6 dB
−12 dB
−24 dB
Magnitude of $\frac{G}{1+G} = -18$ dB
Loop gain-phase diagram
$\frac{G}{1+G}$ versus G
Loop gain G, in decibels
Loop phase, ∡ (G), in degrees

☑ Time-Domain performance Criteria Specified in the Frequency Domain

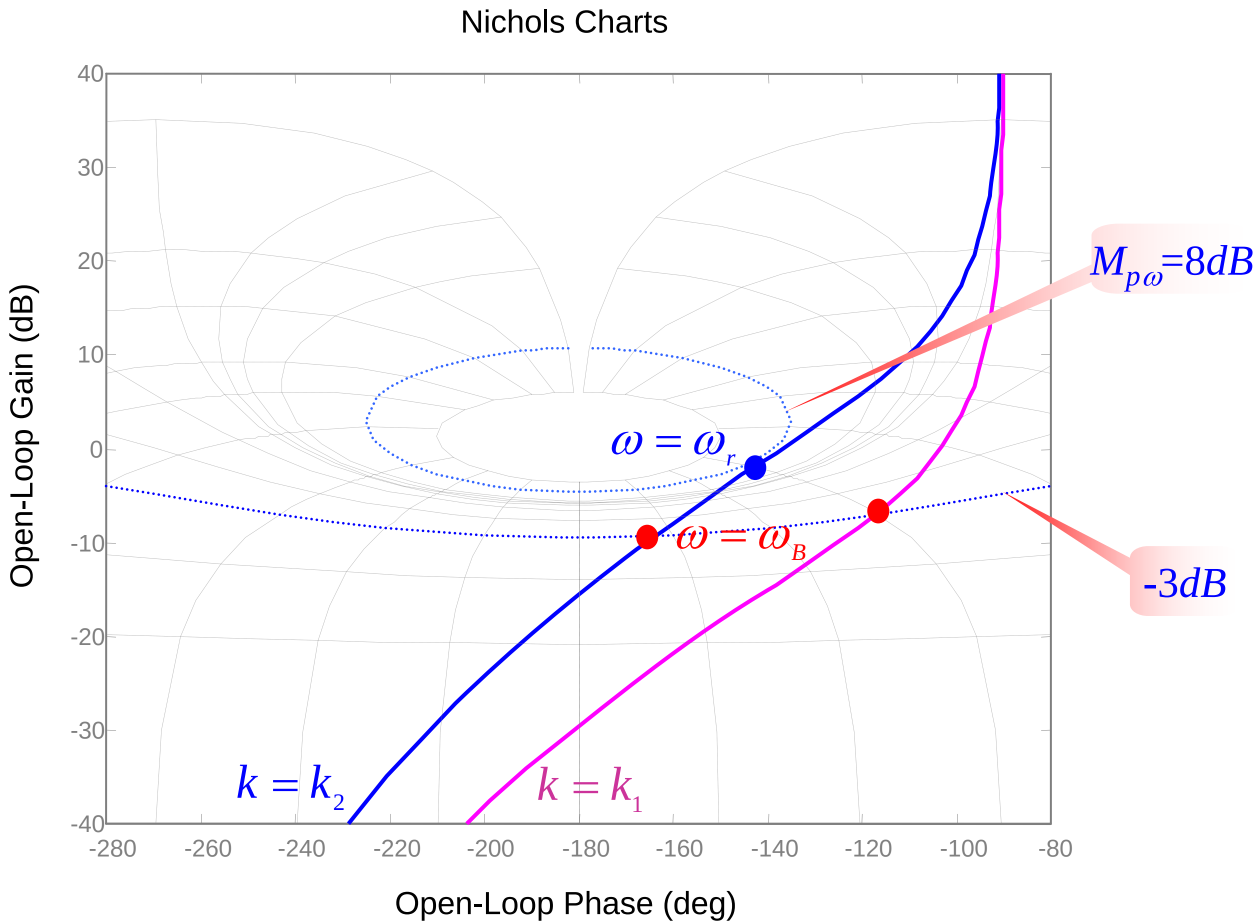
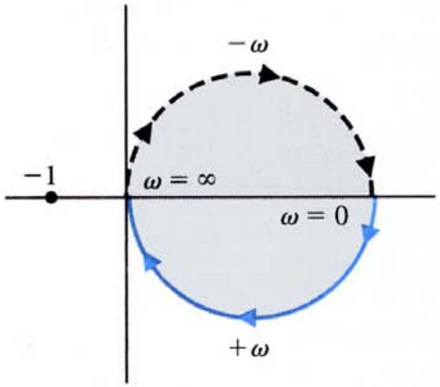
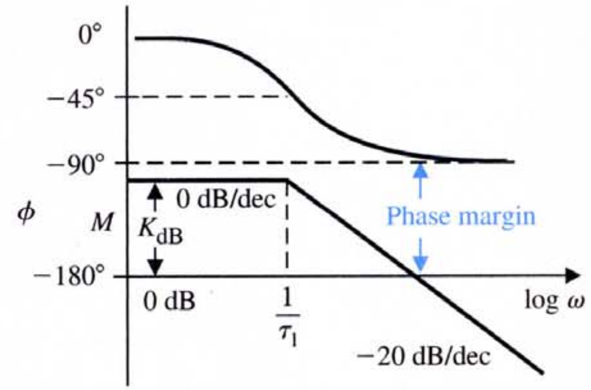
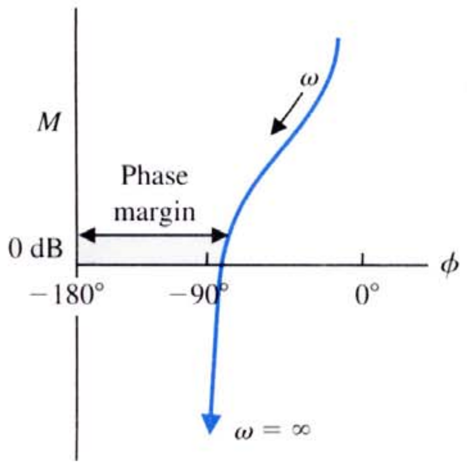


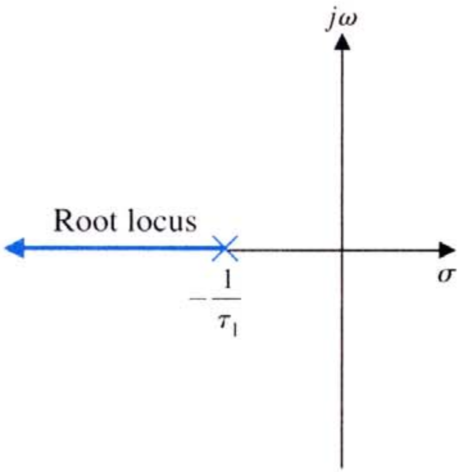
Table 9.6 Transfer Function Plots for Typical Transfer Functions

1. $\dfrac{K}{s\tau_1 + 1}$





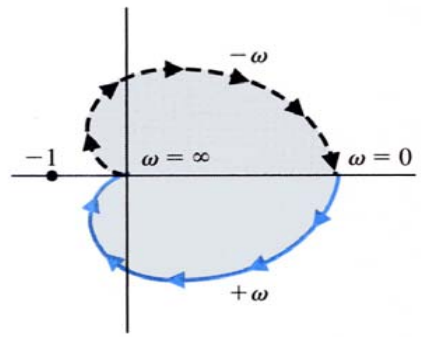


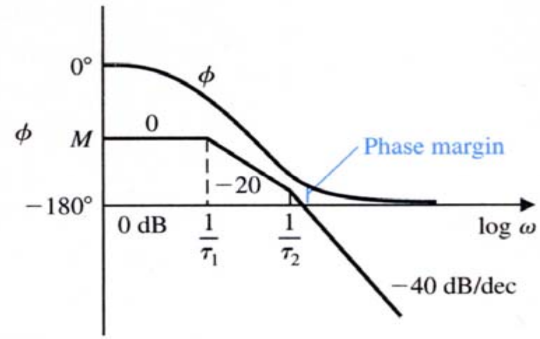


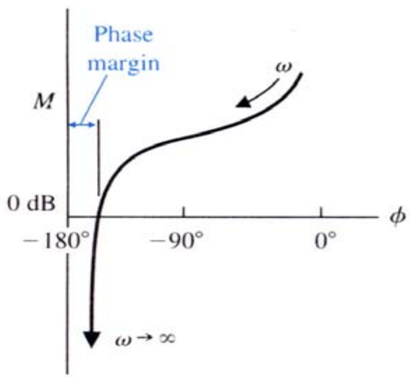
Stable; gain margin $= \infty$

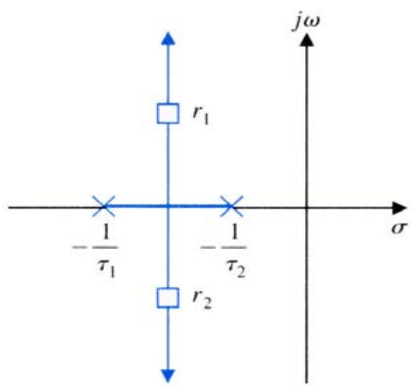
Table 9.6 Transfer Function Plots for Typical Transfer Functions

2. $\dfrac{K}{(s\tau_1 + 1)(s\tau_2 + 1)}$





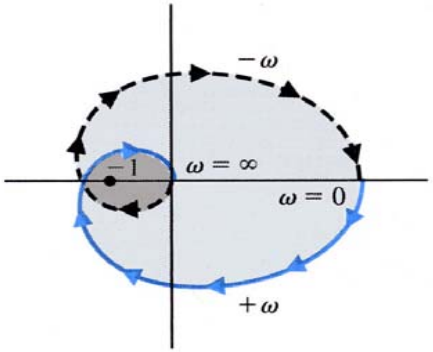


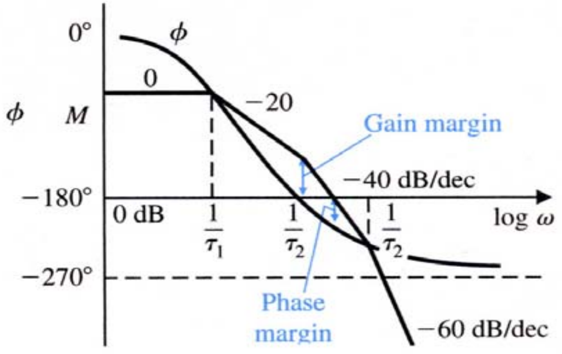


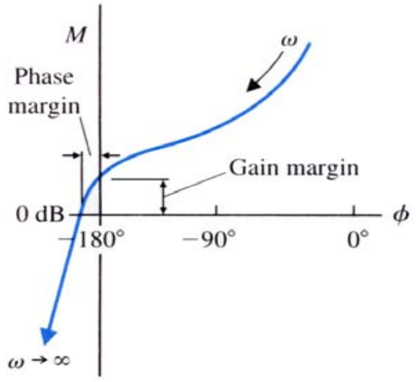
Elementary regulator; stable; gain margin $= \infty$

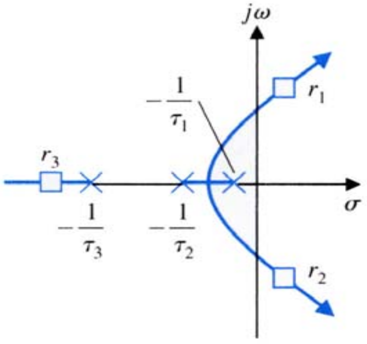
Table 9.6 Transfer Function Plots for Typical Transfer Functions

3. $\dfrac{K}{(s\tau_1+1)(s\tau_2+1)(s\tau_3+1)}$





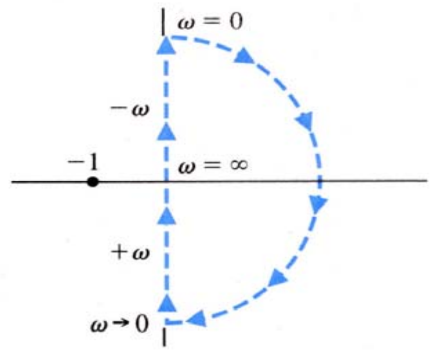


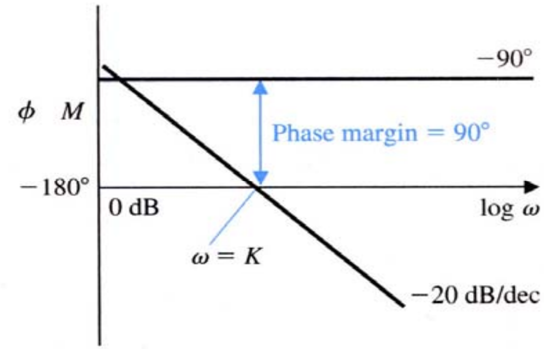


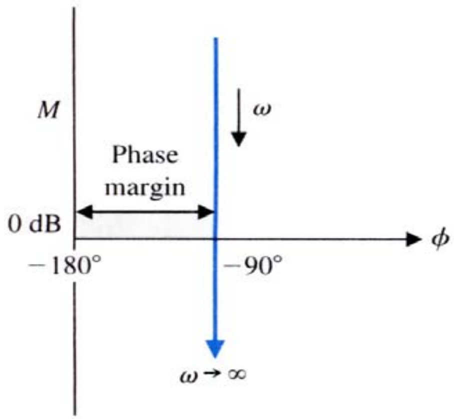
Regulator with additional energy-storage component; unstable, but can be made stable by reducing gain

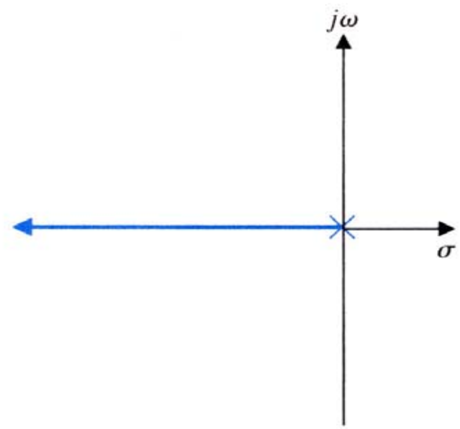
Table 9.6 Transfer Function Plots for Typical Transfer Functions

4. $\dfrac{K}{s}$





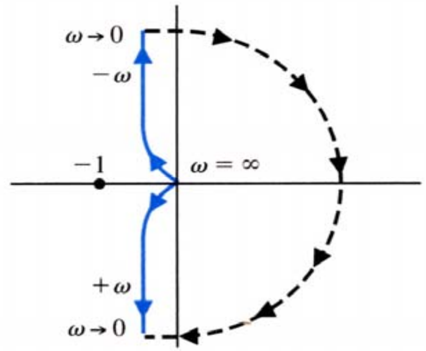


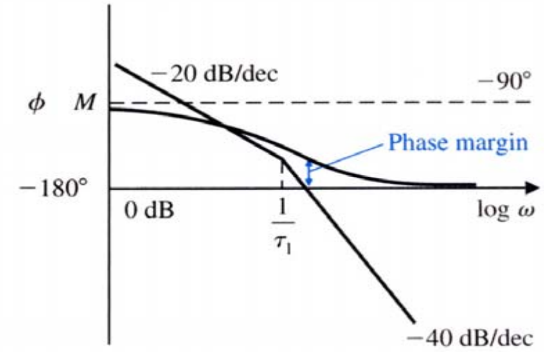


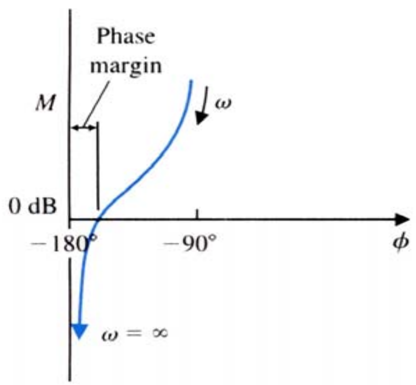
Ideal integrator; stable

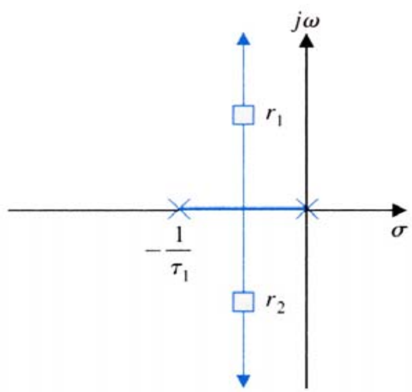
Table 9.6 Transfer Function Plots for Typical Transfer Functions

5. $\dfrac{K}{s(s\tau_1 + 1)}$





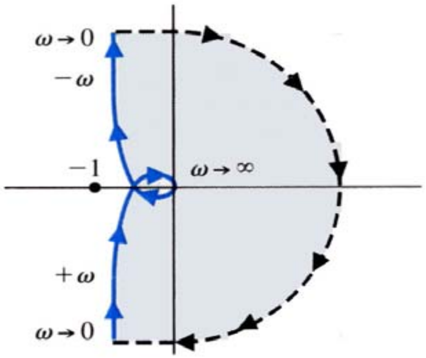


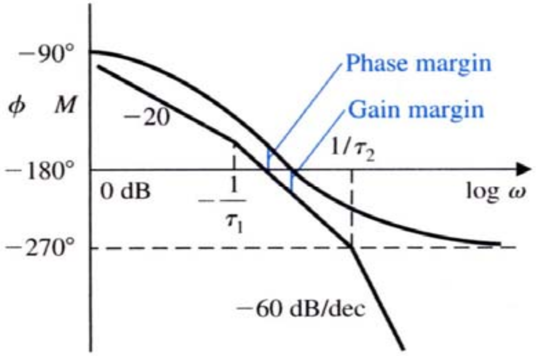


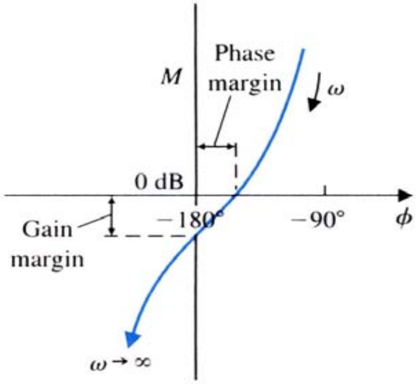
Elementary instrument servo; inherently stable; gain margin $= \infty$

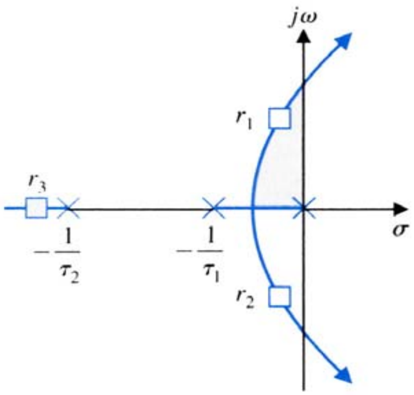
Table 9.6 Transfer Function Plots for Typical Transfer Functions

6. $\dfrac{K}{s(s\tau_1 + 1)(s\tau_2 + 1)}$





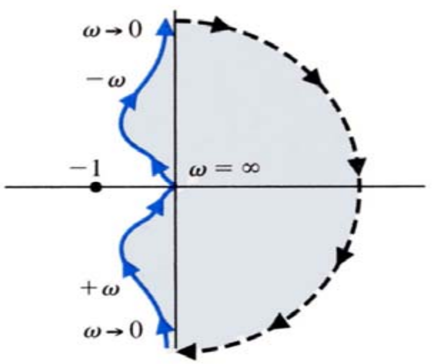


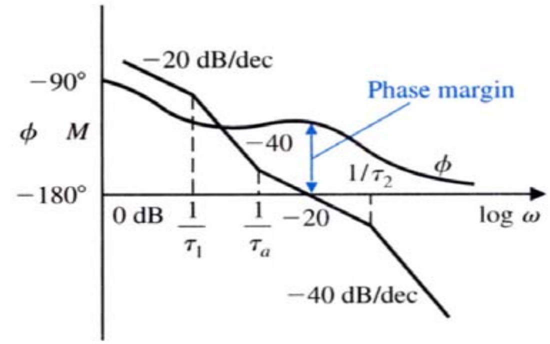


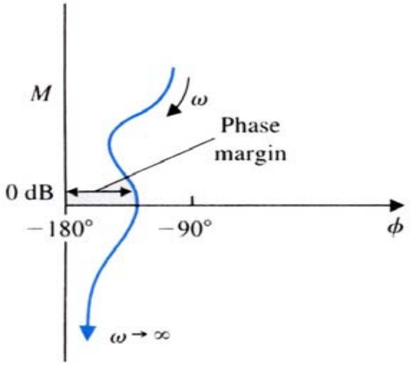
Instrument servo with field control motor or power servo with elementary Wark-Leonard drive; stable as shown, but may become unstable with increased gain

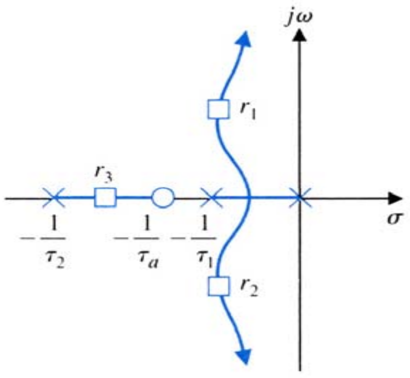
Table 9.6 Transfer Function Plots for Typical Transfer Functions

7. $$\frac{K(s\tau_a + 1)}{(s\tau_1 + 1)(s\tau_2 + 1)}$$





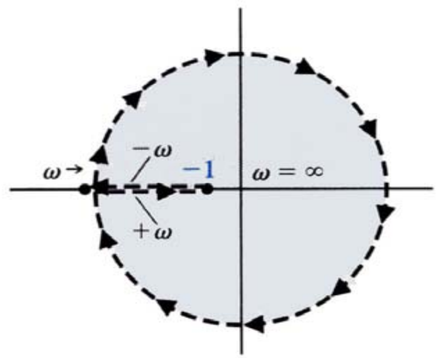


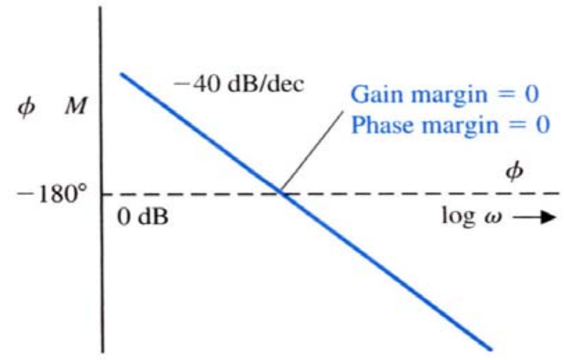


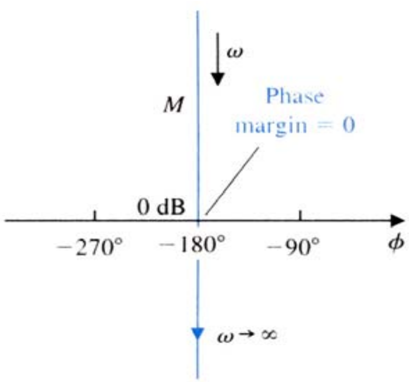
Elementary instrument servo with phase-lead (derivative) compensator; stable

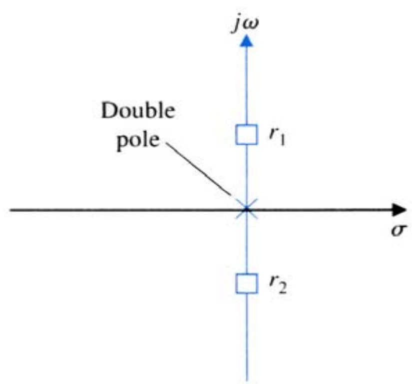
Table 9.6 Transfer Function Plots for Typical Transfer Functions

8. $\frac{K}{s^2}$





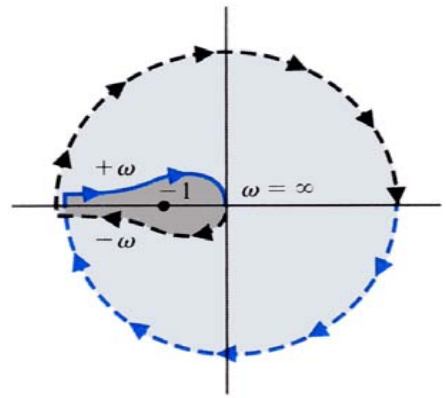


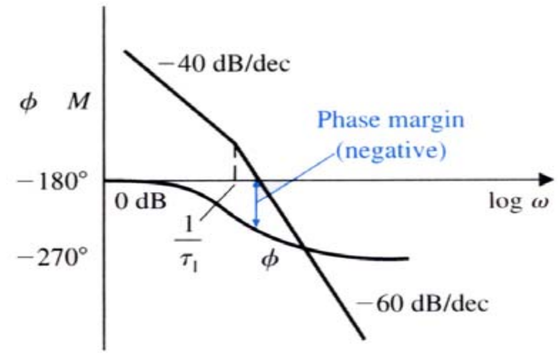


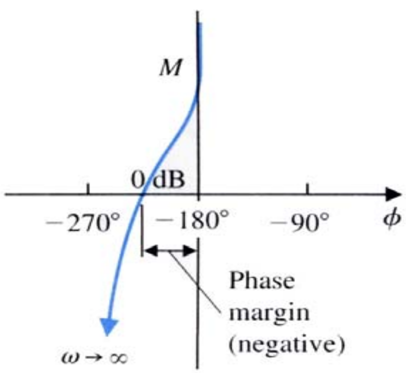
Inherently marginally stable; must be compensated

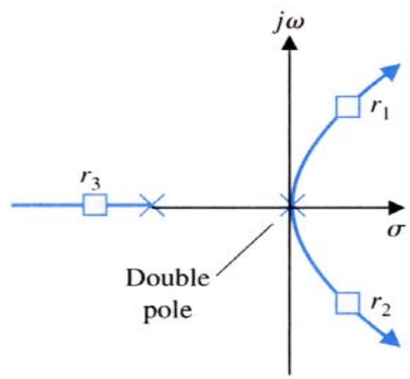
Table 9.6 Transfer Function Plots for Typical Transfer Functions

9. $\dfrac{K}{s^2(s\tau_1 + 1)}$







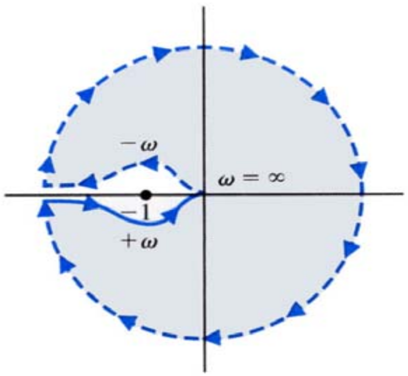


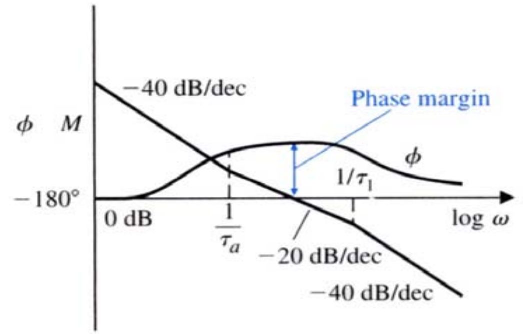
Inherently unstable; must be compensated

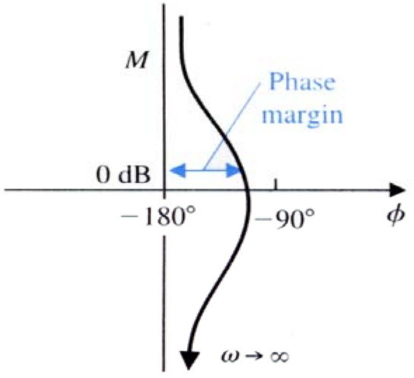
Table 9.6 Transfer Function Plots for Typical Transfer Functions

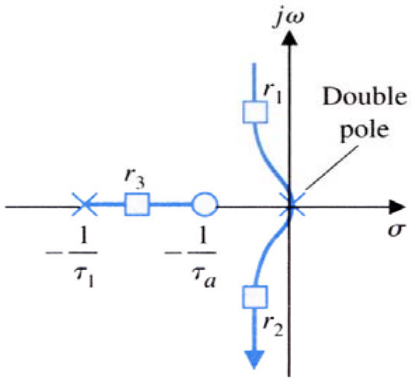
10. $\dfrac{K(s\tau_a + 1)}{s^2(s\tau_1 + 1)}$

$\tau_a > \tau_1$





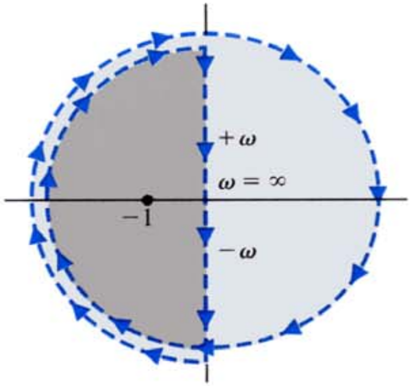


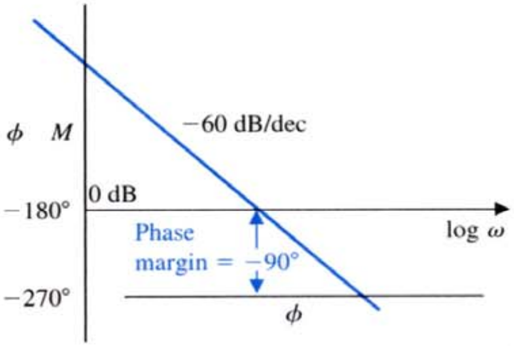


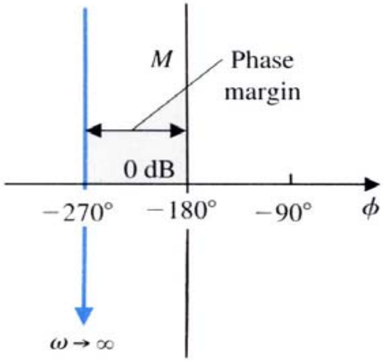
Stable for all gains

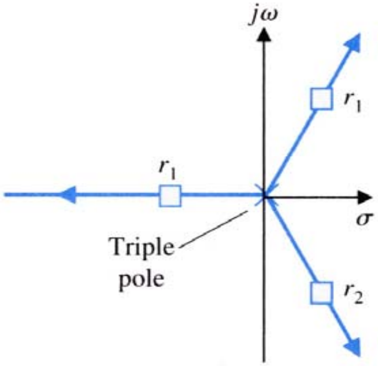
Table 9.6 Transfer Function Plots for Typical Transfer Functions

11. $\frac{K}{s^3}$





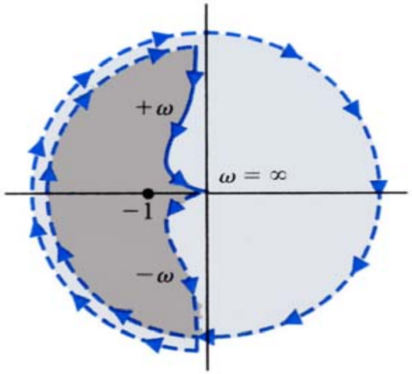


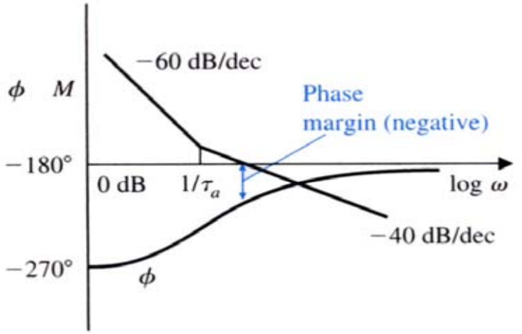


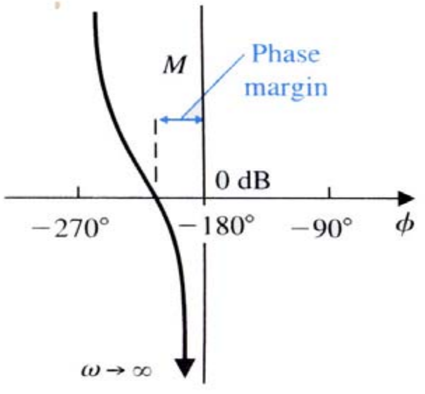
Inherently unstable

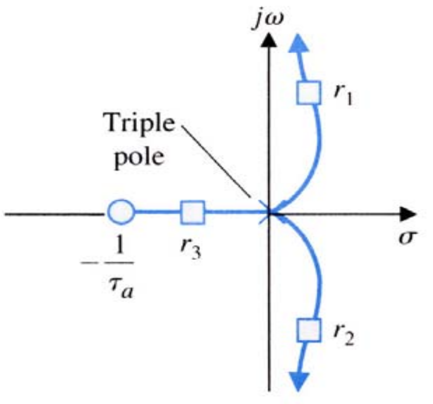
Table 9.6 Transfer Function Plots for Typical Transfer Functions

12. $\dfrac{K(s\tau_a + 1)}{s^3}$





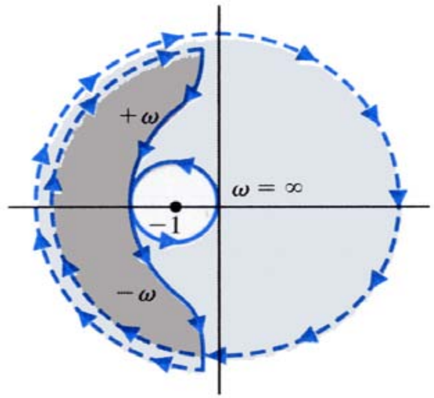


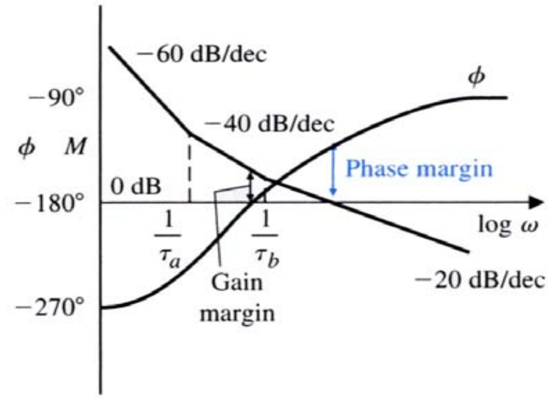


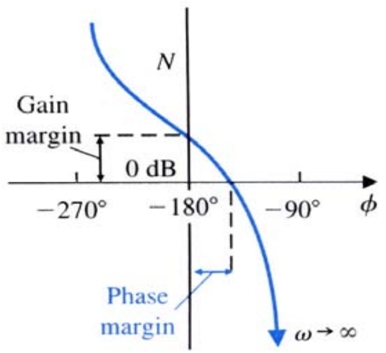
Inherently unstable

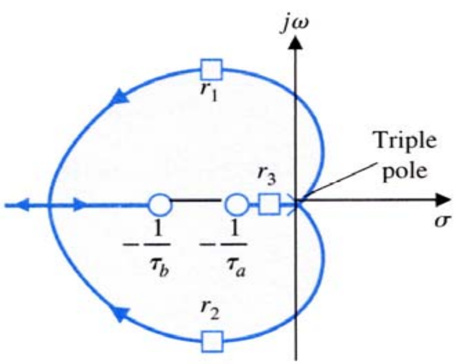
Table 9.6 Transfer Function Plots for Typical Transfer Functions

13. $\dfrac{K(s\tau_a+1)(s\tau_b+1)}{s^3}$





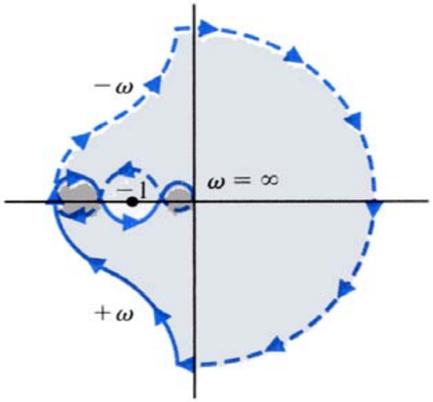


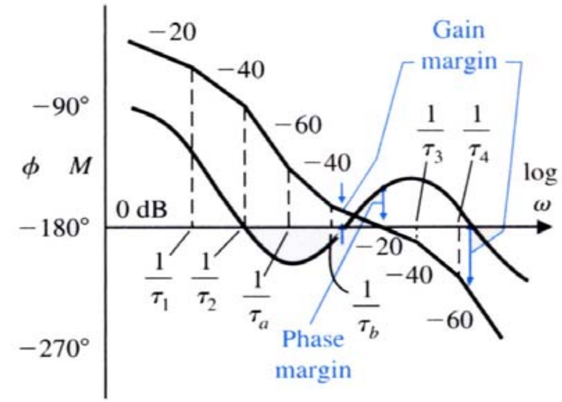


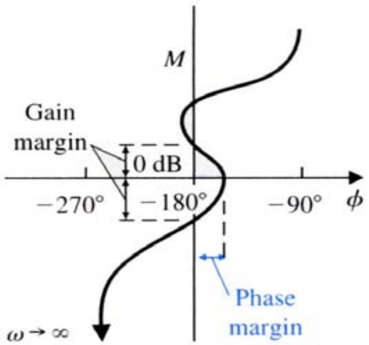
Conditionally stable; becomes unstable if gain is too low

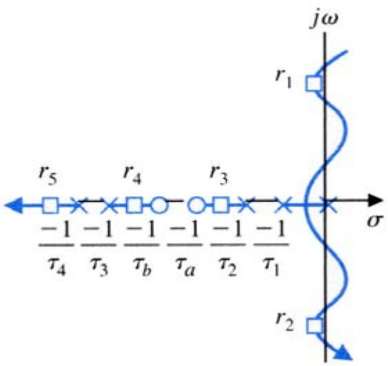
Table 9.6 Transfer Function Plots for Typical Transfer Functions

14. $$\frac{K(s\tau_a+1)(s\tau_b+1)}{s(s\tau_1+1)(s\tau_2+1)(s\tau_3+1)(s\tau_4+1)}$$





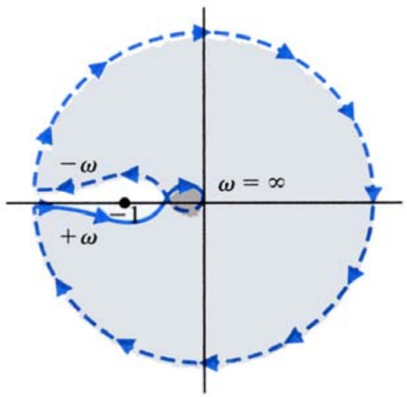


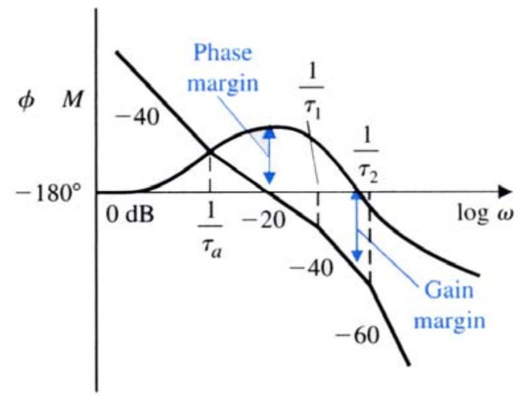


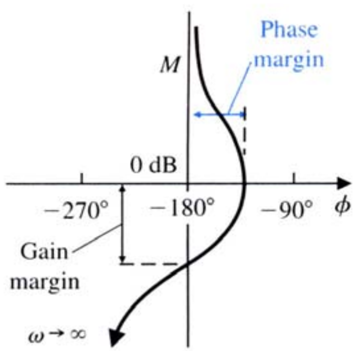
Conditionally stable; stable at low gain, becomes unstable as gain is raised, again becomes stable as gain is further increased, and becomes unstable for very high gains

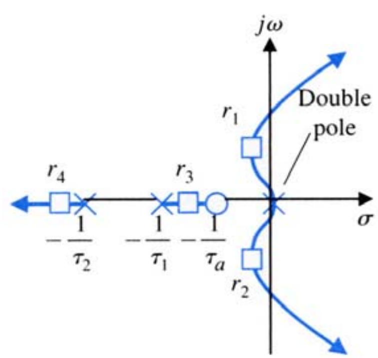
Table 9.6 Transfer Function Plots for Typical Transfer Functions

15. $\dfrac{K(s\tau_a + 1)}{s^2(s\tau_1 + 1)(s\tau_2 + 1)}$









Conditionally stable; becomes unstable at high gain

Exrcises

E9.1,E9.7,E9.8,E9.15,E9.21,P9.2,P9.4,P9.17,P9.21,AP9.4,

CHAPTER 10

The Design of Feedback Control Systems

The Design of Feedback Control System

§ Introduction

§ Time Domain Design

P-Control PI-Control PD-Control PID-Control

Phase-Lead Compensation Design

Phase-Lag Compensation Design

§ Frequency Domain Design

Phase-Lead Design Using Bode Diagram

Phase-Lag Design Using Bode Diagram

§Design for Deadbeat Response

☑Introduction

- ***Design procedure***
  - Determine what the system should do and how to it (design specifications).
  - Determine the controller or compenstaor configuration relative to how it is connected to the controlled plant.
  - Determine the parameter values of the controller or compensator to achieve the design goals.
  - Simulation verifications and recheck the specifications.
  - Experimental results.
- ***Design specification***
  - Time domain: maximum overshoot, rise time, settling time and steady-state accuracy
  - Frequency domain: gain margin, phase margin, resonant peak, bandwidth

- ***Controller (Compensator) configuration***
  - Cascade (series) compensation
  - Feedback compensation
  - Series-feedback compensation
  - Forward compensation with series compensation (two degree of freedom)
  - Feedforward compensation (two degree of freedom)
  - State-feedback compensation

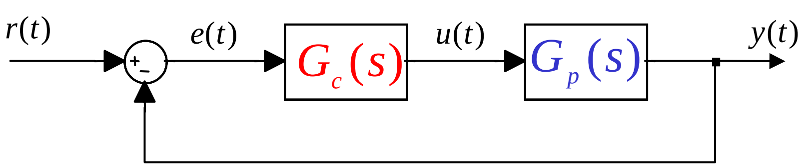


Figure 1 Cascade

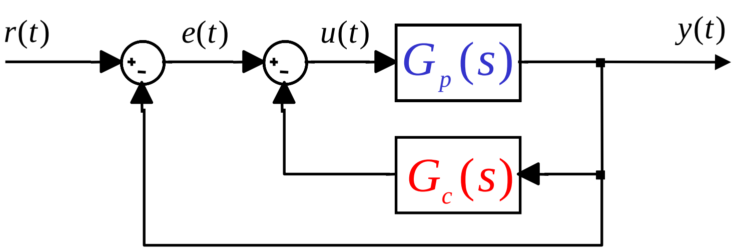


Figure 2 Feedback

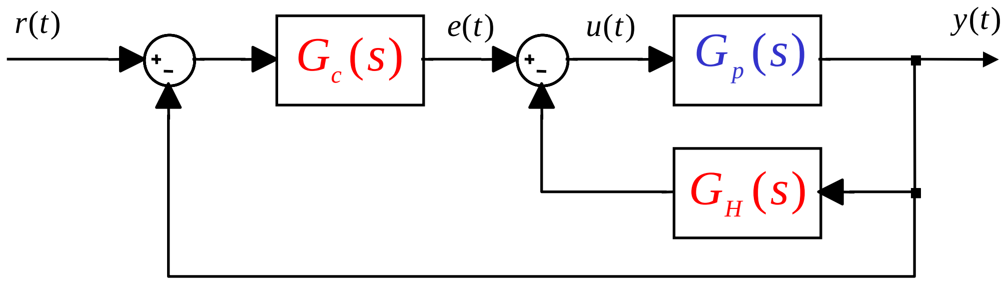


Figure 3 Series-feedback compensation

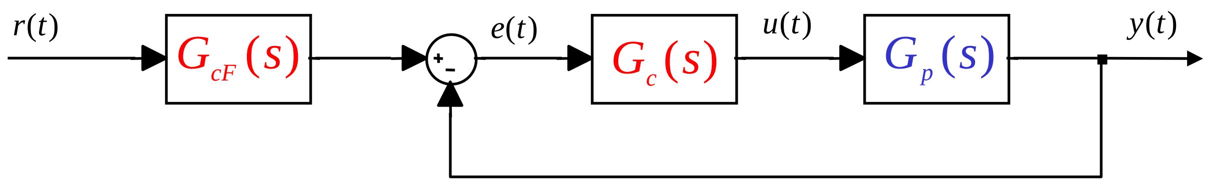


*Figure 4*Forward compensation with series compensation (two degree of freedom)

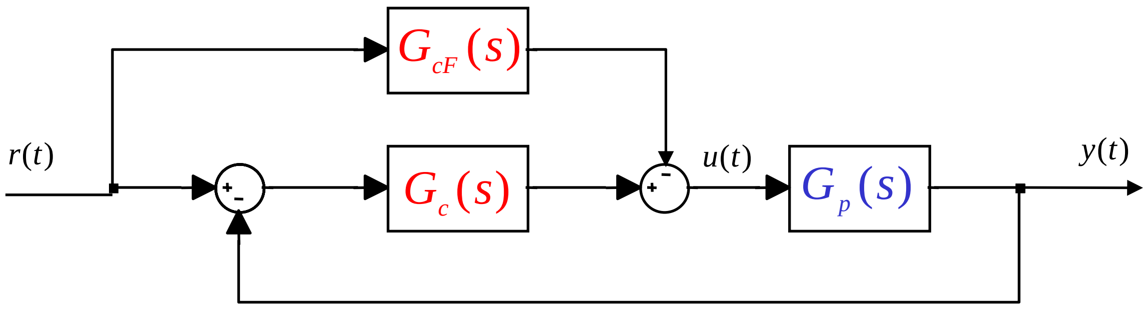


Figure 5 Feedforward compensation (two degree of freedom)

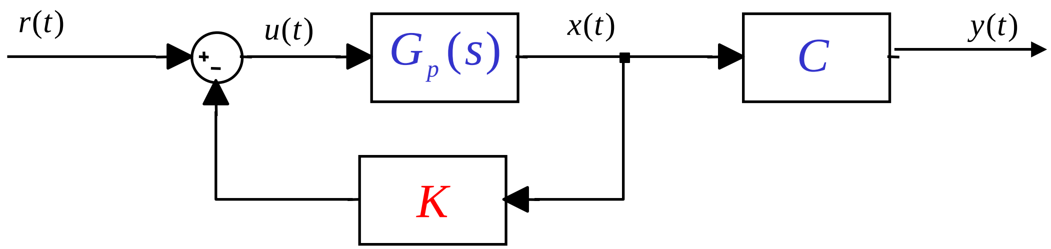
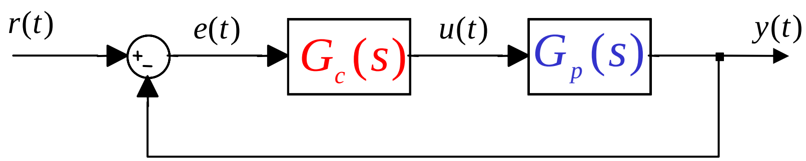


Figure 6 State-feedback compensation

§ Time Domain Design

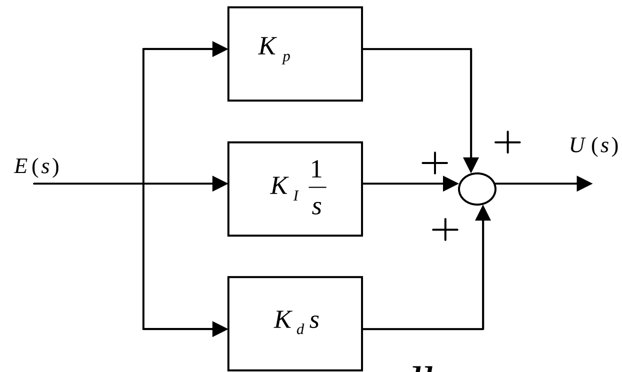


PID Controller Design

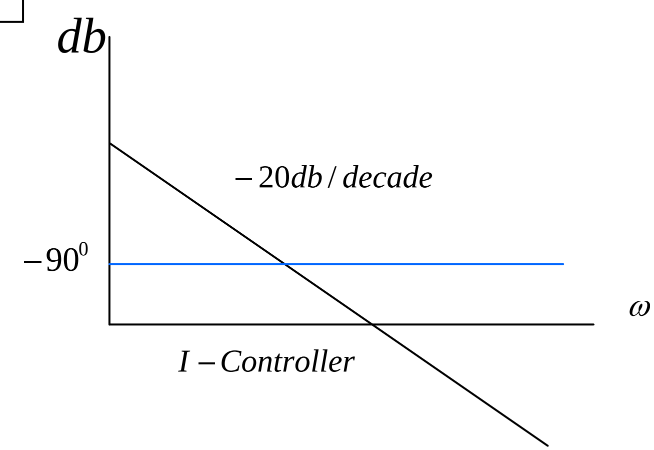
$$u(t) = K_p e(t) + K_I \int e(t) + K_d \frac{d}{dt} e(t)$$

$$U(s) = (K_p + K_i \tfrac{1}{s} + K_d s)E(s)$$

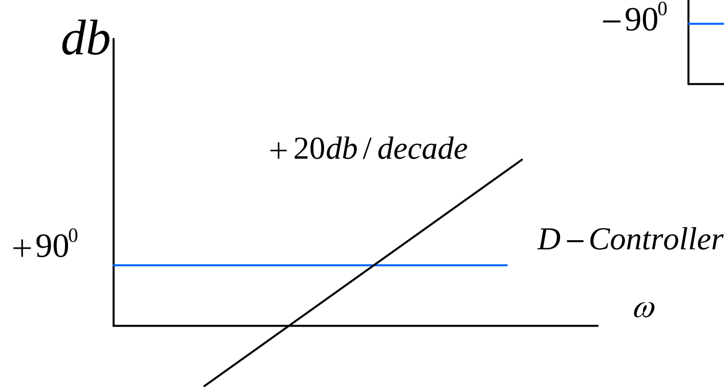
$$G_c(s) = \frac{U(s)}{E(s)} = K_p + K_i \tfrac{1}{s} + K_d s$$



$p - Controller$



$I - Controller$

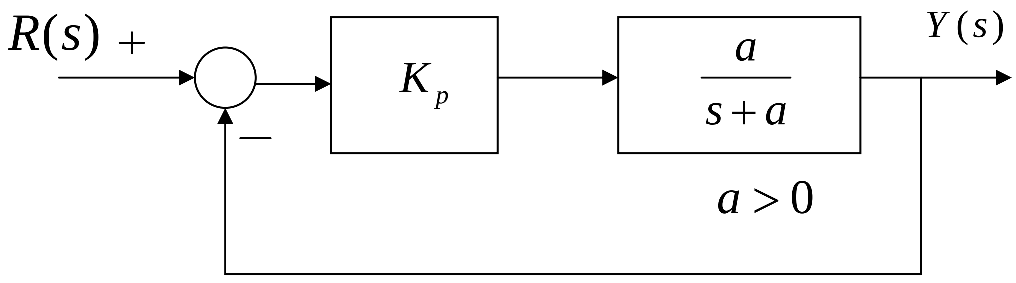


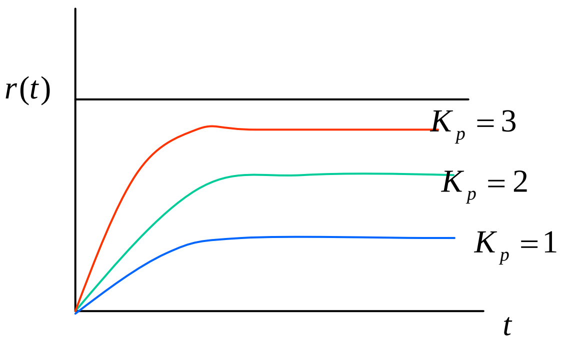
$D - Controller$

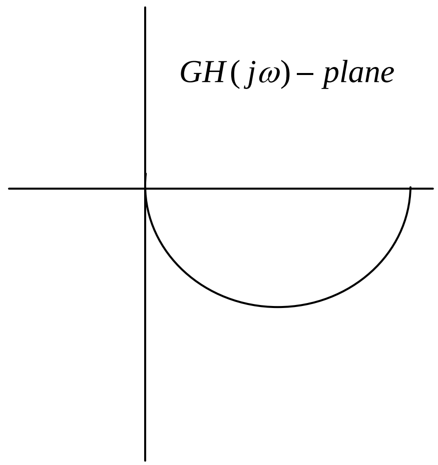
§ Closed-loop control with P-controller

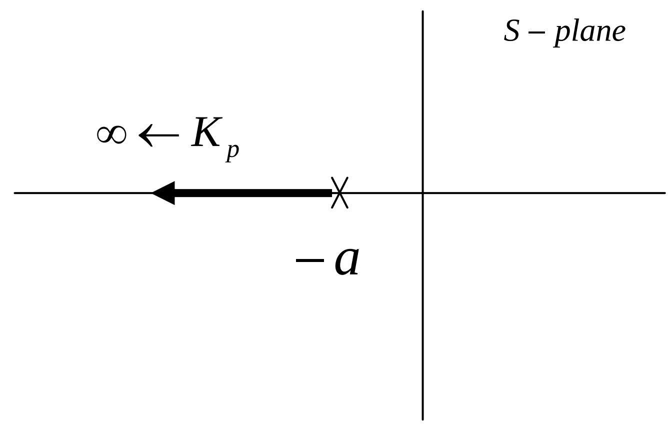
P-control for first order system

$PM \to \infty, GM \to \infty$









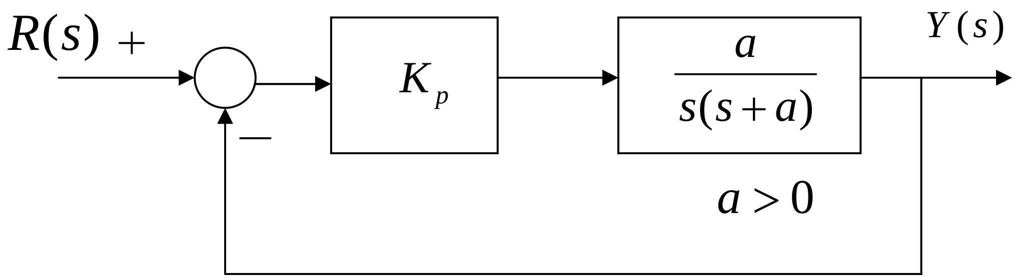
$$\frac{Y(s)}{R(s)} = \frac{aK_p}{s + (aK_p + a)}$$

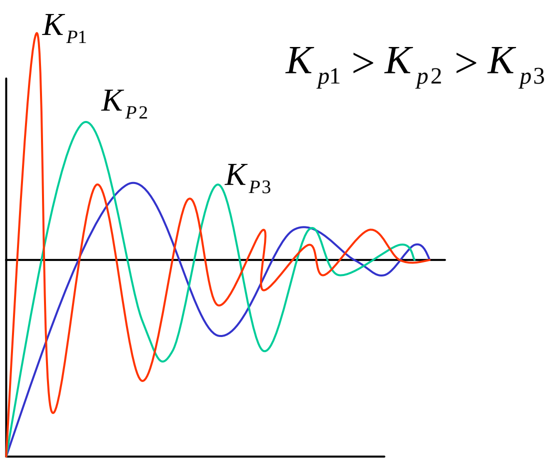
$$y(t) = \frac{K_P}{K_P + 1}(1 - e^{-a(K_P+1)t})$$

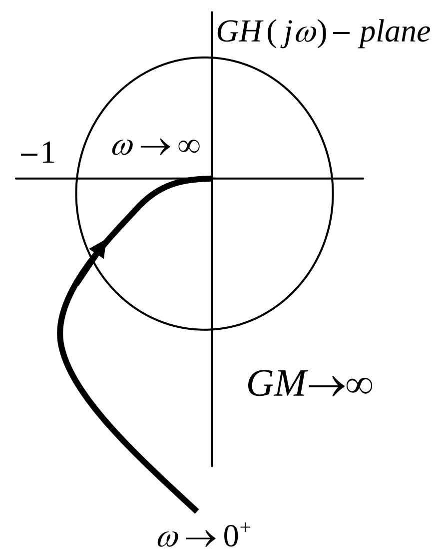
P-control for second order system

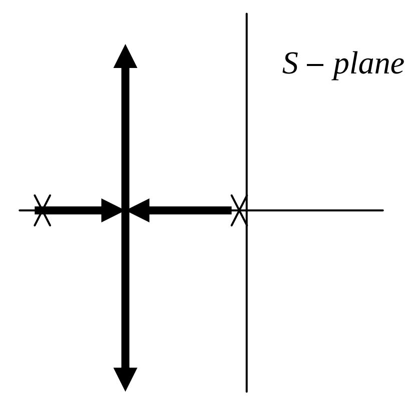
$$\frac{Y(s)}{R(s)} = \frac{aK_p}{s^2 + as + aK_p}$$

$$\omega_n = \sqrt{aK_p}, \qquad \xi = \frac{a}{2\sqrt{aK_p}}$$









PI-control for first order system

$$G_c(s) = K_p + K_I \frac{1}{s}$$

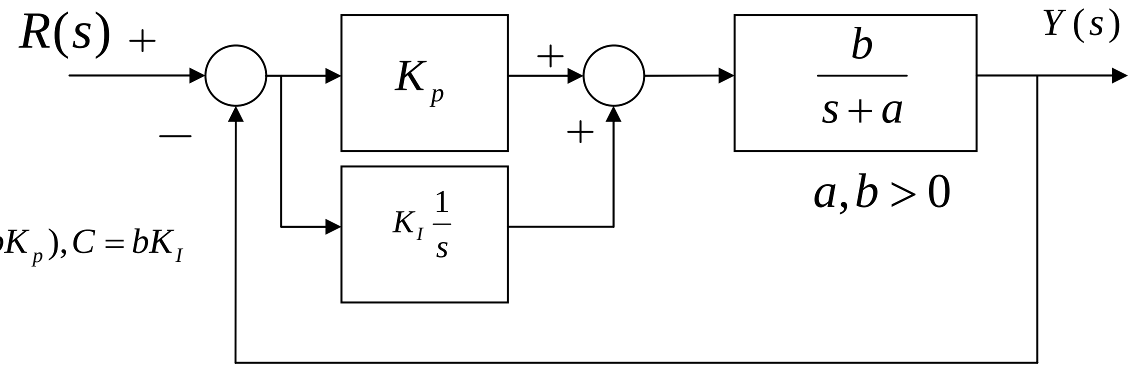
$$\frac{Y(s)}{R(s)} = \frac{a(K_p s + K_I)}{s^2 + (a + bK_p)s + bK_I}$$

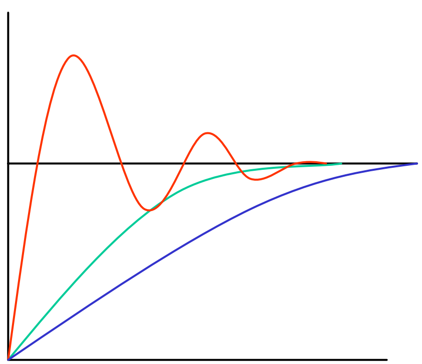
$$\omega_n = \sqrt{bK_I}, \quad \xi = \frac{a + bK_p}{\sqrt{bK_I}}, \quad A = 1, B = (a + bK_p), C = bK_I$$

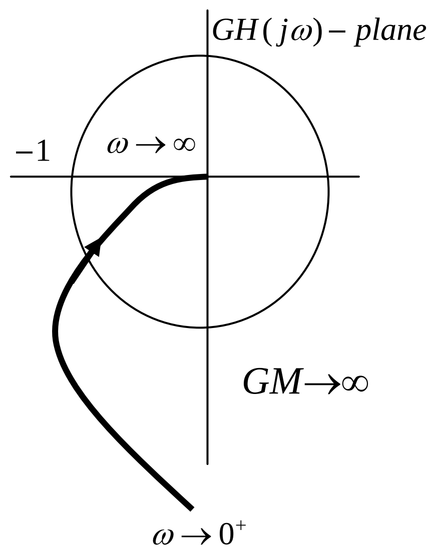
$$B^2 - 4AC > 0, \Rightarrow \xi > 1$$

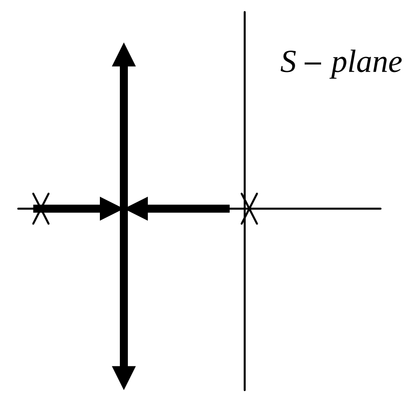
$$B^2 - 4AC = 0, \Rightarrow \xi = 1$$

$$B^2 - 4AC < 0, \Rightarrow \xi < 1$$



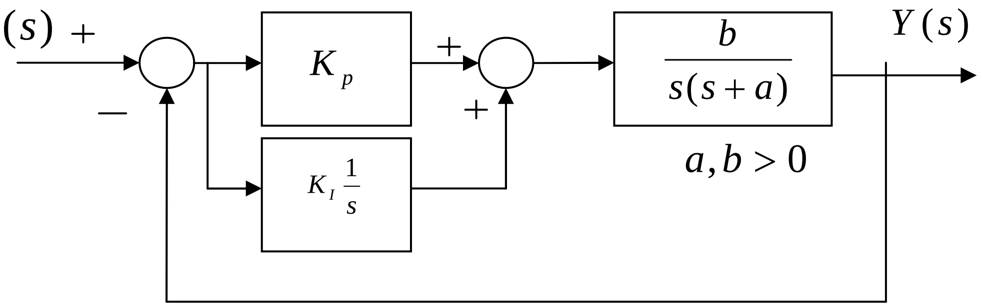




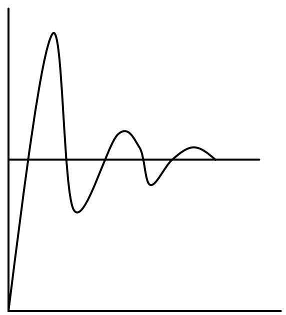


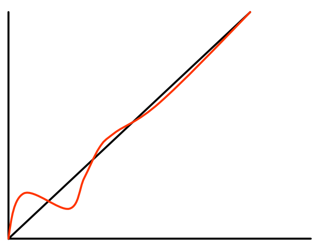
PI-control for second order system

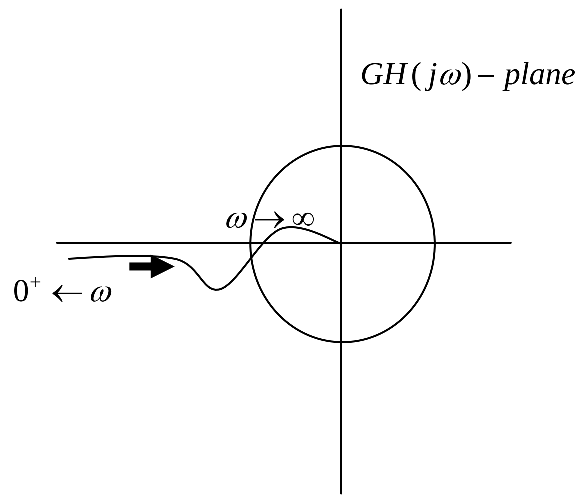
$$\frac{Y(s)}{R(s)}=\frac{b\dfrac{K_p s+K_I}{s}}{1+\dfrac{b(K_p s+K_I)}{s^2(s+a)}},\quad F(s)=1+\frac{bK_p\left(s+\dfrac{K_I}{K_P}\right)}{s^2(s+a)}$$

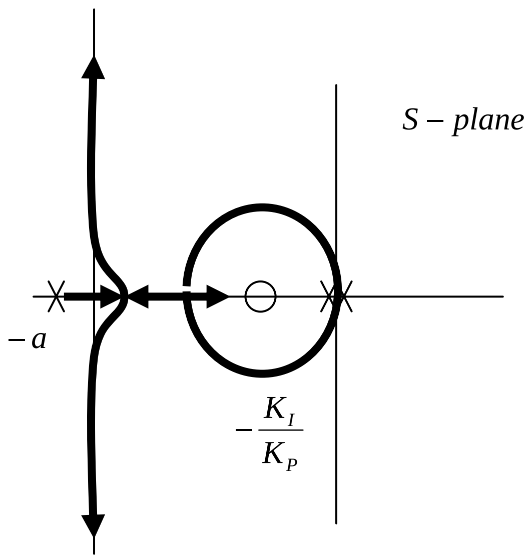


$$\boxed{\frac{K_I}{K_P}<<a}$$







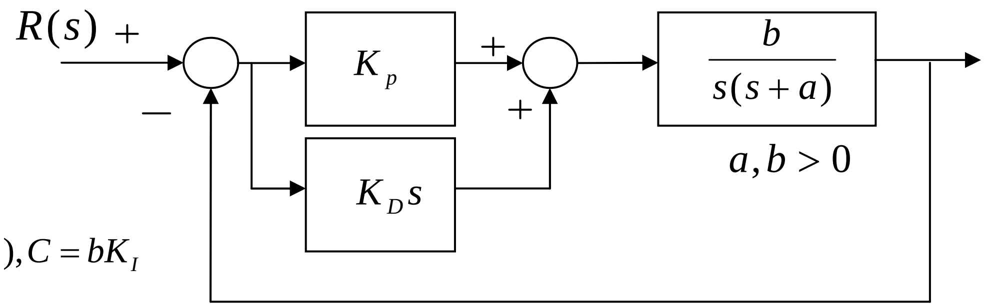


PD-control for second order system

$$G_c(s) = K_p + K_d s$$

$$\frac{Y(s)}{R(s)} = \frac{a(K_d s + K_p)}{s^2 + (a + bK_d)s + bK_p}$$

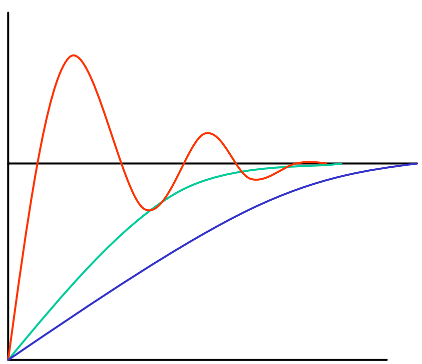
$$\omega_n = \sqrt{bK_p}, \quad \xi = \frac{a + bK_d}{\sqrt{bK_p}}, \quad A = 1, B = (a + bK_d), C = bK_I$$

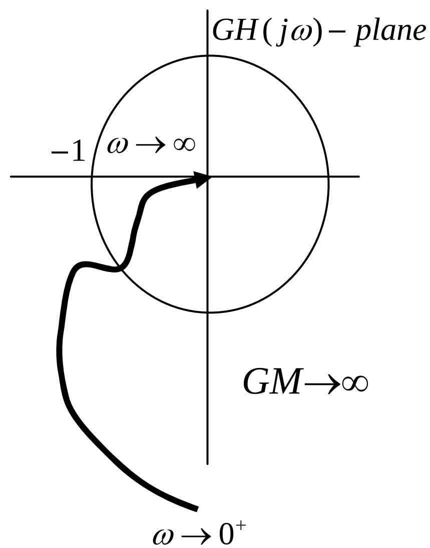


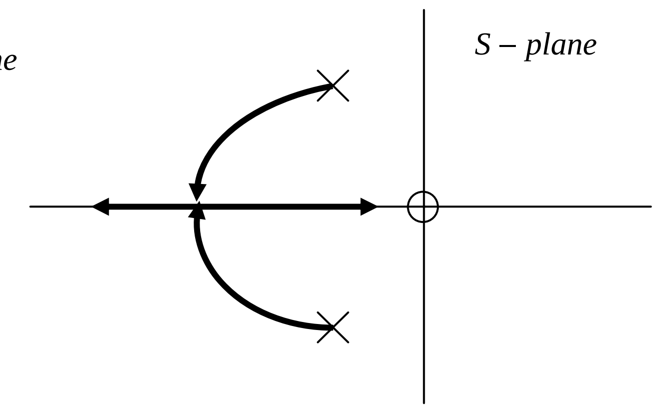
$$B^2 - 4AC > 0, \Rightarrow \xi > 1$$

$$B^2 - 4AC = 0, \Rightarrow \xi = 1$$

$$B^2 - 4AC < 0, \Rightarrow \xi < 1$$





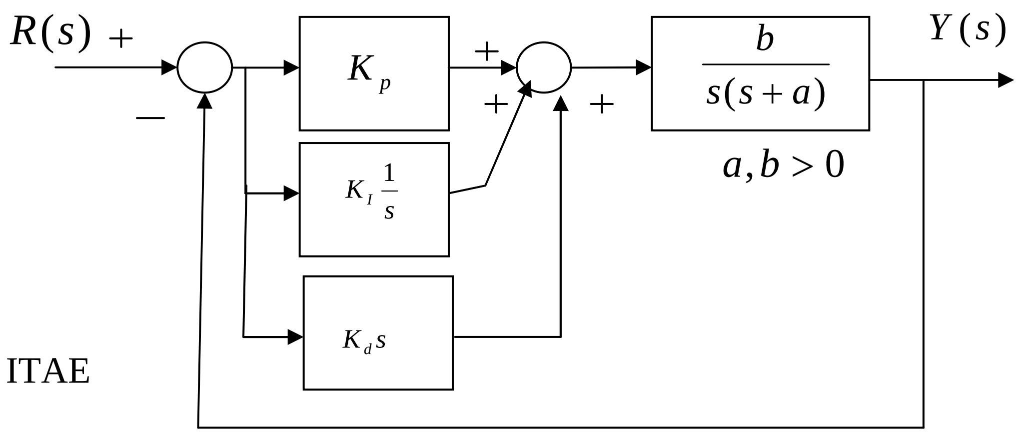


PID-control for second order system

$$G_c(s) = K_p + K_d s + K_I \frac{1}{s}$$

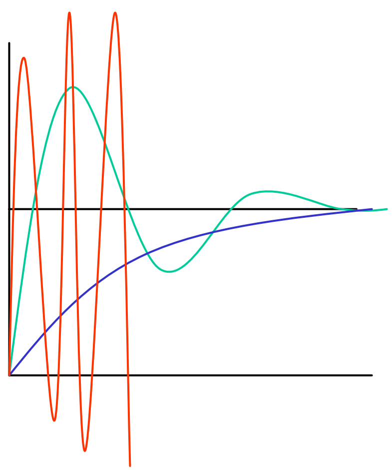
$$\frac{Y(s)}{R(s)} = \frac{b(K_d s^2 + K_p s + K_I)}{s^3 + (a + bK_d)s^2 + bK_p s + bK_I}$$

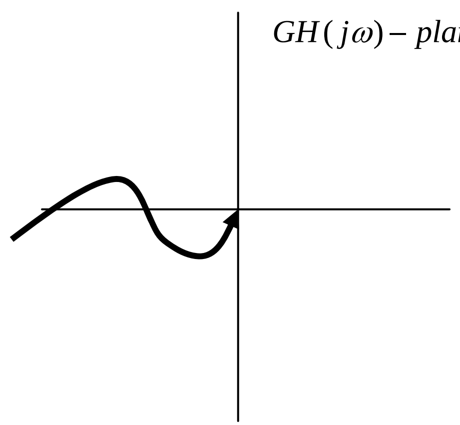
$$(a + bK_d) \bullet bK_p > bK_I$$

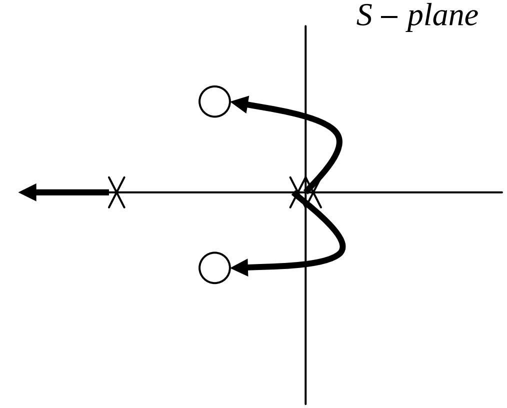


The Optimum Coefficients of T(s) Based on the ITAE Criterion for a step Input (P.252 Table 5.6)

$$s^3 + 1.75\omega_n s^2 + 2.15\omega_n s + \omega_n^3$$







Phase-lead Compensation Network

$$G(s)=\frac{V_2(s)}{V_1(s)}=\frac{R_2}{R_1+R_2}\frac{(R_1Cs+1)}{\{[R_1R_2/(R_1+R_2)]Cs+1\}}$$

$$\tau=\frac{R_1R_2}{R_1+R_2}C,\alpha=\frac{R_1+R_2}{R_2}$$

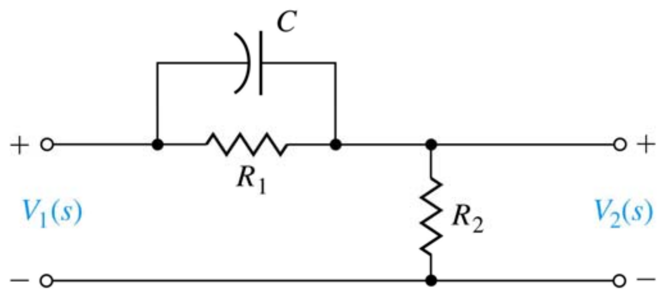
$$G(s)=\frac{(1+\tau\alpha s)}{\alpha(1+\tau s)},\alpha>1$$

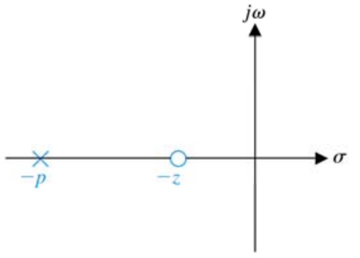
$$\omega_m=\sqrt{zp}=\frac{1}{\tau\sqrt{\alpha}} \qquad z=\frac{1}{\alpha\tau},\ p=\frac{1}{\tau},z<p$$

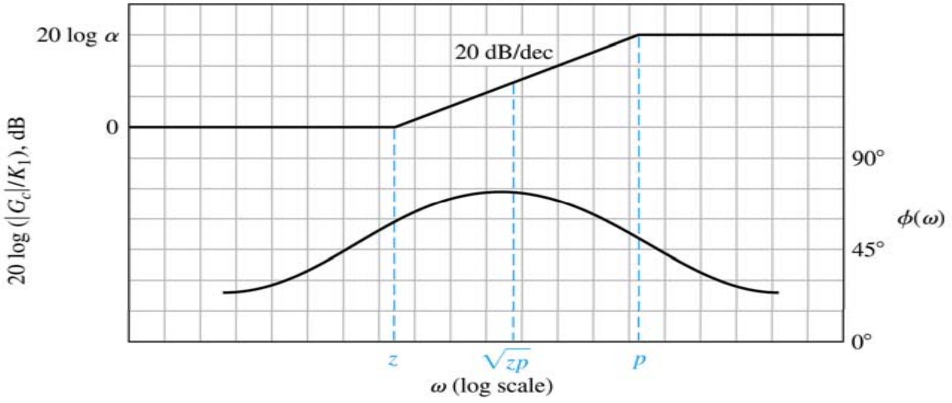
$$\phi=\tan^{-1}\frac{\alpha\omega\tau-\omega\tau}{1+(\omega\tau))^2\alpha},when\quad \omega=\omega_{m,}$$

$$\tan\phi_m=\frac{(\alpha/\sqrt{\alpha}-(1/\sqrt{\alpha})}{1+1}=\frac{\alpha-1}{2\sqrt{\alpha}}$$

$$\boxed{\sin\phi_m=\frac{\alpha-1}{\alpha+1},\alpha=\frac{1+\sin\phi_m}{1-\sin\phi_m}}$$





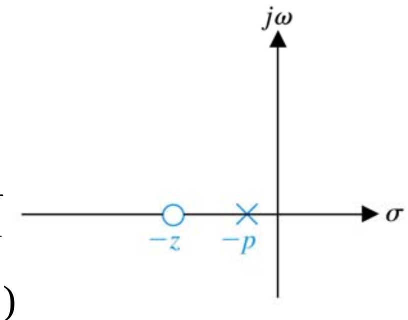


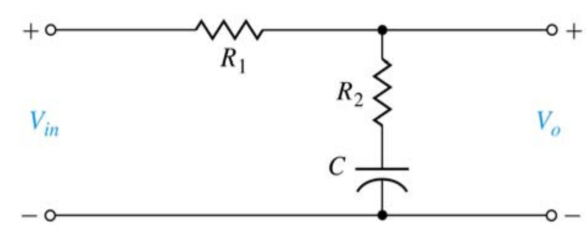
Phase-Lag Compensation Netwo[r]

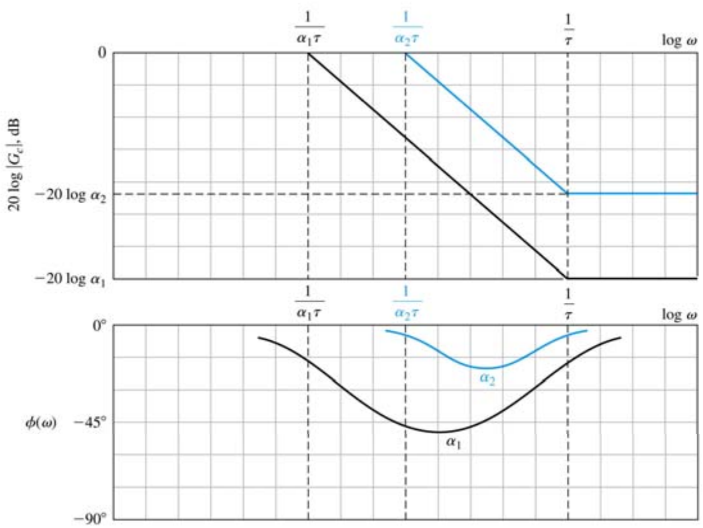
$$G(s)=\frac{V_o}{V_{in}}=\frac{R_2+(1/Cs)}{R_1+R_2+(1/Cs)}=\frac{R_2Cs+1}{(R_1+R_2)Cs+1}$$

$$=\frac{1+\tau s}{1+\alpha\tau s}=\frac{1}{\alpha}\frac{s+z}{s+p},\tau=R_2C,\alpha=\frac{(R_1+R_2)}{R_2}$$

$$z=\frac{1}{\tau},p=\frac{1}{\alpha\tau},\alpha>1,\omega_m=\sqrt{zp}$$

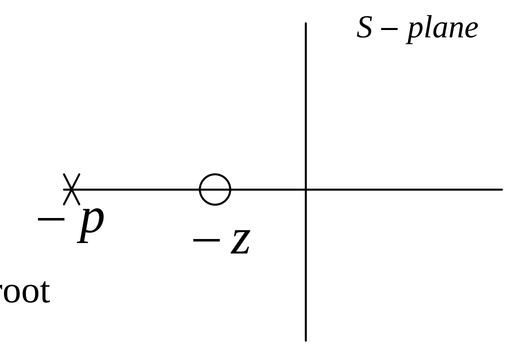






Root Locus Method

$$G_c(s) = \frac{s+z}{s+p}, z < p$$



Phase-Lead Compensation Design

The s-plane root locus method is as follows:

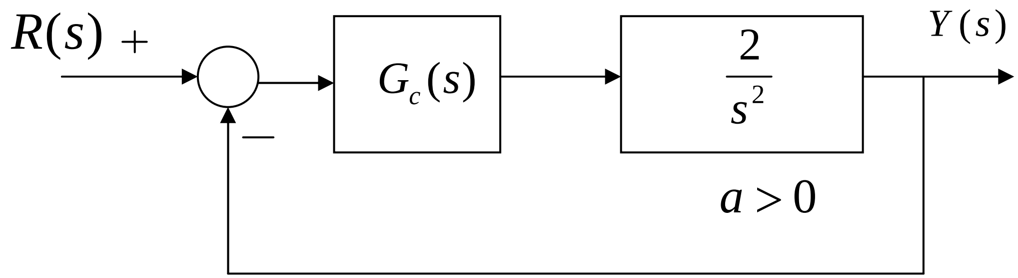
1. List the system specifications and translate them into a desired root location for the dominant roots.
2. Sketch the uncompensated root locus, and determine whether the desired root locations can be realized with an uncompensated system.
3. If a compensator is necessary, place the zero of the phase-lead network directly below the desired root location (or to the left of the first two real poles).
4. Determine the pole location so that the total angle at the desired root location is 180^0
5. Evaluate the total system gain at the desired root location and then calculate the error constant.
6. Repeat the steps if the error constant is not satisfactory.

$$G_c(s) = K\frac{s+z}{s+p}$$

The specification for the system are

Settling time(2% criterion) $T_s \le 4\sec onds$

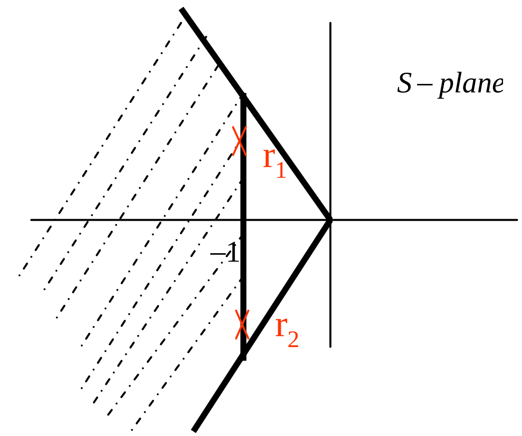
Percent overshoot for a step input, $\le 35\%$



Sol:

$$P.O = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \le 0.35 \Rightarrow \xi \ge 0.32, \theta = \cos^{-1}\xi = 71^0$$

$$T_s \le \frac{4}{\xi\omega_n} = 4 \Rightarrow \xi\omega_n \ge 1$$

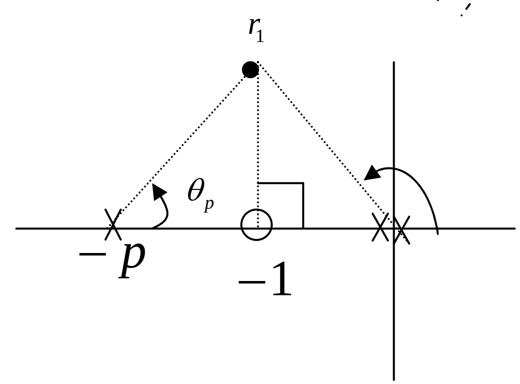


Select the desired roots location are

$$r_1 r_2 = -1 \pm j2 \quad \theta = 63.4^0 \Rightarrow \xi = 0.45$$

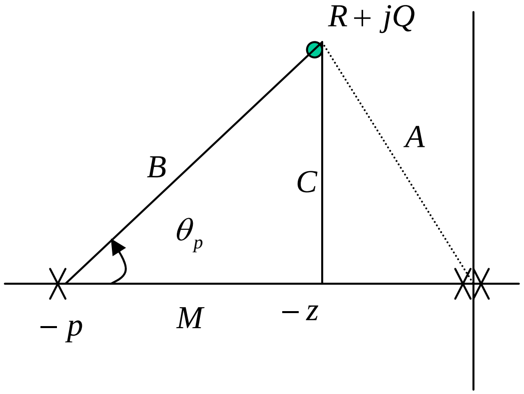
$$\phi = -2(180^0 - \tan^{-1}2) + 90^0 = -2(116^0) + 90^0 = -142$$

$$-180 = \phi - \theta_p \Rightarrow \theta_p = 38^0$$



$$\frac{Q}{M} = \tan\theta_p, M = \frac{Q}{\tan\theta_p}, p = z + m$$

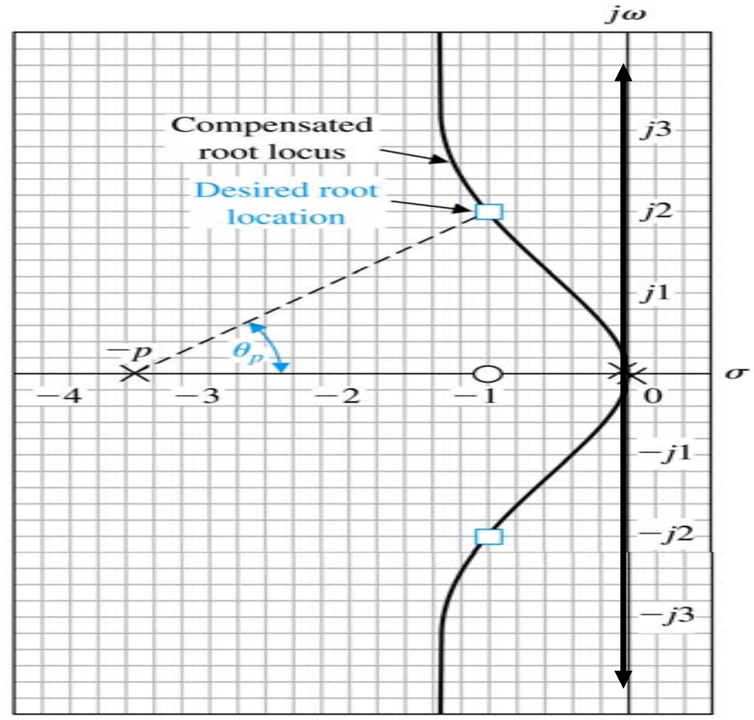
$$M = \frac{2}{0.7812} = 2.6 \Rightarrow p = 1 + 2.6 = 3.6$$



$$G_c G_p(s) = \frac{2K(s+1)}{s^2(s+3.6)}, F(s) = 1 + KG_1H_1(s) = 0$$

$$K = \left|\frac{1}{G_1H_1(s)}\right|_{s=r_1} = \frac{A*A*B}{2C} = \frac{2.23^2*3.25}{2*2} = 4.1$$

$$G_c(s) = 4.1\frac{s+1}{s+3.6}$$

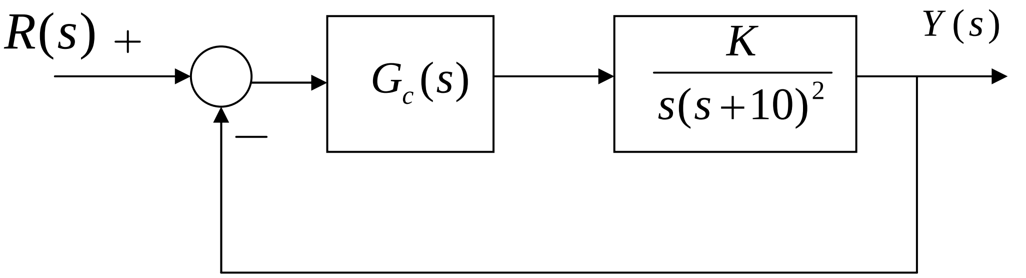


Phase-Lag Compensation Design

The steps necessary for the design of a phase-lag network on the s-plane are as follows:

1. Obtain the root locus of the uncompensated system.
2. Determine the transient performance specifications for the system and locate suitable dominant root location on the uncompensated root locus that will satisfy the specifications.
3. Calculate the loop gain at the desired root location and thus the system error constant,
4. Compare the uncompensated error constant with the desired error constant, and calculate the necessary increase that must result from the pole-zero ratio of the compensator, α .
5. With the known ratio of the pole-zero combination of the compensator, determine a suitable location of the pole and zero of the compensator so that the compensated root locus will still pass through the desired root location. Locate the pole and zero near the origin of the s-plane in comparison to ω_n .

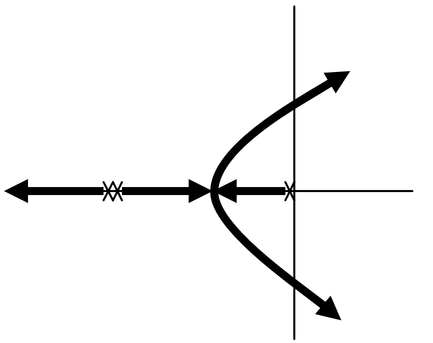
Sol:



$Spec.: \xi = 0.707, K_v = 20$

$K_v = 20 = \lim_{s \to 0} G(s) = \frac{K}{10^2}, K = 2000$

If K=2000, the system will be unstable



$\xi = 0.707 \Rightarrow s_{1,2} = -2.9 \pm j2.9$

$\Rightarrow \frac{K}{\alpha} = 236$

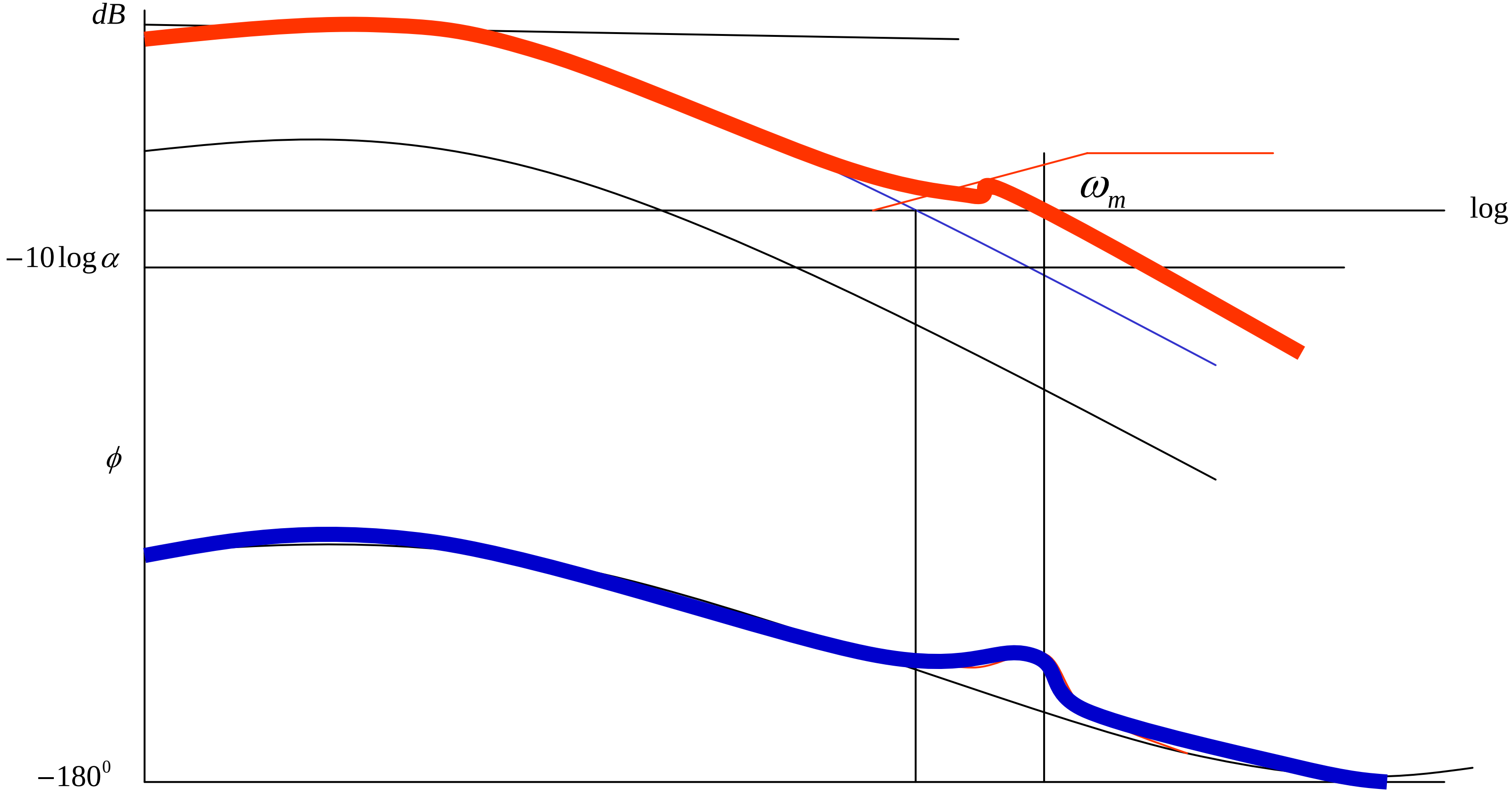
$$\alpha = \left| \frac{z}{p} \right| = \frac{2000}{236} = 8.5$$

$select \quad z = 0.1, p = 0.1/9 = 0.0111$

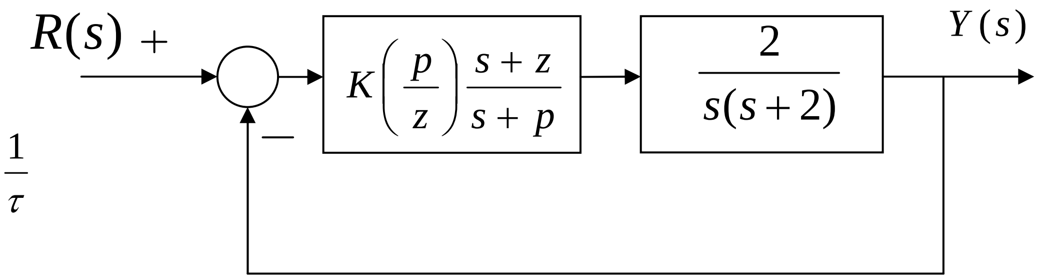
§ Frequency Domain Design

Phase-Lead Design Using Bode Diagram

1. Determined the system DC-gain such that the error constant can be satisfied.
2. Evaluate the uncompensated system phase margin when the error constant are satisfied
3. Allowing for a small of safety(5~10 degree), determine the necessary additional phase lead, ϕ_m $\phi_m = \phi_d - \phi_n + 3^0 \sim 10^0$ $\alpha = \dfrac{\sin\phi_m + 1}{1 - \sin\phi_m}$
4. Evaluate α from Eq.(10.11).
5. Evaluate 10 logα and determined the frequency where the uncompensated magnitude curve is equal to -10 logα . Because the compensation network provides a gain of 10 logα atω_m ,this frequency is the new 0-dB crossover frequency and simultaneously.
6. Calculate the pole $p = \omega_m \sqrt{\alpha}$ and $z = p/\alpha$
7. Draw the compensated frequency response, check the resulting phase margin, and repeat the steps if necessary. Finally, for an acceptable design, raise the gain of the amplifier in order to account for the attenuation($1/\alpha$)


dB
ω_m
$\log\omega$
$-10\log\alpha$
ϕ
-180^0

Example 10.1



$$G_c(s) = K\frac{p}{z}\frac{s+z}{s+p}, \qquad z = \frac{1}{\alpha\tau}, \qquad p = \frac{1}{\tau}$$

$Spec.: K_v < 5\%, \xi \ge 0.4$

The system dc-gain>=20

Phase margin:45^0

$$GH(jw) = \frac{20}{jw(0.5jw+1)}$$

$$0-dB = 20\log 20 - 20\log w - 20\log[(0.5w)^2+1]^{1/2}$$

$$= 20\log 20 - 20\log w - 10\log[(0.5w)^2+1]$$

$$w_c \approx 6.2$$

$$\phi_n = 180 - 90 - \tan^{-1}(0.5w_c) = 180 - 162 = 18^0$$

$$\phi_m = \phi_d - \phi_n + 3^0 \sim 10^0 = 45^0 - 18^0 + 3^0 = 30^0$$

$$\alpha = \frac{1+\sin\phi_m}{1-\sin\phi_m} = \frac{1+0.5}{1-0.5} = 3$$

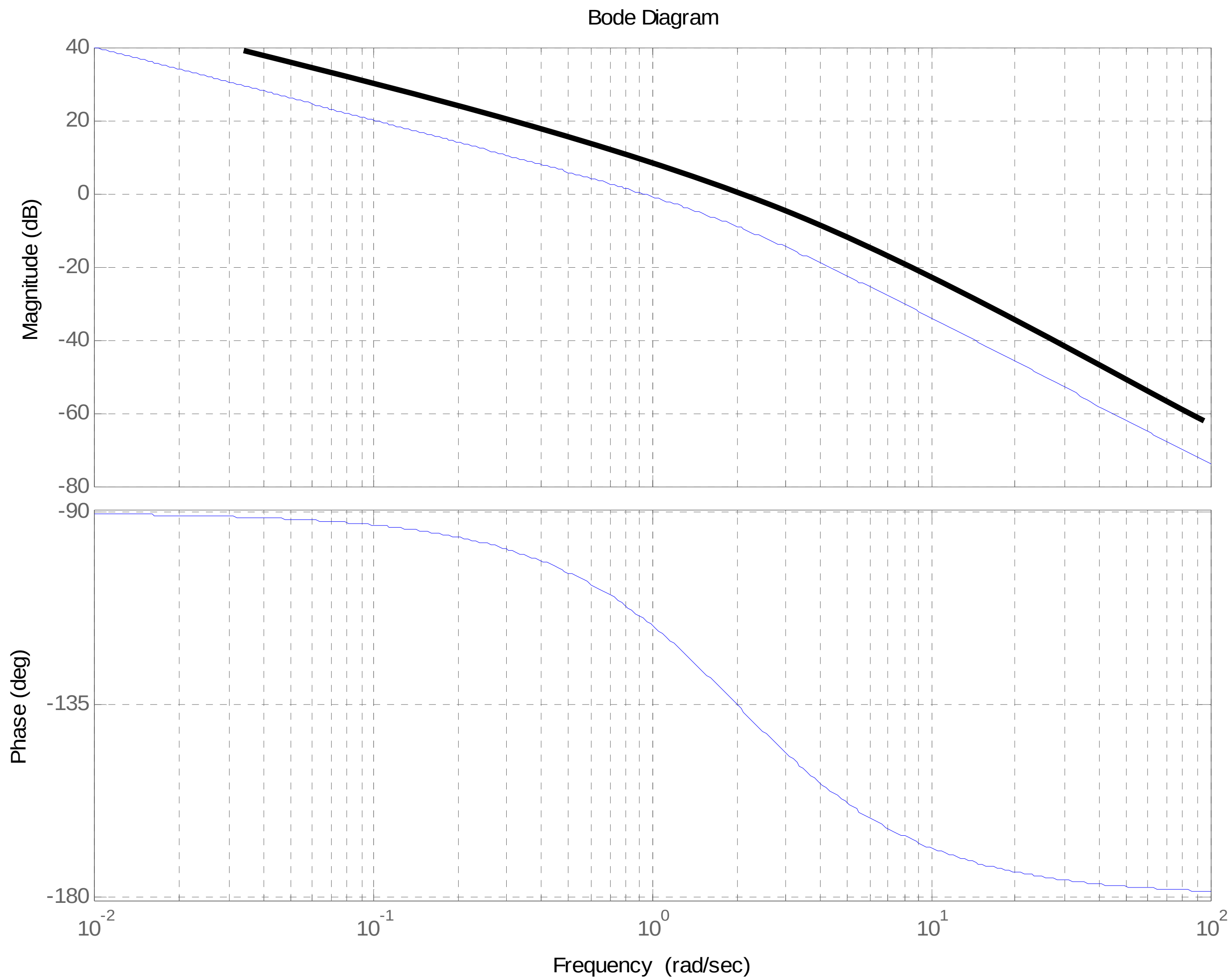
$$-10\log\alpha = -4.77dB$$

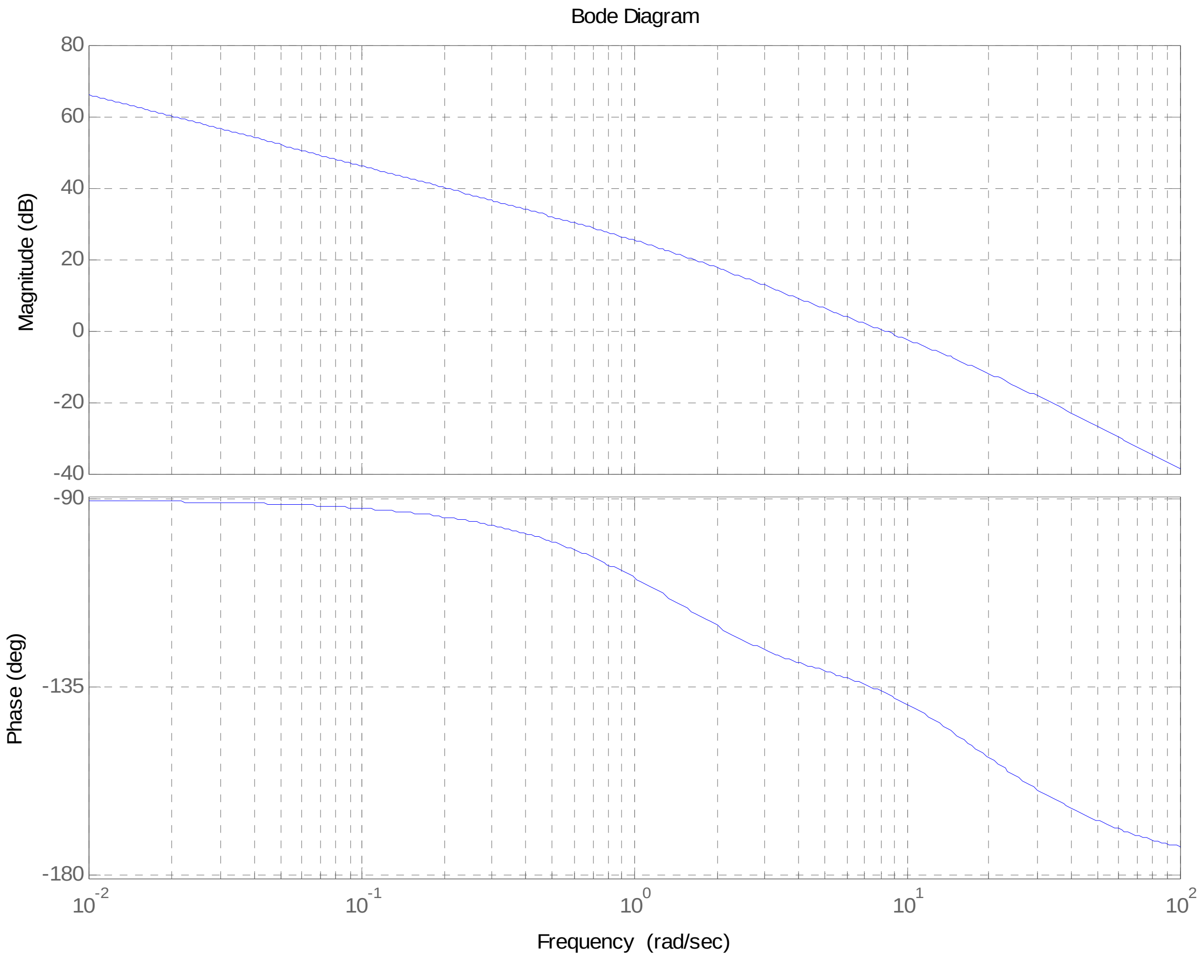
find the new crossover frequency from the bode diagram

$$\omega_m = 8.4 \Rightarrow p = \omega_m\sqrt{\alpha} = 14.4, z = \frac{p}{\alpha} = 4.8$$

$$K*(\frac{p}{z}) = 20*3 = 60$$

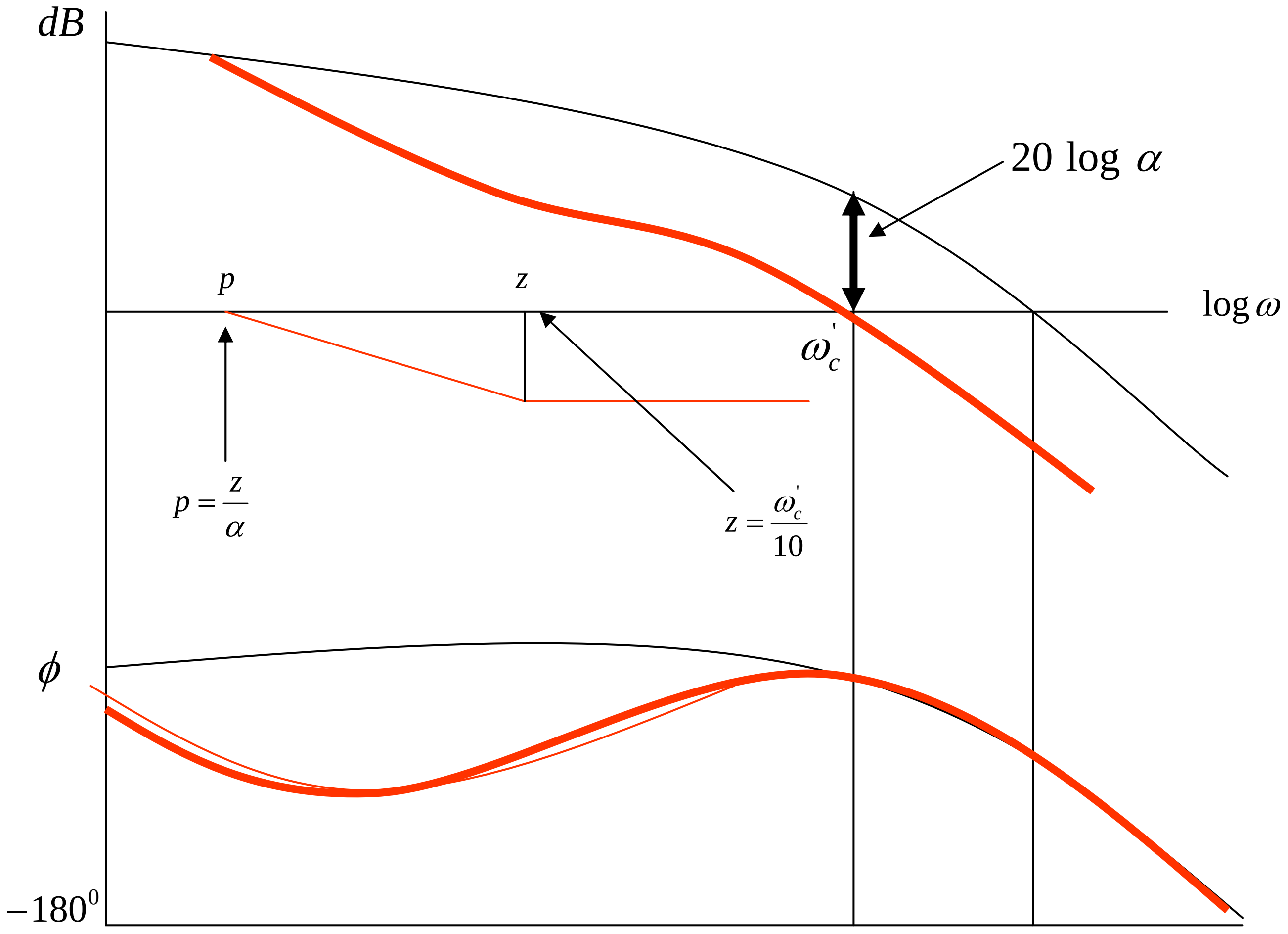
$$G_c(s) = 60\frac{s+4.8}{s+14.4}$$


Bode Diagram
Magnitude (dB)
40
20
0
-20
-40
-60
-80
Phase (deg)
-90
-135
-180
10^-2
10^-1
10^0
10^1
10^2
Frequency (rad/sec)


Bode Diagram
Magnitude (dB)
80
60
40
20
0
-20
-40
Phase (deg)
-90
-135
-180
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/sec)

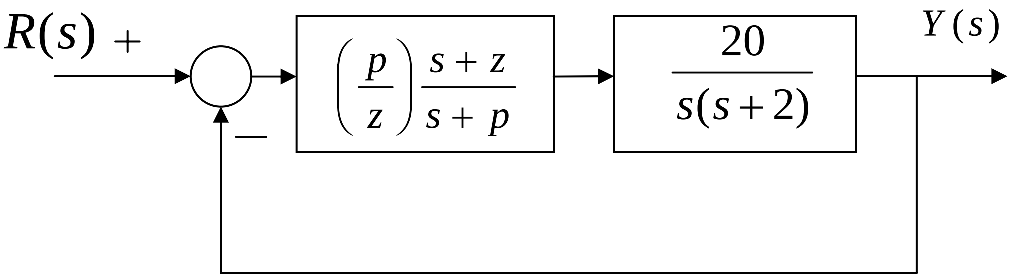
Phase-lag Design Using the Bode Diagram

1. Obtain the Bode diagram of the uncompensated system with the gain adjusted for the desired error constant.
2. Determined the phase margin of the uncompensated system and , if it is insufficient, proceed with the following steps.
3. Determined the frequency where the phase margin requirement would be satisfied if the magnitude curve crossed the 0-dB line at this frequency. (allow 5^0 phase lag from the phase-lag network when determining the new crossover frequency.
4. Place the zero of the compensator one decade below the new crossover frequency and thus ensure only 5^0 of additional phase lag at $\omega_c^{'}$ due to the lag network.
5. Measure the necessary attenuation at $\omega_c^{'}$ to ensure that the magnitude curve crosses at this frequency.
6. Calculate by noting that the attenuation introduced by the phase-lag network is -20logα at .
7. Calculate the pole as $\omega_p = 1/\alpha\tau = \omega_z / \alpha$,and the design is completed.



dB
$20 \log \alpha$
p
z
$\log \omega$
ω_c'
$p = \frac{z}{\alpha}$
$z = \frac{\omega_c'}{10}$
ϕ
-180^0

Example:



$$G_c(s)=\frac{1+\tau s}{1+\alpha\tau s}=\frac{\tau(\frac{1}{\tau}+s)}{\alpha\tau(\frac{1}{\alpha\tau}+s)}=\frac{p}{z}\frac{s+z}{s+p}$$

$$p=\frac{1}{\alpha\tau},z=\frac{1}{\tau}$$

$$GH(jw)=\frac{20}{jw(jw+2)}=\frac{K_v}{jw(j0.5w+1)},K_v=20/2$$

Spec. :phase margin=45^0 the uncompensated system has a phase margin of 20^0

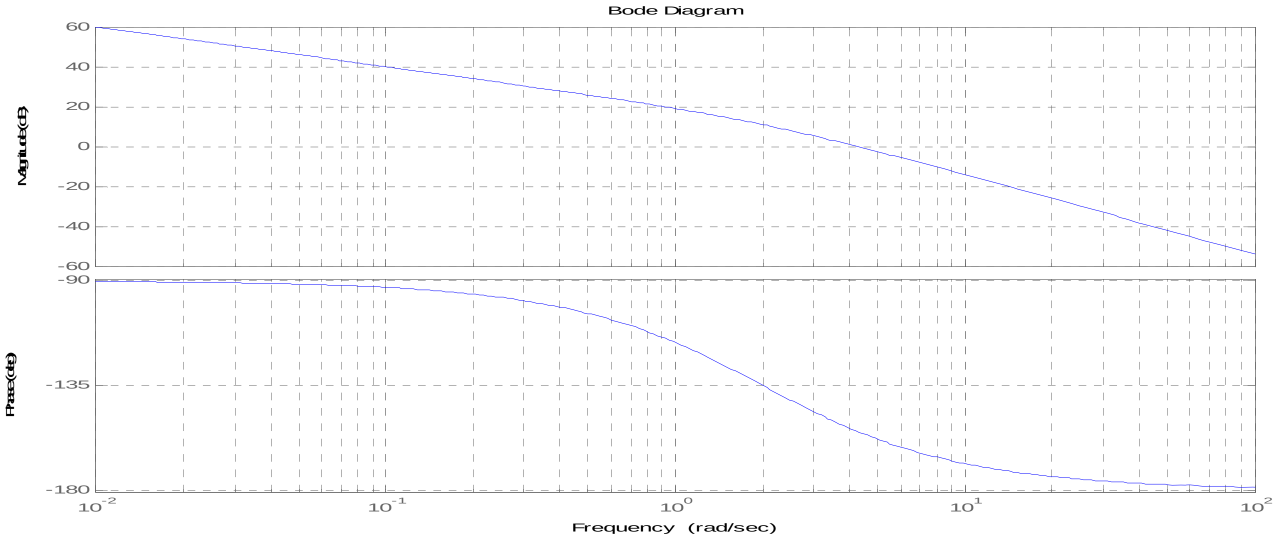
Allowing   for the phase-lag compensator, we locate the frequency ω where $\phi(\omega_c^{'})=-130^0$ which is to be our new crossover frequency $\omega_c=1.5$

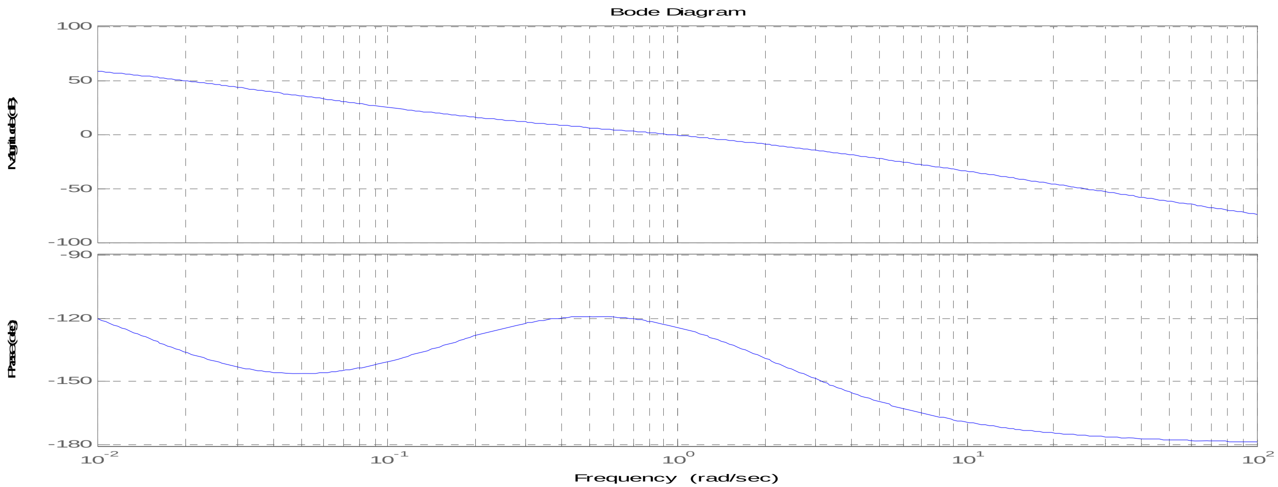
The attenuation necessary to cause   to be the new crossover frequency is equal to 20 dB.

$$20dB=20\log\alpha\Rightarrow\alpha=10$$

$$z=\omega_z=\frac{\omega_c^{'}}{\alpha}=\frac{1.5}{10}=0.15,\ p=\omega_p=\frac{\omega_z}{10}=0.015$$

$$G_c(s)=\frac{1}{10}\frac{s+0.15}{s+0.015}$$


Bode Diagram
60
40
20
0
-20
-40
-60
-90
-135
-180
10^-2
10^-1
10^0
10^1
10^2
Frequency (rad/sec)


Bode Diagram
100
50
0
-50
-100
-90
-120
-150
-180
10^-2
10^-1
10^0
10^1
10^2
Frequency (rad/sec)

§Design for Deadbeat Response

Deadbeat response: proceeds rapidly to the desired level and holds at that level with minimal overshoot.

$$F(s) = s^3 + \alpha\omega_n s^2 + \beta\omega_n^2 s + \omega_n^3$$

$$F(s) = s^4 + \alpha\omega_n s^3 + \beta\omega_n^2 s^2 + \gamma\omega_n^3 s + \omega_n^4$$

A deadbeat response has the following characteristics:

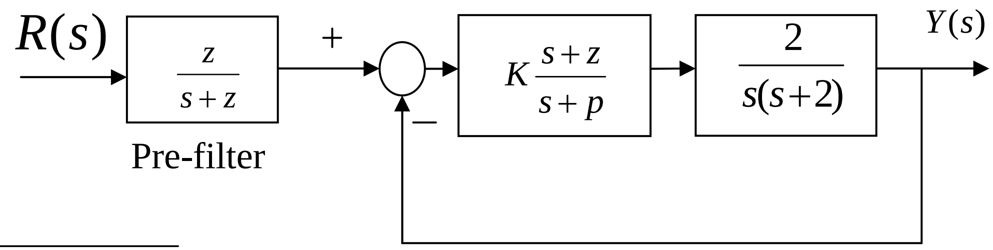
1. steady-state error=0
2. Fast response: minimum rise time and settling time
3. 0.1%<= percent overshoot<2%
4. Percent undershoot<2%

Table 10.2 Coefficients and Response Measures of a Deadbeat System

System Order	Coefficients α	β	γ	δ	ϵ	Percent Over-shoot, P.O.	Percent Under-shoot, P.U.	90% Rise Time, T_{r90}	100% Rise Time, T_r	Settling Time, T_s
2nd	1.82					0.10%	0.00%	3.47	6.58	4.82
3rd	1.90	2.20				1.65%	1.36%	3.48	4.32	4.04
4th	2.20	3.50	2.80			0.89%	0.95%	4.16	5.29	4.81
5th	2.70	4.90	5.40	3.40		1.29%	0.37%	4.84	5.73	5.43
6th	3.15	6.50	8.70	7.55	4.05	1.63%	0.94%	5.49	6.31	6.04

Note: All time is normalized.

Spec. : settling time 1.2 sec



$$G_cG_p(s)=\frac{K(s+z)}{s+p}\frac{2}{s(s+2)}$$

$$T(s)=\frac{z}{s+z}\frac{G_cG_p(s)}{1+G_cG_p(s)}=\frac{2Kz}{s^3+(2+p)s^2+(2p+2k)s+2kz}$$

$$sol:\omega_nT_s=4.04,\omega_n=\frac{4.04}{1.2}=3.37$$

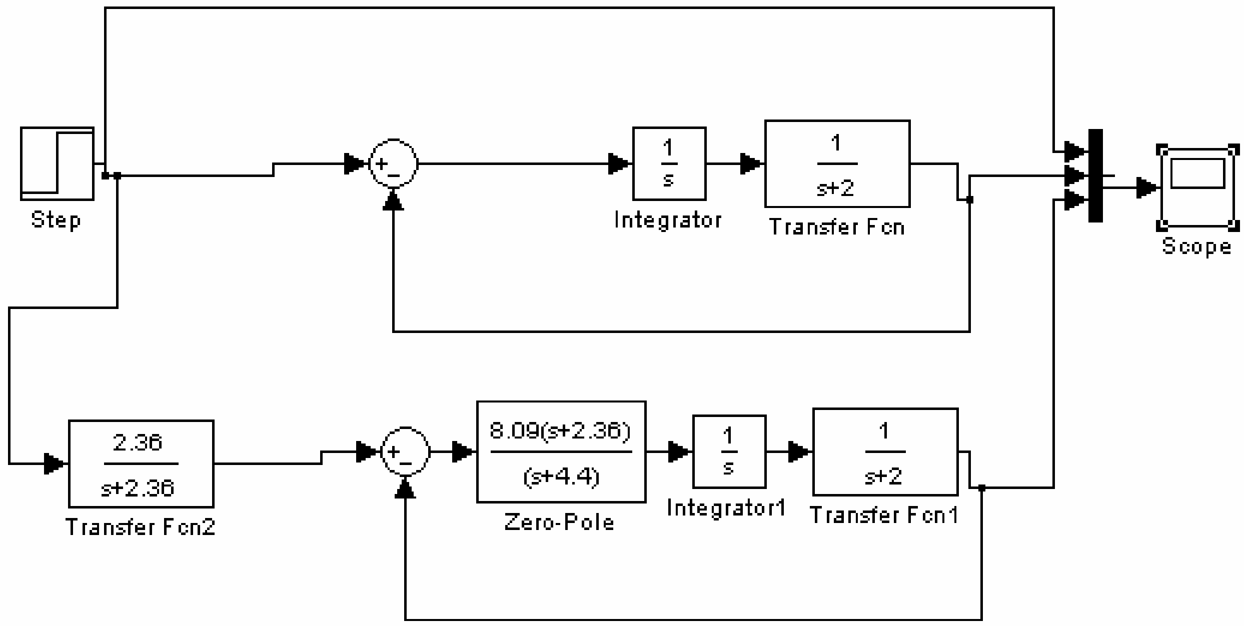
$$T(s)=s^3+\alpha\omega_ns^2+\beta\omega_n^2s+\omega_n^3$$

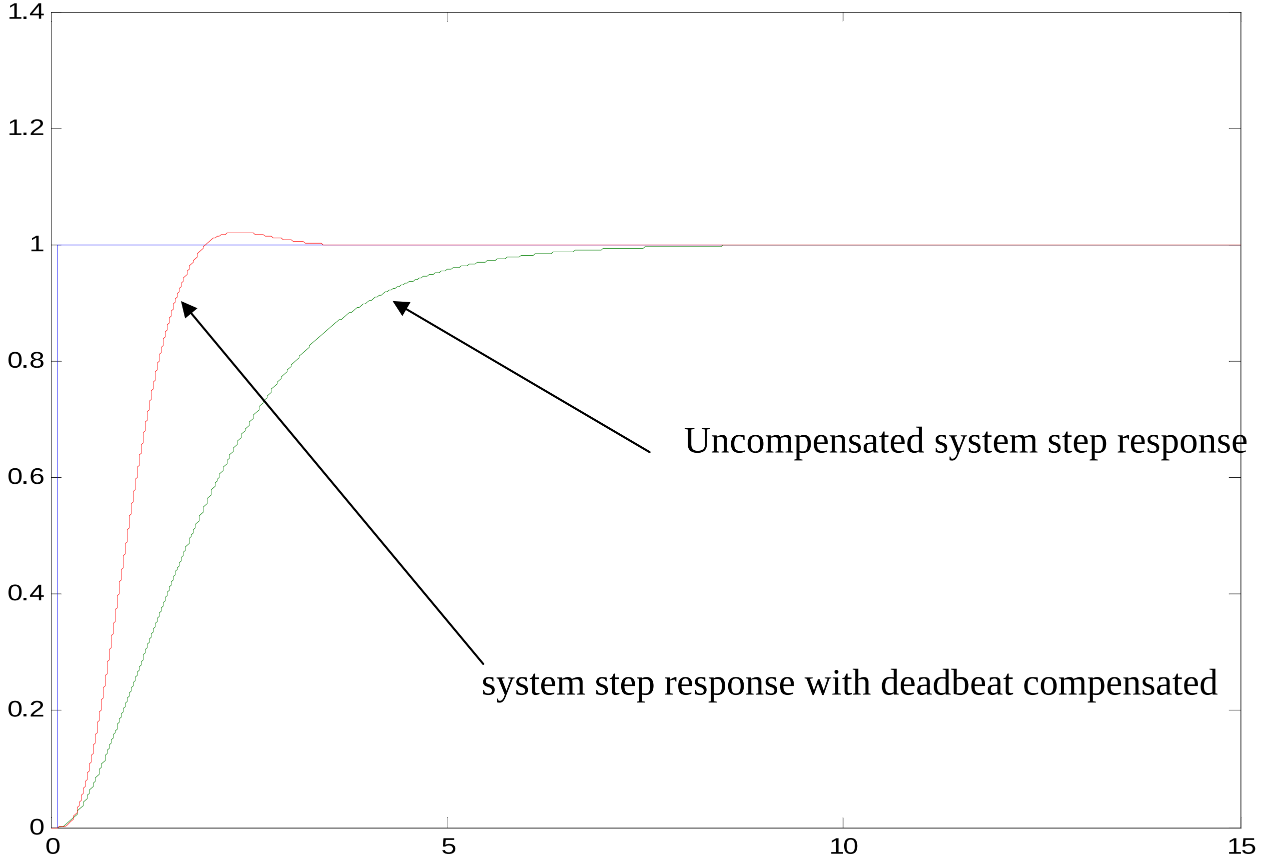
$$\alpha=1.9,\beta=2.2$$

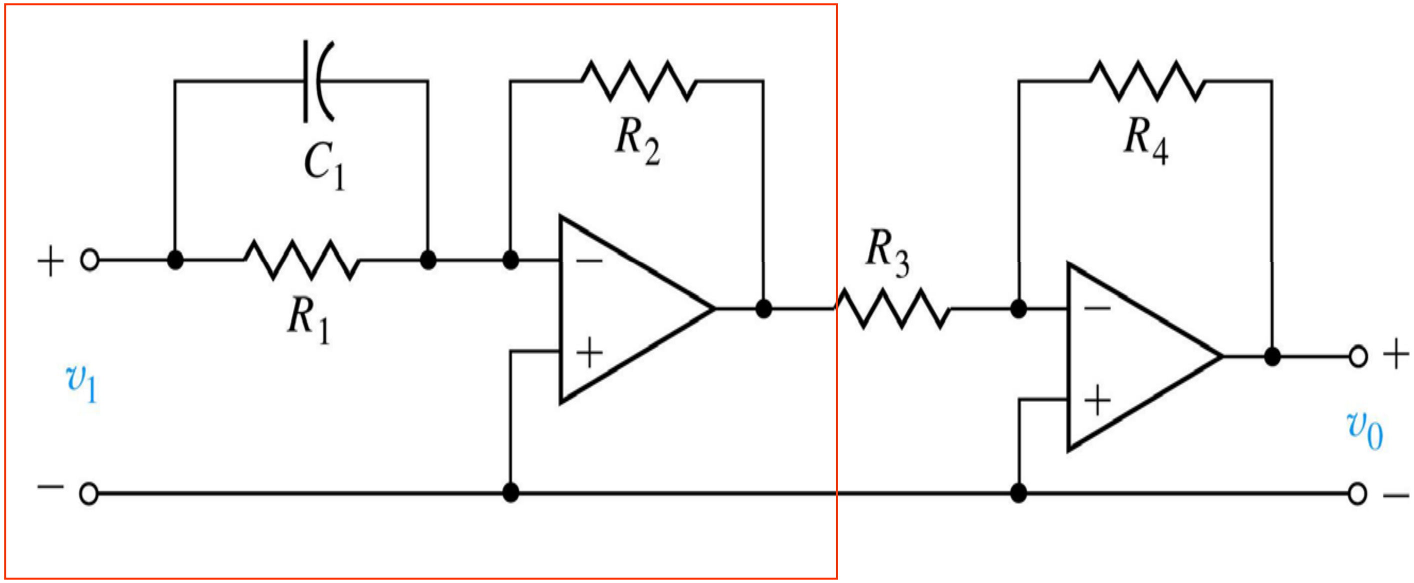
$$T(s)=s^3+6.4s^2+24.98s+38.27=s^3+(2+p)s^2+(2p+2k)s+2kz$$

$$p=4.4,K=8.09,z=2.36$$

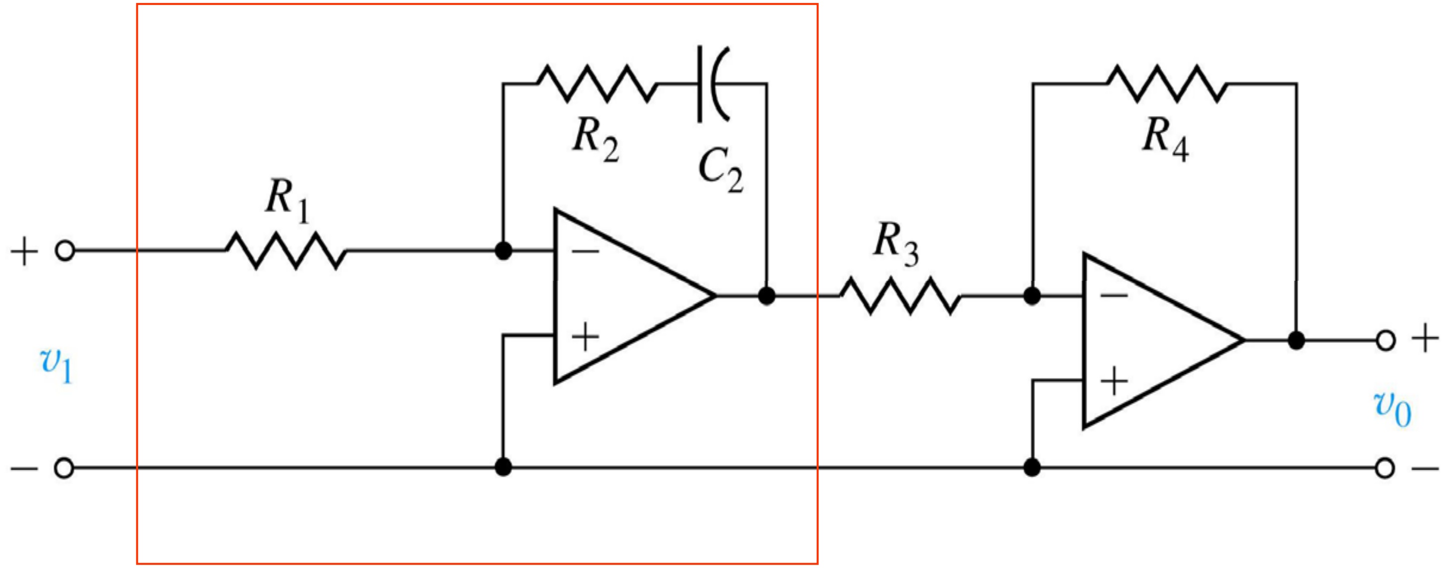
$$G_c(s)=8.09\frac{s+2.36}{s+4.4}$$


Step
1
s
Integrator
1
s+2
Transfer Fcn
Scope
2.36
s+2.36
Transfer Fcn2
8.09(s+2.36)
(s+4.4)
Zero-Pole
1
s
Integrator1
1
s+2
Transfer Fcn1

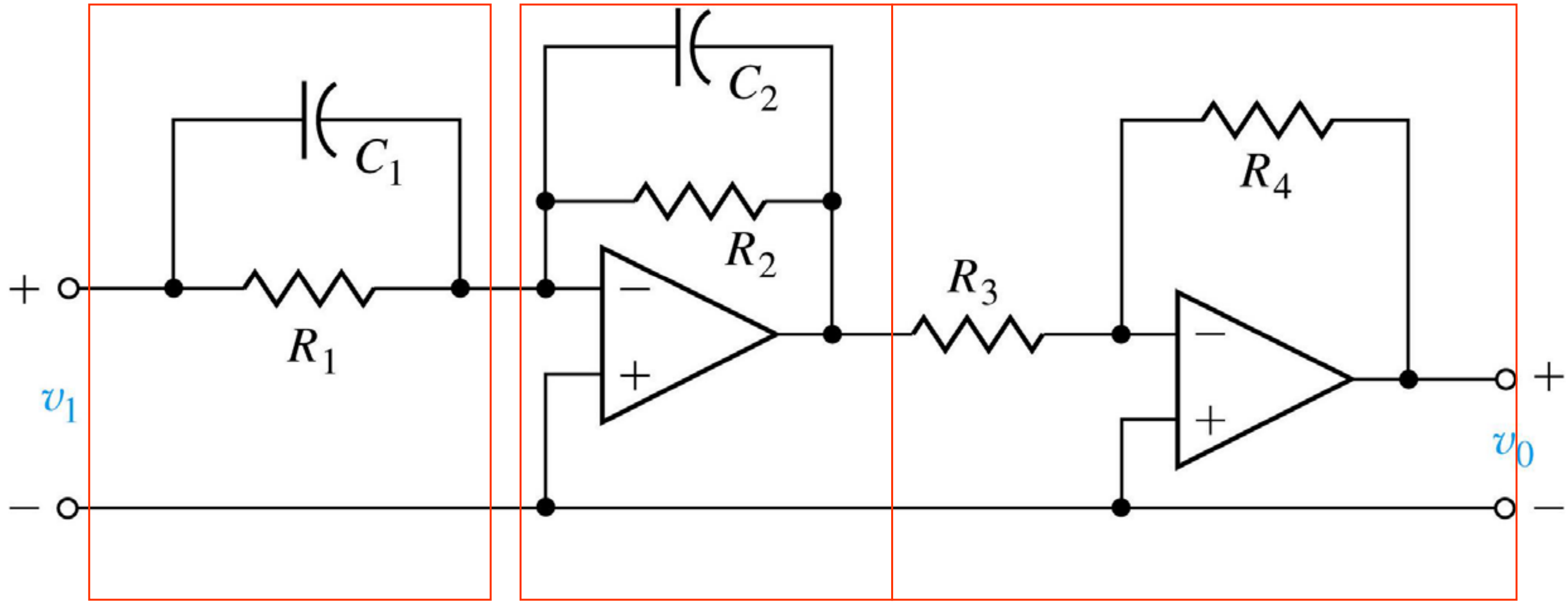

1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
5
10
15
Uncompensated system step response
system step response with deadbeat compensated



PD controller



PI controller



PID controller

Exercises:

E10.1, E10.2, E10.4, E10.8, P10.9 P10.11,

P10.14, P10.21, P10.39 ,AP10.9 AP10.10, DP10.4

DP10.5